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Strategic Sourcing in Procurement

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Abstract

In this paper, we study how a big buyer owing many smallish units may coordinate procurement demands in order to obtain a low price. We show that when the capacity of the big buyer is relatively large, the optimal policy consists in procuring (part of) its requirements through lots. The optimal number and sizes of these lots would depend on the number of potential suppliers as well as the total requirements of this buyer; and in general, a single lot is not optimal. Moreover, we also show that it may be optimal to demand some requirements through smallish units.

JEL Classification: C72, D44, L14
Key words: Procurement policy; Auctions; Buyer groups; Single-sourcing; Lots.

1 Introduction

Procurement is an important the public policy issue. According to the OECD (2013), "in 2011, on average, general government procurement spending represented 29% of total general government expenditures (or 13% of GDP)". As an example, the OECD highlights that "on average in OECD member countries, a decrease in procurement spending by 10% through improvements in efficiency (e.g. keeping the same basket of goods and services procured) would amount to a reduction of 2.9% of total general government expenditure, representing 1.3% of GDP in 2011". Observing also the widespread use of purchasing divisions by firms reflects the importance of procurement policies in the private sector, too.[1] Hence, although many OECD countries declare to include many objectives into their strategic procurement (as innovation, environmental considerations or to promote participation of small and medium size firms), selecting the adequate procurement tools might entail important costs saving both in the private and public sector. These procurement policies are highly heterogeneous regarding its degree of centralization and the structure of the supply contracts. As reported by Dimitri et al. (2006), while public sector tendency is to use centralized systems, the private sector is more heterogenous in this respect centralization and decentralization coexist.[2] There are arguments both, against or in favor of centralization (see Munson,

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[1] As reported by Cousins and Spekman, (2003), the manufacturing sector spends more than 65% in purchasing goods and services.

[2] Baldi and Vannoni (2014) provide some examples of such heterogeneity. "During the 90's, many big companies went through important reorganizations of their activities, including purchasing, and adopted different combinations of centralized

2007). One of them relates to the nature of the requests: while it is argued that there are no gains from centralizing heterogenous products, the homogeneity of the products is an argument in favor of centralization (as pooling requirements confers large bargaining power to the buyer). The ability to receive quantity discounts is one of the most cited reasons in favor of centralization, which somehow presumes single-sourcing policies and rely on assumptions on the economies of scale of suppliers. In this paper, we study a auction procurement game of an homogeneous good and show how centralizing purchases might be an optimal policy when suppliers face strictly convex costs; i.e., when there is no economies of scope. In particular, we claim that the optimal procurement policy consists in demanding procurement requirements through many lots, and that this can be done only if this decision is centralized.

We follow the line started by Anton and Yao (1989) and Inderst (2008), and we study a static procurement game where a strategic buyer must choose his procurement policy in order to reduce costs.[3] Specifically, we consider the following scenario: A big buyer owns a large set of smallish units, each requiring a fixed amount of services or goods. Other smallish units are also present in the market, with the same requirements. We study how this buyer may coordinate the demands of its smallish units in order to obtain a low procurement price. As an example, think on a public Administration that owns a large number of hospitals. Each hospital requires a certain amount of goods. The decision of the Administration is how to group the requirements of its smallish units into single-sourcing lots. Hence, centralizing purchases must be though as an instrument that allows to coordinate procurement demands rather than an instrument to demanding all requirements into a single-sourcing contract in order to increase competition among suppliers.

An assumption made in Anton and Yao (1989) and Inderst (2008) is that the procurement policy of the strategic buyer is restricted to either single or multiple sourcing for the whole demand. Once selected, sellers submit bids for the whole capacity of the buyer in a procurement bidding game. Anton and Yao (1989) study the optimal choice of a single buyer when deciding between single sourcing or multiple sourcing to procure a fixed amount of an homogenous good. They show that, under complete information and strictly convex costs of (two) suppliers, single sourcing is the best policy.[4] Inderst (2008) amend this result by specifying that when there is more than one buyer then single sourcing is optimal if and only if the buyer controls a sufficiently large share of the procurement market. The intuition is that single-sourcing would increase competition among suppliers only when the alternative to winning is small enough (i.e., the residual demand is low). Otherwise, because of convex costs, the losing supplier will be able to obtain a large share of the residual demand and therefore he will bid less aggressively for the single-sourcing contract. Our starting point is different. We study the situations where a big buyer owns a large set of smallish firms (not necessarily all the market share, as in Inderst, 2008), and his choice consists in grouping smallish units in lots in order to offer contracts. Procuring through lots has two consequences: First, if the residual demand is supplied by more than one seller, then it increases competition among sellers to obtain these contracts; and second, if production costs are convex efficiency is reduced since the winner would produce at a marginal costs higher than that of the losers. While the first effect reduces procurement costs, the second effect goes in opposite. Thus, the (optimal) decision would depend on how these effects counterbalance. We show that when the capacity of the strategic buyer is relatively large, the optimal policy will consist in creating some purchasing groups that will be committed to single-sourcing contracts. The number of these (optimal) groups would depend on the number of potential suppliers as well as the total requirements of this (large) buyer; and in general, a

and decentralized procurement. Some of them, as Motorola, General Electric, United Technologies and Fiat, decentralized this function, while some others, such as Honda and General Motors, centralized it."

[3] In O'Brien and Shaffer (1997) exclusivity is offered by suppliers and not by the retailer.

[4] The case of decreasing marginal costs is studied by Gans and King (2012). Also Spector (2011) considers minimum viable size of firms.

unique single-sourcing contract is not the optimal policy. Moreover, we also show that it may be optimal to keep some smallish units to procure their goods independently.

Other papers focus on different aspects of the procurement policy. Specifically they study how this policy alters either (i) downstream competition and foreclosure (see e.g., Chen and Riordan 2007, Abito and Wright 2008, or Kitamura *et al.* 2014), or (ii) investment decisions (see e.g., Segal and Whinston 2000, de Meza and Selvaggi 2004, Dalen et al. 2006, Wilken 2011 or Gong *et al.* 2012). Our work also relates to the recent literature addressing the impact of group purchasing organizations (GPOs). These GPOs have also recently received some special attention, specially in what refers to the health care reform debate in the US. In particular, the analysis is concerned about their effects on the total health care costs and the competitive effects of these groups. While Gong *et al.* (2012) cite examples in Canada and Australia that see significantly lower prices vis-à-vis the U.S. for the same drugs, Saha et al. (2014) claim that "the question whether purchasing through group purchasing organizations (GPOs) saves hospitals money remains unanswered". Our results suggest that the fact that the GPO members obtain a large unitary cost than non-organized buyers is not inconsistent with the fact that the presence of the GPO benefits all its members: In fact, the presence of a sufficiently large GPO reduces procurement costs of their members. However, in many circumstances non-members obtain an even lower unitary cost.[5] This is the reason why, it might not be optimal for the big buyer to centralize all its requirement through a single GPO.

In the next section, we present the model and derive some properties of the bidding equilibrium. Section 3 analyzes the optimal policy of the strategic buyer, and Section 4 applies our results to study the stability of a GPO in a specific setting. Finally, Section 5 concludes.

2 The model

Consider n symmetric suppliers $S = \{1, ..., n\}$ producing an homogeneous good with a strictly increasing convex and twice continuously differentiable cost function $c(x)$, with $c(0) = 0$. At a downstream level, there is bundle of smallish buyers, referred as units, with procurement requirements of b each. Total demand is normalized to 1. In this market, there is one big buyer B owing a proportion of smallish units with total requirements $a \in (0, 1]$. We study the following two-stage procurement game: First, buyer B strategically commits to demand part of his requirements a through m single-sourcing contracts (or lots) of sizes $z_1, ..., z_m$, where $\sum_{i=1}^{m} z_i \leq a$, and the rest is demanded through multiple-sourcing contracts offered by the smallish units. Other smallish units demand their requirements b (non-strategically) using multiple-sourcing.[6] Once B strategically selects $z = (z_1, ..., z_m)$, the bidding game consists of suppliers making offers for all these contracts. Denote by $t_j^i(z_j)$ the bid of supplier $i \in S$ for lot z_j and $t_k^i(x)$ the price offered for x units of non-exclusive demands to buyer $k > m$, referred as competitive units. There is complete information.

It is worth to remark the difference between our model and Anton and Yao (1989) or Inderst (2008). In these papers, the buyer faces a binary choice: either single-sourcing (winner takes all) or multiple-sourcing (split-award auction). In our model, before the competitive tendering the buyer fixes both the number and sizes of the single-sourcing lots by grouping small units and the part of his requirements that would be acquired independently by smallish units. This is different from split-awards auctions where

[5] In a different model with spatial competition Hu and Schwarz (2011a) show that in the presence of contract adminstration fees (CAFs) this might also happen under some parameter configurations.

[6] One may thing on a buyer owing a proportion of smallish firms of size b, that may (or not) group the requirements of some smallish units into lots. Grouping some of these units and demanding its requirements using multiple-sourcing contracts is dominated by keeping these units to act independently (see footnote in Lemma 2).

suppliers submit bids for every possible split of the demand.

Consider a bidding equilibrium and denote by Z_i and x^i the amount supplied by seller i through single-sourcing lots and multiple-sourcing contracts, respectively. Assume w.l.o.g. $Z_1 \geq ... \geq Z_n$ and let define $Z = Z_1 + ... + Z_n \leq a$ and $\overline{Z}_i = \sum_{j=1}^{i} Z_j$. As in Inderst (2008), we consider *truthful equilibria.* These equilibria restrict the equilibrium strategies of the bidders to *two-part tariff bidding functions* where the marginal price of supplying x units to any buyer is equal to the difference in the costs that this generates to the supplier.[7] In these cases, *the equilibrium* bidding functions over competitive (smallish) units satisfy:

$$t_k^i(x) = t_k^i(0) + c\left(Z_i + x_{-k}^i + x\right) - c\left(Z_i + x_{-k}^i\right),$$

where $x_{-k}^i = x^i - x_k^i$, with x_k^i denoting the equilibrium supply of bidder i to competitive buyer k.

Next lemma shows that in any truthful equilibrium, if some sellers supply a positive amount to smallish units, then they must produce the same amount.

Lemma 1 *In any a truthful equilibrium, $x^i > 0$ and $x^j > 0$ for some $i, j \in S$ implies $Z_i + x^i = Z_j + x^j = q$. Moreover, in these cases $t_k^i(0) = t_k^j(0) \leq c(q+b) - c(q)$ for all smallish units k.*

Proof. Suppose there is a truthful equilibrium where $x^i > 0$, $x^j > 0$ for some $i, j \in S$ and $Z_i + x^i > Z_j + x^j$. Consider buyer k satisfying $x_k^i > 0$. She pays

$$t_k^i\left(x_k^i\right) = t_k^i(0) + c\left(Z_i + x_{-k}^i + x_k^i\right) - c\left(Z_i + x_{-k}^i\right)$$

for these x_k^i units. If $x_k^j > 0$ then she also pays $t_k^j\left(x_k^j\right) = t_k^j(0) + c\left(Z_j + x_{-k}^j + x_k^j\right) - c\left(Z_j + x_{-k}^j\right)$. Thus, as $Z_i + x_{-k}^i + x_k^i > Z_j + x_{-k}^j + x_k^j$ by choosing $dx_k^j = -dx_k^i > 0$ the change in costs would be $-c'\left(Z_j + x_{-k}^i + x_k^i\right)dx_k^i + c'\left(Z_j + x_{-k}^j + x_k^j\right)dx_k^j$, which by the convexity of c is negative, implying that x_k^i and x_k^j are not optimal.

Suppose now that $x_k^j = 0$. Then by bidding

$$t_k^j(x) = c\left(Z_j + x^j + x\right) - c\left(Z_j + x^j\right) < t_k^i(x)$$

seller j will obtain at least x_k^j from buyer k. So, profits would increase, a contradiction again.

To see that $t_k^i(0) = t_k^j(0) \leq c\left(Z_i + x^i + b\right) - c\left(Z_i + x^i\right)$ for all k, suppose this is not the case. Suppose first $x_k^i, x_k^j > 0$. The costs of $x_k^i + x_k^j$ are smaller than demanding $x_k^i + x_k^j$ to a unique supplier if

$$\begin{aligned} & t_k^i(0) + c\left(Z_i + x_{-k}^i + x_k^i + x_k^j\right) - c\left(Z_i + x_{-k}^i\right) \\ > \;& t_k^i(0) + c\left(Z_i + x_{-k}^i + x_k^i\right) - c\left(Z_i + x_{-k}^i\right) + t_k^j(0) + c\left(Z_i + x_{-k}^j + x_k^j\right) - c\left(Z_i + x_{-k}^j\right) \end{aligned}$$

As $c\left(q + x_k^j\right) > c\left(Z_i + x_{-k}^i + x_k^i + x_k^j\right)$ and $c(q) - c\left(Z_i + x_{-k}^i\right) > 0$, this yields

$$t_k^i(0) < c\left(q + x_k^j\right) - c(q) < c(q+b) - c(q).$$

In case that $x_k^i = 0$ and $x_k^j > 0$ it is immediate that seller i may bid $t_k^i(x) = c\left(Z_i + x^i + x\right) - c\left(Z_i + x^i\right)$ in order to obtain all the demand required by k. ■

[7] It is worth to note that, although $a - Z$ units are not demanded through exclusivity contracts, sellers cannot submit bids to B for the total amount of $a - Z$, since these requirements are demanded by smallish (independent) units.

The previous result shows that in any equilibrium, if two sellers can compete for smallish units then they must do it at the same marginal cost; thus producing the same amount, say q. Otherwise, price competition among them would allocate these units to the seller facing a smaller marginal cost. Next definition will be useful to determine the number of sellers that supply q and its value, which would determine the unitary cost of smallish units.

Definition 1 *Given $(Z_1, ..., Z_n)$ there uniquely exists a unique g and $\left(x^{g+1}, ..., x^n\right) \geq 0$ satisfying (i) $x^{g+1} + ... + x^n = 1 - Z$ and (ii) $Z_{g+1} + x^{g+1} = ... = Z_n + x^n$. Define $q(Z_1, ..., Z_n) = \left(1 - \overline{Z}_g\right)/(n-g) = Z_i + x^i$ for all $i \geq g+1$.*

Lemma 2 *As $b \to 0$, in a truthful equilibrium the unitary price of competitive units is $p = c'(Z_{n-1})$ if $Z_{n-1} - Z_n \geq 1 - Z$ and $p = c'(q)$ otherwise. Thus, the unitary price when $m = 0$ is $c'(1/n)$.*

Proof. In a truthful equilibrium, suppliers would compete for the $1-Z$ competitive units. We denote by g the number of suppliers that are able to compete for these units. Note that when $Z_{n-1} \geq Z_n + 1 - Z$, then the costs of supplying all the competitive units is always smaller for supplier n. So, this seller will obtain all these units. In these cases, for any smallish unit with requirements b,

$$t_k^n(b) = t_k^n(0) + c(Z_n + 1 - Z) - c(Z_n + 1 - Z - b),$$

and

$$t_k^{n-1}(b) = t_k^{n-1}(0) + c(Z_{n-1} + b) - c(Z_{n-1}).$$

Moreover,

$$t_k^n(b) \leq t_k^{n-1}(b) = t_k^{n-1}(0) + c(Z_{n-1} + b) - c(Z_{n-1}).$$

Hence, in the equilibrium seller n would set $t_k^n(b) = c(Z_{n-1} + b) - c(Z_{n-1})$, since by selecting $t_k^n(b) > c(Z_{n-1} + b) - c(Z_{n-1})$ seller $n-1$ could obtain b by choosing $t_k^{n-1}(0) = 0$. Taking the limit, we obtain

$$\lim_{b \to 0} \frac{t_k^n(b)}{b} = c'(Z_{n-1}).$$

Otherwise, when $Z_{n-1} - Z_n < 1 - Z$, then $c(Z_{n-1} + b) < c(Z_n + 1 - Z)$ so that at least two suppliers, say $n-g$, would compete for the smallish units. Moreover, (i) the total amount supplied by each of these sellers is $q = \left(1 - \overline{Z}_g\right)/(n-g)$so that the marginal costs of these suppliers equalize and (ii) the fixed part for the competitive units must be $t_k^i(0) \leq c(q+b) - c(q)$.

In case that two sellers supply to the same competitive buyer, the latter must be indifferent between buying the last unit to any of the suppliers. That is, $t_k^i\left(x_k^i\right) + t_k^i\left(x_k^j\right) = t_k^i\left(x_k^i + x_k^j\right) = t_k^j\left(x_k^j + x_k^i\right)$. Moreover, competition leads to $t_k^i(0) = t_k^j(0) = 0$. Hence,

$$c(q) - c\left(q - x_k^i\right) + c(q) - c\left(q - x_k^j\right) = c\left(q + x_k^i\right) - c\left(q - x_k^j\right).$$

Therefore, the unitary price of a smallish unit k is

$$\begin{aligned} \frac{t_k^i\left(x_k^i + x_k^j\right)}{x_k^i + x_k^j} &= \frac{c\left(q + x_k^i\right) - c\left(q - x_k^j\right)}{x_k^i + x_k^j} \\ &= \frac{c\left(q + \left[x_k^i + x_k^j\right] - x_k^j\right) - c\left(q - x_k^j\right)}{x_k^i + x_k^j} \end{aligned} \tag{1}$$

which, when z is small enough so that x_k^i and x_k^j converge to zero yield

$$\lim \frac{t_k^i\left(x_k^i + x_k^j\right)}{x_k^i + x_k^j} = c'(q).$$

When $x_k^i = b$, from Lemma 1, it can be easily derived that $t_k^i(0) = c(q+b) - c(q)$. Thus,

$$\frac{t_k^i(b)}{b} = \frac{t_k^i(0)}{b} + \frac{c(q) - c(q-b)}{b} = \frac{c(q+b) - c(q-b)}{b}$$

that converges to $c'(q)$ as $b \to 0$.[8] ■

The previous result characterizes the unitary cost of the smallish units when b is infinitessimal, which, for tractability, is assumed henceforth. In these cases, and not surprisingly, when competing for these small buyers, there is no difference between making offers to the buyers and/or submitting anonymous offers to the market, where competition à la Bertrand would determine the price. From now on, we will refer to this unitary cost as the *competitive price* p.

Equipped with this result, we can now address the strategic policy of B in selecting the number and sizes of (large) single-sourcing lots. Next result shows that any policy where he selects $m \geq n$ (single-sourcing) lots is (weakly) dominated by a policy with $m < n$ exclusivity lots. Before, we claim that in any equilibrium, the revenues of all suppliers must coincide.

Lemma 3 *In any bidding equilibrium, all suppliers have the same expected profits.*

Proof. Immediate. Just note that in the bidding game suppliers submit bids simultaneously. Hence, each supplier can obtain the profits of any other by mimicking the strategies of the latter (with $\pm\varepsilon$) in order to win/lose. ■

Lemmata 2 and 3 provide the tools that allow us to determine the total costs of B when selecting $z = (z_1, ..., z_m)$; thus, the optimal procurement policy.

Proposition 1 *Any policy where either $m \geq n$ or some seller obtains more than one lot is (weakly) dominated.*

Proof. See Appendix A. ■

The logic of the result is as follows. Choosing $m > n$ lots is equivalent to select a number of lots equal to the numbers of winning sellers. Moreover, by choosing $m = n$ winning sellers reduces competition among suppliers. Thus, the strategic buyer would divide its demand using $m < n$ lots of sizes $z_1, ...z_m$ and the rest is $a - Z$ would be demanded through an infinite stream of smallish units. Next results use the previous Lemmata to characterize the optimal policy of the large buyer B.

Lemma 4 *If $m < n$ then the unitary price of any lot of size $z_i \leq \frac{1-Z}{n-m}$ is the competitive one.*

Proof. Suppose there is $g \in \{1, ..., m\}$ such that $z_i \leq \frac{1-Z}{n-m}$ for all $i > g$. Since $z_i \leq \frac{1-Z}{n-m}$, the any seller that obtained exclusivity with lot z_i is able to compete with non-exclusivity sellers. That is, they will sell

[8] From the previous analysis, it is obvious, then, that the unitary cost increases in b. Thus, any buyer would benefit from demanding b through two buyers of size $b/2$ (ad infinitum).

also part of $1-Z$ to other (smallish) buyers. In particular all these sellers sell $x^i = q - z_i$ competitive units so that they supply a total amount of q satisfying

$$(n-m)\,q + \sum\nolimits_{j=g+1}^{m} (q - Z_j) = 1 - Z \Longrightarrow q = \frac{1-\overline{Z}_g}{n-g}$$

Moreover, since supplier n only sells to competitive units and profits are the same for all sellers, we have that

$$T(z_i) + (q - z_i)\,c'(q) - c(q) = c'(q)\,q - c(q) \Longrightarrow T(z_i) = z_i c'(q) \text{ for all } i > g$$

and

$$T(z_i) - c(z_i) = c'(q)\,q - c(q) \Longrightarrow T(z_i) = c'(q)\,q + c(z_i) - c(q) \text{ for all } i \leq g$$

In case that $z_1 = Z_1, ...z_g = Z_g$ then $n-g$ buyers would supply q units to the competitive stream. Hence, total costs with these g lots are identical to those obtained with m (exclusivity) lots. Thus, this is equivalent to demand z_i using infinite multiple-sourcing contracts. ■

This result is in line of Inderst (2008) who stresses that single-sourcing is not optimal for small buyers, as their decisions would never affect their procurement prices. As a simplifying assumption (as it is irrelevant), we assume throughout that no lot $z_i \leq \frac{1-Z}{n-m}$ is never observed.

Lemma 5 *If $m \leq n-2$ then all lots have the same size.*

Proof. Given m and $Z = \sum\nolimits_{i=1}^{m} z_i$ with $z_i > \frac{1-Z}{n-m}$, we know that competition to supply $1-Z$ is reduced to $n-m$ sellers. Thus, the competitive price is independent of sizes and depends only on Z and m. Hence, since $T(z_i) = c'(q)\,q + c(z_i) - c(q)$ for all $i \leq m$ and the unitary competitive price is $c'(q)$ we have that

$$\begin{aligned} TC &= \sum_{i=1}^{m} \left(c'(q)\,q + c(z_i) - c(q)\right) + (a - Z)\,c'(q) \\ &= \sum_{i=1}^{m} c(z_i) + (a - Z + mq)\,c'(q) - mc(q)\,. \end{aligned}$$

Therefore, since $c' > 0$ the costs are reduced if all lots are equal. That is, $z_i = ... = z_m$. ■

From the previous lemmata we know that the optimal policy of B is such that when she selects $m \leq n-2$ (single-sourcing) contracts then all lots would have the same size z satisfying $z = \frac{Z}{m} > \frac{1-Z}{n-m} = q$. The logic of this result is as follows: Once (Z, m) is selected, then the competitive price is uniquely determined by the competitive behavior of $n-m$ suppliers for a demand of $1-Z$. Hence, (Z, m) determines completely the profits of losing bidders. Therefore, the choice of B would only affect competition at the tendering stage. By the concavity of costs this is achieved when all lots have the same size, as this minimizes the total costs of the winning bidders. Nevertheless, both the optimal number of lots and its size, thus Z, would depend on how (m, Z) affects competition for the requirements of smallish units, which in turn would affect the profits of the sellers obtaining these lots z. Additionally, a policy consisting in $n-1$ lots might also be optimal.

3 The optimal policy under homogeneous cost functions

Two aspects determine the optimal procurement policy: competition and efficiency. Given Z, a reduction in the number of lots m has two effects. First, the number os sellers supplying to the smallish units is large. Second, due to the strict convexity of the cost functions, the winning sellers support a large (marginal) cost. The first effect reduces the competitive price and therefore the profits of all sellers, which implies a reduction in the procurement costs. However, the second effect increases the inefficient allocation of production, which translates into larger procurement costs. Thus, the optimal policy must account on how these effects counterbalance. Both, competition and efficiency loses would finally depend on the cost function of the providers. In this section, in order to deep into the analysis, we would assume homogeneous cost functions. That is, $c(q) = q^k$, with $k > 1$.[9]

To highlight the differences between our model and Inderst (2008) or Anton and Yao (1989), we start by characterizing the optimal policy when $n = 2$. In these cases, according to Proposition 1 either no lot or one lot is optimal. If the buyer procures his requirements in the competitive market at a unitary price $p = c'(1/2)$ yielding total costs of $c'(1/2) \cdot a$. By selecting a lot of size $z \in (1/2, a]$ the equilibrium bid of the winning seller, say i, satisfies

$$t_1^i(z) - c(z) = c'(z)(1-z) - c(1-z)$$

yielding total procurement costs of

$$\begin{aligned} TC(z,1;k,a,2) &= c'(z)(1-z) - c(1-z) + c(z) + (a-z)c'(z) \\ &= kz^{k-1}(1-z) - (1-z)^k + z^k + kz^{k-1}(a-z). \end{aligned}$$

Evaluating $\frac{\partial^2 TC(z,1;k,a,2)}{\partial z^2}$ at any critical point yields

$$k(k-1)z^{-1}\left\{-k(1+a-2z)z^{k-2} - z(1-z)^{k-2} - 2(1-z)^{k-1}\right\} < 0$$

meaning that the unique candidate to minimizing total costs is $z = a$. In these cases,

$$TC(z,1;k,a,2) = c'(a)(1-a) - c(1-a) + c(a) \geq ac'(1/2)$$

This policy may be preferred to *no-lot* only if $k \leq 2$. Otherwise, (i) $c(a) - c(1-a) = \int_{1-a}^{a} c'(s)\,ds > c'(1/2)$ as c' is convex and (ii) $ac'(1/2) - c'(a)(1-a) < [a-(1-a)]c'(1/2) = [2a-1]c'(1/2) < c'(1/2)$ we obtain

$$c(a) - c(1-a) = \int_{1-a}^{a} c'(s)\,ds > c'(1/2) > ac'(1/2) - c'(a)(1-a)$$

Thus, the next result follows.

Proposition 2 *When $n = 2$ and $k > 2$, single-sourcing is never an optimal procurement policy if the buyer has the possibility to obtain the competitive price.*

This result bears on the fact that we allow the dominant buyer to demand all requirements through smallish units of size b, which imply a unitary cost equal to the marginal costs of the suppliers when b tends to zero. In contrast, in Anton and Yao (1989) or Inderst (2008), the alternative to single-sourcing is a split-award auction over a, which generates a unitary price above the competitive one. Nevertheless,

[9] It is obvious that under constant marginal costs the unitary price is equal to this cost. Under decreasing marginal costs, as assumed in some papers (e.g., Gans and King, 2012 and Spector, 2011) the forces driving our results change completely: In our model, the possibilities to compete for some marginal units are reduced with the total sales of the retailer. However, the effect would be the opposite under decreasing marginal costs. Hence, when bidding for a lot, sellers would be more aggressive under decreasing marginal costs, because this will allow them to reduce their unitary cost. Clearly, these effects on the bidding behavior might change the optimal policy of the big buyer.

as shown next, when there are more than two suppliers the dominant buyer may obtain procurement costs above the competitive ones, thus making single-sourcing lots profitable.

Consider first an optimal procurement policy consisting in $m \leq n-2$ lots, which by Proposition 5 must have all equal size. As the competitive price is $p = c'\left(\frac{1-Z}{n-m}\right)$, total costs are given by

$$TC(Z,m;k,a,n) = m\left(c\left(\frac{Z}{m}\right) + c'\left(\frac{1-Z}{n-m}\right)\left(\frac{1-Z}{n-m}\right) - c\left(\frac{1-Z}{n-m}\right)\right) + (a-Z)\,c'\left(\frac{1-Z}{n-m}\right),$$

where (Z,m) satisfies (i) feasibility; that is, $Z \leq a$ and (ii) effectivity of the policy (see Lemma 5); i.e., $mz = Z \geq m/n$.

Minimizing this function under these restrictions, will allow us to characterize the optimal procurement policy. In deriving the results we consider m as a continuous variable. However, when reading the results (in particular, part (1) in the next proposition) we must account that m is a natural number.

Proposition 3 *Under homogeneous costs and $a > 1/n$, if the optimal choice (m^*, Z^*) satisfies $m^* \leq n-2$ then there exist $\underline{a}(k,n), \overline{a}(k,n)$ with $0 < \underline{a}(k,n) < \overline{a}(k,n) < 1$ such that*

1. *If $a < \underline{a}(k,n)$ then $Z^* = a$ and $m^* < n-2$ solves $TC_m(a, m^*; k, a, n) = 0$.*
2. *If $\underline{a}(k,n) \leq a \leq \overline{a}(k,n)$ then $Z^* = a$ and $m^* = n-2$.*
3. *If $a > \overline{a}(k,n)$ then $m^* = n-2$ and $Z^* < a$ solves $TC_Z(Z^*, n-2; k, a, n) = 0$.*

where TC_m and TC_Z denote partial derivatives.

Proof. See the Appendix. ■

It is worth to note the effects of the inefficiency generated when procuring though large lots. First, it implies that a unique lot is not generally the optimal policy, which contrasts with Anton and Yao (1989). Second, it shows that even when $a = 1$ (so that the buyer would have the possibility to "penalize" the losing bidders and thus increase competition) the amount procured through lots would be smaller than 1. In this case, given that $m = n-2$ an increase in Z would affect the costs of suppliers, which would translate into larger total costs for the buyer. Notably, because of the competitive bidding process and the fact that when $m \leq n-2$ the unitary price of competitive units is $p = c'(q)$ with $q = \frac{1-Z}{n-m} < z$, the strategic buyer will have large unitary costs than the competitive one.[10]

Corollary 1 *When $m^* \leq n-2$ then the unitary cost of smallish buyers is below the unitary cost of the strategic buyer.*

Proof. From Lemma 3 all bidders have the same expected profits. That is, $c'(q)\,q - c(q) = p_z z - c(z)$ where p_z denotes the unitary price of a lot z. Hence,

$$p_z = \frac{c'(q)\,q + c(z) - c(q)}{z} > \frac{c'(q)\,q + c'(q)(z-q)}{z} = \frac{c'(q)\,z}{z} = c'(q).$$

[10] As reported by Saha at al. (2014), "A small GAO study (GAO-02-690T, 2002) found that GPO prices for specialized items such as safety syringes and pacemakers were in fact higher than the prices outside." Nevertheless, as our results show, this is not inconsistent with the fact that the cenrtal purchasing allows for a reduction in the unitary costs of the big buyer.

■

As showed (see Proposition 1) the optimal policy is such that $m < n$. However, we still must consider the possibility that B selects $m = n-1$. In these cases, it is not generally the case that all lots have equal size. Note that the sellers that obtain a lot do not sell at the competitive price (as $z > 1-Z$). Moreover, because of a monopolistic situation, the remaining seller would increase the price upwards until the threat of competition enters into play. Thus, the competitive price would be $p = c'(z_{\min})$ where $z_{\min}$ is the size of the smallest lot. This implies that B would prevent the competitive price to be too large selecting $z_{\min}$ appropriately. Once done, she optimally would divide the remaining demand $Z - z_{\min}$ into lots of equal size.[11] Thus, when $m^* = n-1$ the optimal policy would consist in selecting (s, Z) where $s = z_{\min}$ and demand $Z - s$ using $n-2$ symmetric exclusivity contracts of size $z = \frac{Z-s}{n-2} \geq s$. Now, total costs are

$$\overline{TC} = (n-2)\left(\left(\frac{Z-s}{n-2}\right)^2 + 2s(1-Z) - (1-Z)^2\right) + s^2 + 2s(1-Z) - (1-Z)^2 + 2s(a-Z)$$

and we require (i) $Z \leq a$, (ii) $s \geq 1-Z$ and (iii) $s \leq \frac{Z-s}{n-2} \Longrightarrow s \leq Z/(n-1)$, which imply

$$1 - Z \leq s \leq Z/(n-1) \Longleftrightarrow Z \geq (n-1)/n \Longrightarrow a \geq (n-1)/n$$

Thus, a necessary condition for $m = n-1$ to be an optimal policy is that $a \geq (n-1)/n$; that is, the share of B must be sufficiently large relatively to the number of suppliers. In those cases, we obtain the following result.

Proposition 4 *When $m^* = n-1$ then $Z^* = a$ and s^* solves*

$$\left(\frac{a-s}{n-2}\right)^{k-1} = s^{k-1} + (n-1)(k-1)s^{k-2}(1-a).$$

Proof. See the Appendix. ■

The proof of the previous proposition builds on the fact that for any given z, it must be that either $s = 1-Z$ or that $s = a-(n-2)z$. Moreover, by Lemma 4 $s = 1-Z$ yields the same costs as $s = 0$ and therefore the optimal policy might be $m^* = n-1$ only in case that $s = a-(n-2)z$. Moreover, total costs at the optimal procurement policy when restricted to $m \leq n-2$ must coincide with the total costs at $s = 1-Z$.

The optimal policy would result from the comparison between $\overline{TC}^*$ and TC^*, which strongly depend on the specification of the parameters. We next provide some examples that illustrate it.

Example 1 *Let $k = 2$ and $n = 3$. In this case, $\underline{a}(k,n) = \sqrt{2}-1$ and $\overline{a}(k,n) = 3/7$. We obtain that, restricted to $m \leq n-2 = 1$, $m^* = 0$ when $a < 1/3$ and $m^* = 1$ otherwise. In the latter, $z = \min\left\{a, \frac{2a+3}{9}\right\}$, meaning that $Z = z < a$ when $a > 3/7$. In case that $m = n-1 = 2$, which might be the case only when $a > 2/3$, the optimal policy is $z + s = a$ where s solves*

$$\left(\frac{a-s}{n-2}\right)^{k-1} = s^{k-1} + (n-1)(k-1)s^{k-2}(1-a),$$

yielding

$$s^* = \left(\frac{3}{2}a - 1\right) \text{ and } z^* = 1 - \frac{1}{2}a.$$

[11] The same reasoning as in Lemma 5 applies.

which satisfies $s > 1 - Z$ if $a > 4/5$.

Evaluating total costs TC^* and $\overline{TC}^*$ for $a > 4/5$ we obtain

$$TC^* = -\frac{1}{9}a\,(a-6) \text{ and } \overline{TC}^* = -\frac{11}{2}a^2 + 10a - 4$$

so that

$$\overline{TC}^* \leq TC^* \iff a \geq 0.953\,46$$

Therefore,

1. *If $a \leq 1/3$ then no single-sourcing contract is established.*
2. *If $a \in (1/3, 0.95346]$ then it is optimal to fix one single-sourcing contract with size $z = \min\left\{a, \frac{2a+3}{9}\right\}$, and*
3. *If $a > 0.95346$ then it is optimal to fix two single-sourcing contracts with sizes $z = (2-a)/2$ and $s = (3a-2)/2$.*

Example 2 *Let $k = 3$ and $n = 3$. In these case, replicating the same calculations as in the previous example, we obtain that when $m \leq n-2 = 1$, $m^* = 0$ when $a \leq 1/3$, $m^* = 1$ and $z^* = a$ in case that $a \in (1/3, 0.414\,21)$ and $m^* = 1$ with $z^* = (a+1)/(a+3) < a$ for $a \geq 0.414\,21$. In case that $m = n-1 = 2$, we would obtain $s = \frac{a^2}{2(2-a)}$ which is larger than $1-a$ iff $a > 0.763\,93$. For these cases,*

$$TC^* = \frac{a}{a+3} \text{ and } \overline{TC}^* \frac{1}{4\,(a-2)}\left(15a^4 - 48a^3 + 72a^2 - 56a + 16\right)$$

and it can be checked that $TC^* < \overline{TC}^*$ for all $a \in (0.763\,93, 1)$ so that $m = n-1 = 2$ can never be optimal.

Hence,

1. *If $a \leq 1/3$ then no single-sourcing lot is established.*
2. *If $a \geq 1/3$ the optimal policy consists in one single-sourcing lot with size $z = \min\left\{a, \frac{a+1}{a+3}\right\}$.*

It is worth to note that, a change in the optimal policy from $m \leq n-2$ to $n = n-1$ introduces a discontinuity in the unitary costs suffered by competitive firms. Next pictures display the costs obtained in the previous examples, both for the strategic buyer and the smallish units. While under cubic costs the competitive price is always below the unitary costs of B, it grows upwards when B chooses $m = n-1$. That is, externalities become negative. This fact fact yields the surprising result that, in such cases, also the total costs of B are affected negatively by its size.

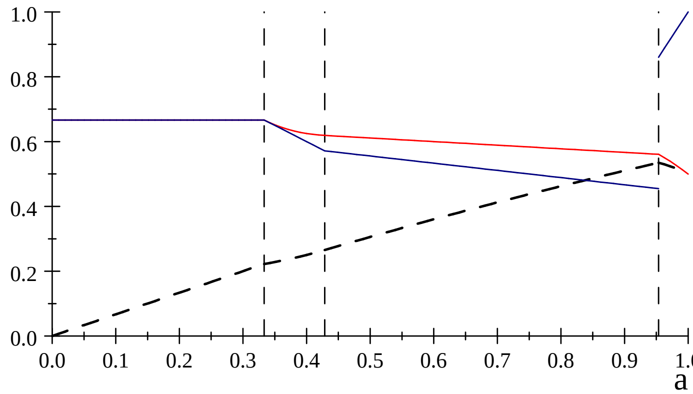

Figure 1: Quadratic cost functions. Dashed Black: Total costs for a demand of a; Red: unitary costs of B; and Blue: competitive price.

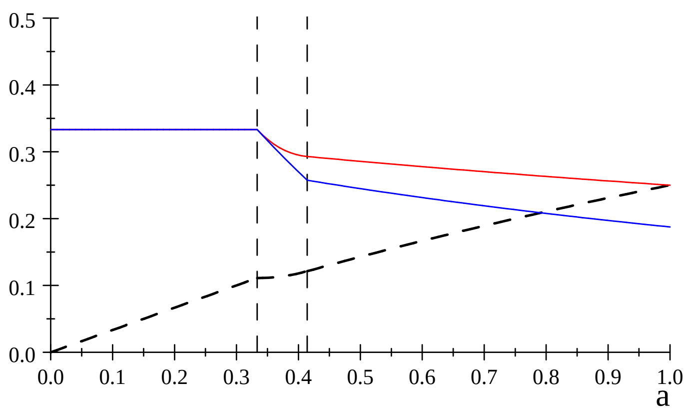

Figure 2: Cubic cost functions. Dashed Black: Total costs for a demand of a; Red: unitary costs of B; and Blue: competitive price.

Example 3 *Let $k = 2$ and $n = 6$. We obtain $\overline{a}(k, n) = 6/7 = 0.857\,14$ and $\underline{a}(k, n) = 0.817\,26$. Under the restriction that $m \leq 4$, we know that $m = 4$ and $Z = a$ whenever $a \in (0.817\,26, 0.857\,14)$ and that $m = 4$ and $Z = \frac{1}{n^2}(2a + n)(n - 2) = \frac{2}{9}a + \frac{2}{3} < a$ if $a > 0.857\,14$ where, in these cases, $z = \frac{1}{18}a + \frac{1}{6}$. The case where $a < 0.817\,26$ is more complicated due to the fact that m is discrete. In these cases, finding the optimal procurement policy m requires to evaluate TC at $Z = a$ and then pick (optimally) either $m^* = \lfloor m \rfloor$ or $m^* = \lceil m \rceil$. Comparing the total costs when $a \in [1/6, 0.817\,26]$ for different values of $m \in \{1, 2, 3, 4\}$ we obtain that $m^* = 1$ when $a \leq 0.291\,94$, $m^* = 2$ when $a \in [0.291\,94, 0.527\,86]$, $m^* = 3$ when $a \in [0.527\,86, 0.738\,80]$ and $m^* = 4$ otherwise.*

With respect to the possible optimal policy $m = n - 1$, we know that s solves

$$\left(\frac{a-s}{n-2}\right)^{k-1} = \left(s^{k-1} + (n-1)(k-1)\, s^{k-2}(1-a)\right)$$

This yields $s = \frac{21}{5}a - 4$ and $z = 1 - \frac{4}{5}a$. We require $s > 1 - a$. I.e., $a > \frac{25}{26} = 0.961\,54$. For these cases,

$$TC^* = \frac{1}{9}a(3-a) \text{ and } \overline{TC}^* = -\frac{134}{5}a^2 + 52a - 25$$

so that

$$\overline{TC}^* \leq TC^* \Longleftrightarrow a \geq 0.981\,91$$

Hence, the optimal procurement policy is as follows:

1. *If $a \leq 1/6$ then no single-sourcing contract is established.*
2. *If $a \in [1/6, 0.291\,94]$ then $m^* = 1$ and $z^* = a$.*
3. *If $a \in [0.291\,94, 0.527\,86]$ then $m^* = 2$ and $z^* = a/2$.*
4. *If $a \in [0.527\,86, 0.738\,80]$ then $m^* = 3$ and $z^* = a/3$.*
5. *If $a \in [0.738\,80, 0.857\,14]$ then $m^* = 4$ and $z^* = a/4$.*
6. *If $a \in [0.857\,14, 0.981\,91]$ then $m^* = 4$ and $z^* = \frac{1}{18}a + \frac{1}{6} - \frac{a}{4} < a/4$, and*
7. *If $a > 0.95346$ then $m^* = 5$ with $s^* = \frac{21}{5}a - 4$ and $z^* = 1 - \frac{4}{5}a$.*

Overall, three things must be noted. First, the unitary prices might be affected by the policy of B only if $a > 1/n$ (see Proposition 4). Otherwise, $p = c'(1/n)$ is attained. Second, buyer B will never choose $m \geq n$ since otherwise, competition for the exclusivity lots is reduced (see Proposition 1). Such competition is increased by selecting either (i) $m \leq n - 2$, where prevents the monopolistic position of one seller over the non-exclusivity demands, or (ii) $m = n - 1$, she creates a (restricted) monopoly over $1 - Z$ whose maximal price is determined by the size of the smallest lot. Third, one might think that when $a = 1$, B is not affected by such non-competitive scenario since she can demand all units using lots. However, this is not necessarily the optimal policy, which may involve also $Z < a$ when $m^* \leq n - 2$. The reason is that demanding all procurement requirements through exclusive lots generates inefficiency because suppliers produce at $c'(z) > c'(1/n)$. Hence, although competition is enhanced by procuring all demands through lots by leaving the losing suppliers at zero profits, such policy might imply so large production costs that overcome the positive effects of competition.

4 The strategic choice of capacity: Group purchasing organizations

Group purchasing organizations (GPO) consist of buyers forming a consortium with the objective of obtaining lower prices. These groups (either for-profit or not-for-profit) act as intermediaries establishing contracts with suppliers and retailers. One of the main arguments in favor of these organizations is that a large demand increases their bargaining strength with suppliers. In our model, prices are settled through competitive tendering and we showed that a strategic buyer, say a GPO, with large demand would generally divide (part of) his procurement requirements into many lots. Hence, having large requirements does not necessarily imply using a large single-sourcing contract. Although one might think that such an optimal policy is equivalent to forming many GPOs, this is not the case, because when the strategic buyer chooses many single-sourcing contracts he is internalizing (at least partially) the positive externalities that a single-sourcing contract generates on the unitary cost of other demands. Hence, the presence of a unique GPO acts as a coordination device.[12]

Although coordinating all requirements would reduce total procurement costs, the competitive price might be lower than the unitary cost of this strategic buyer. Thus, it is not clear whether a GPO grouping (all or some) buyers would be formed, because either individual or coalitional deviations may allow agents outside the group to obtain such a better competitive price. To address this issue, let consider the possibility of B representing a subset $\alpha \leq 1-a$ of smallish buyers by offering them a unitary price $\widehat{p}$. It is obvious that the unitary cost of this new GPO would decrease when $\alpha > 0$, because the GPO may always procure the new α units through smallish units. That is,

$$TC^{*}(a+\alpha) \leq TC^{*}(a) + p(a)\alpha,$$

where $p(a)$ denotes the unitary cost of the (originally) α smallish units when B has capacity a. However, it is not clear that such a decrease is enough to offer a unitary prices $\widehat{p}$ to smallish units guaranteeing that they accept it. We will say that a big buyer may form a *stable GPO* if he is able to obtain a lower procurement cost by coordinating the requirements of all their members, who can freely to leave the group. Formally,

Definition 2 *A pair $(\alpha, \widehat{p})$ is (individually) stable if $\widehat{p} \leq p(a+\alpha)$ and $TC^{*}(a+\alpha) \leq TC^{*}(a) + \alpha\widehat{p}$.*

Among all stable pairs, we assume w.l.o.g. that B chooses $(\alpha, \widehat{p}) \in \arg\min TC^{*}(a+\alpha) - \widehat{p}\alpha$; that is, for any α the large buyer would fix $\widehat{p} = p(a+\alpha)$. Hence, $\alpha \in \arg\min TC^{*}(a+\alpha) - \alpha p(a+\alpha)$.

From Example 1, it is immediate that a GPO containing all smallish units is stable when costs are quadratic, as $p(a+\alpha) > TC^{*}(a+\alpha)/(a+\alpha)$ when $a+\alpha$ is sufficiently large and $TC^{*}(a+\alpha)/(a+\alpha) \leq TC^{*}(a)/a$, smallish units can always be compensated to avoid individual deviations. Nevertheless, the results would differ when allowing for coalitional deviations. This is not the case when costs are cubic, where (as the competitive price is decreasing in a) the concept of individual stability is equivalent to coalitional stability. Hence, in order to clearly expose our arguments, we study the case of three suppliers and cubic costs, where the optimal policy (see Example 2) is the following:

1. If $a \leq 1/3$ then $m^{*} = 0$.
2. If $a \geq 1/3$ then $m^{*} = 1$ and $z = \min\left\{a, \frac{a+1}{a+3}\right\}$.

This yields competitive prices of $p(a) = c'(1/3) = 1/3$ when $a \leq 1/3$, $p(a) = c'\left(\frac{1-a}{2}\right) = 3\left(\frac{1-a}{2}\right)^{2}$ when $a \in \left[1/3, \sqrt{2}-1\right]$, and $p(a) = c'\left(\frac{1-\frac{a+1}{a+3}}{2}\right) = 3\left(\frac{1}{a+3}\right)^{2}$, which is decreasing in a. Likewise, it is immediate to see that $TC^{*}(a) = a/3$ when $a \leq 1/3$, $TC^{*}(a) = \frac{3}{4}a^{3} + \frac{3}{4}a^{2} - \frac{3}{4}a + \frac{1}{4}$ when $a \in \left[1/3, \sqrt{2}-1\right]$, and $TC^{*}(a) = \frac{a}{a+3}$ when $a \geq \sqrt{2}-1$.

Two possibilities must be considered by B: (i) $a+\alpha \in (1/3, 0.4142)$ or (ii) $a+\alpha \geq 0.4142$. In the first case, direct computations yield $\alpha^{*}_{(i)} = \frac{1}{a+3}\left(1-2a-a^{2}\right)$, implying $a+\alpha^{*}_{(i)} = \frac{1+a}{3+a}$, whereas in case (ii) we get $\alpha^{*}_{(ii)} = 0.4142 - a$.

[12] A question that remains open is the optimal policy of a big buyer when grouped smallish units decide independently their procurement policies.

We obtain

$$\begin{aligned} TC^*\left(a+\alpha^*_{(i)}\right)-\alpha^*_{(i)}p\left(a+\alpha^*_{(i)}\right) &= \frac{a}{a+3}, \text{ and} \\ TC^*\left(a+\alpha^*_{(ii)}\right)-\alpha^*_{(ii)}p\left(a+\alpha^*_{(ii)}\right) &= \frac{9}{2}a-3\sqrt{2}a-6\sqrt{2}+\frac{17}{2}, \end{aligned}$$

where $TC^*\left(a+\alpha^*_{(i)}\right)-\alpha^*_{(i)}p\left(a+\alpha^*_{(i)}\right) \leq TC^*\left(a+\alpha^*_{(ii)}\right)-\alpha^*_{(ii)}p\left(a+\alpha^*_{(ii)}\right)$ iff $a \leq 0.414\,2$.

When $a \leq 0.4142$ it is immediate that $\alpha^* = \alpha^*_{(i)}$. Also, including the constraint $\alpha^*_{(ii)} \geq 0$, we obtain the same α^* otherwise.

Proposition 5 *Under cubic costs and $n = 3$, it is never the case that a unique GPO is formed. In particular, a stable pair $(\alpha, \widehat{p})$ satisfies $\alpha = a - 0.4142$ when $a \leq 0.4142$ and $\alpha = 0$ otherwise.*

The proposed example of group formation puts forward the difficulties to coordinate procurement requirements. It is obvious that the unitary cost is reduced when such demands are coordinated. However, the fact that the presence of a strategic buyer creates positive externalities to smallish buyers (its price is smaller than the unitary cost of the strategic one) might make impossible to prevent individual deviations and keep stable the purchasing group.

5 Conclusions

We studied we optimal procurement policy of a strategic buyer owing a large set of smallish units. In contrast to Anton and Yao (1989) or Inderst (2008), in our model this buyer has the option to divide procurement requirements into smallish units that act independently. We showed that in general it is never optimal to create a unique single-sourcing lot. Instead, under strictly convex production costs, the optimal policy would involve creating $m \leq n-2$ symmetric lots and the rest is demanded either though another lot of smaller size or leaving some of the smallish units to act independently in the market. We noted that the selection of many lots is not identical to choosing many GPOs acting independently. This is because centralizing decisions allows to internalize the positive externalities that single-sourcing contracts generate on other buyers. These externalities may prevent the formation of a large purchasing group.

References

[1] Abito, J.M. and Wright, J. (2008) Exclusive dealing with imperfect downstream competition. International Journal of Industrial Organization 26, 227-246.

[2] Anton, J.J and Yao, D.A. (1989). Split Awards, Procurement, and Innovation. The RAND Journal of Economics 20 (4) 538-552.

[3] Blair, R.D. and Durrance, C.P. (2014) Group Purchasing Organizations, Monopsony, and Antitrust Policy. Managerial and Decision Economics 35, 433-443.

[4] Chen, Y. and Riordan, M.H. (2007) Vertical integration, exclusive dealing, and ex post cartelization. The RAND Journal of Economics 38 (1) 1-21.

[5] Cousins, P. and Spekman, R. (2003), Strategic Supply and the Management of Inter- and Intra-Organizational Relationships. Journal of Purchasing & Supply Management, 9, 19–29.

[6] Dalen, D.M., Moen, E.R. and Riis, C. (2006) Contract renewal and incentives in public procurement. International Journal of Industrial Organization 24, 269-285.

[7] de Meza, D. and Selvaggi, M. (2004). Exclusive Contracts Foster Relationship-Specific Investment. CMPO Working Paper Series No. 04/105.

[8] Dimitri, N., Dini, F. and Piga, G. (2006). When should procurement be centralized? In Handbook of Procurement. Edited by Dimitri, Piga and Spagnolo. Cambridge University Press.

[9] Gans, J.S. and King, S.P. (2002). Exclusionary contracts and competition for large buyers. International Journal of Industrial Organization 20, 1363-1381.

[10] Gong, J., Li, J. and McAfee, R.P. (2012). Split-award contracts with investment. Journal of Public Economics 96, 188-197.

[11] Guo, L. and Iyer, G. (2013) Multilateral Bargaining and Downstream Competition. Marketing Science 32 (3), 411-430.

[12] Hu, Q. and Schwarz, L.B. (2011a) Controversial Role of GPOs in Healthcare-Product Supply Chains. Production and Operations Management 20 (1) 1-15.

[13] Hu, Q., Schwarz, L.B. and Uhan, N.A. (2011b) The Impact of Group Purchasing Organizations on Healthcare-Product Supply Chains

[14] Inderst, R. (2008). Single sourcing versus multiple sourcing, RAND Journal of Economics 39 (1), 199-213.

[15] Jeon, D-S. and Menicucci, D. (2011) Bundling and Competition for Slots. Unpublished work.

[16] Jeon, D-S. and Menicucci, D. (2014) Buyer Group Without Exclusive Purchase: With Applications to Library Consortium. Unpublished work.

[17] Kitamura, H. Matsushima, N. and Sato, M. (2014) Exclusive Contracts with Complementary Inputs. Unpublished work.

[18] Marvel, H.P. and Yang, H. (2008) Group purchasing, nonlinear tariffs, and oligopoly. International Journal of Industrial Organization 26, 1090-1105.

[19] Munson, C.L. (2007) The appeal of centralised purchasing policies. International Journal of Procurement Management 1 (1-2), 117-143.

[20] O'Brien, D.P. and Shaffer, G. (1997) Nonlinear supply contracts, exclusive dealing, and equilibrium market foreclosure. Journal of Economics & Management Strategy 6 (4), 755-785.

[21] OECD (2013), Public procurement spending. In Government at Glance 2013, OECD Publishing.

[22] Saha, R.L.,Seidmann, A. and Tilson, V. (2014).The Impact of Custom Contracting on the Key Information Roles of Group Purchasing Organizations (GPOs) in the Healthcare Supply Chain. Unpublished work.

[23] Spector, D. (2011) Exclusive contracts and demand foreclosure. The RAND Journal of Economics 42 (4) 619-638.

[24] Wilken, F. (2011) Strategic Sourcing to Sustain Supplier Competition: Theoretical and Experimental Analyses. Dissertation, Universitat Heidelberg.

Appendix

A Proof of Proposition 1

Consider the procurement policy consisting in selecting $m \geq n$ lots. We first show that if a supplier obtains more than one lot, there is an equivalent policy consisting in bundling all these lots. Let $Z_i = z_i^1 + ... + z_i^{k_i}$. By Lemma 2 $p = c'(Z_{n-1})$ if $Z_{n-1} - Z_n \geq 1 - Z$, $p = c'(q)$ otherwise.[13] Let $q_i = \max\{Z_i, q\}$ that is, the total amount provided by seller i. We distinguish two cases:

Case 1: $Z_n = 0$. In this case, seller n obtains (expected) profits $qc'(q_{n-1}) - c(q)$. Since all sellers obtain the same profits we must have that

$$qc'(q_{n-1}) - c(q) = T^1 - c(q_1) = ... = T^{n-1} - c(q_{n-1})$$

Thus, by choosing $z_1 = Z_1, ..., z_{n-1} = Z_{n-1}$, in a truthful equilibrium at least one seller obtains no lot and therefore obtains profits $qc'(q_{n-1}) - c(q)$. Hence, the total procurement costs $T^1 + ... + T^{n-1} + qc'(q_{n-1})$ remain constant.

Case 2: $Z_n > 0$. In these cases the profits of seller n are $T^n + (q - Z_n)c'(q_{n-1}) - c(q) = T^1 - c(q_1) = ... = T^{n-1} - c(q_{n-1})$. By bundling lots $z_1 = Z_1, ..., z_n = Z_n$, the same costs are obtained. So, assume each seller obtains just one lot. Since $q_i \geq q$ we have that in any truthful equilibrium $T^n(Z_n) = t_n^n(0) + c(q) - c(q - Z_n)$ and $T^{n-1}(Z_n) = c(q_{n-1} + Z_n) - c(q_{n-1}) \geq c(q + Z_n) - c(q)$. Therefore, as $T^n(Z_n) = T^{n-1}(Z_n)$ we obtain

$$t_n^n(0) \geq c(q + Z_n) - c(q) - [c(q) - c(q - Z_n)],$$

implying (due to strict convexity of costs) $T^n(Z_n) = c(q + Z_n) - c(q) > Z_n c'(q)$. Thus, by proposing $Z_1, ..., Z_{n-1}$ we have that profits of seller n are given by $qc'(q_{n-1}) - c(q) \leq T^n + (q - Z_n)c'(q_{n-1}) - c(q)$, implying that total procurement costs are small.

Hence, we have shown that any optimal policy is at least weakly dominated by another one where each seller obtains just one lot and that $Z_n > 0$ cannot be optimal.

B Proof of Proposition 3

Total costs are

$$TC = \sum_{i=1}^{m} (c(z_i) + c'(q)q - c(q)) + (a - Z)c'(q)$$

where, by symmetry (Lemma), $z_i = Z/m$. Thus assuming $c(q) = q^k$, costs can be written as

$$TC = m\left((k-1)\left(\frac{1-Z}{n-m}\right)^k + \left(\frac{Z}{m}\right)^k\right) + (a - Z)k\left(\frac{1-Z}{n-m}\right)^{k-1}$$

[13] See Definition 1.

Minimization without restrictions yield partial derivatives

$$
\begin{aligned}
TC_m &= k(k-1)\left[m\left(\frac{1-Z}{n-m}\right)+(a-Z)\right]\left(\frac{1-Z}{n-m}\right)^{k-1}\left(\frac{1}{n-m}\right)+(k-1)\left(\frac{1-Z}{n-m}\right)^{k}-(k-1)\left(\frac{Z}{m}\right)^{k} \\
TC_Z &= -k(k-1)\left[m\left(\frac{1-Z}{n-m}\right)+(a-Z)\right]\left(\frac{1-Z}{n-m}\right)^{k-2}\frac{1}{n-m}-k\left(\frac{1-Z}{n-m}\right)^{k-1}+k\left(\frac{Z}{m}\right)^{k-1}
\end{aligned}
$$

To prove our result, we use a geometric argument of the minimization problem. Let denote by $m^m(Z)$ and $Z^Z(m)$ the values of m and Z that solve $TC_m = 0$ and $TC_Z = 0$, respectively. Denote also by $Z^m(m)$ the inverse of $m^m(Z)$. We will next show that for any m, we have that $m/n < Z^m(m) < Z^Z(m)$.

Claim. $m/n < Z^m(m) < Z^Z(m)$.

Substituting $Z = m/n$ into the FOC, we obtain

$$
\begin{aligned}
TC_m &= ak\frac{n}{n-m}(k-1)\left(\frac{1}{n}\right)^k > 0 \Longrightarrow m^m(Z) \le m/n \Longrightarrow Z^m(m) > m/n \\
TC_Z &= -ak\frac{n^2}{n-m}(k-1)\left(\frac{1}{n}\right)^k < 0 \Longrightarrow Z^Z(m) > m/n
\end{aligned}
$$

At $Z = Z^Z(m)$; i.e., at Z satisfying $TC_Z = 0$ we obtain

$$
\begin{aligned}
0 &= k(k-1)\left(m\left(\frac{1-Z}{n-m}\right)+(a-Z)\right)\left(\frac{1-Z}{n-m}\right)^{k-2}\left(-\frac{1}{n-m}\right)-k\left(\frac{1-Z}{n-m}\right)^{k-1}+k\left(\frac{Z}{m}\right)^{k-1} \\
\Longleftrightarrow & \; k(k-1)\left(m\left(\frac{1-Z}{n-m}\right)+(a-Z)\right)\left(\frac{1-Z}{n-m}\right)^{k-2}\left(\frac{1}{n-m}\right) = -k\left(\frac{1-Z}{n-m}\right)^{k-1}+k\left(\frac{Z}{m}\right)^{k-1}
\end{aligned}
$$

Hence, evaluating TC_m at $Z^Z(m)$ we obtain

$$
TC_m = k\left(\frac{Z}{m}\right)^{k-1}\left(\frac{1-Z}{n-m}\right)-\left(\frac{1-Z}{n-m}\right)^{k}-(k-1)\left(\frac{Z}{m}\right)^{k}
$$

The previous expression can be written as

$$
\begin{aligned}
TC_m &= kz^{k-1}q - q^k - (k-1)z^k \\
&= kz^{k-1}q - q^k - kz^{k-1}z + z^k \\
&= z^k - q^k - kz^{k-1}(z-q)
\end{aligned}
$$

Since $z > q$, and $k > 1$ we have that

$$
kq^{k-1} < \frac{z^k - q^k}{z-q} < kz^{k-1}
$$

Hence,

$$
TC_m = \left(z^k - q^k\right) - kz^{k-1}(z-q) < 0
$$

and the claim follows. □

From the previous claim it is immediate that no interior solution can be optimal. Noting that $Z^m(m) < Z^Z(m)$, it is immediate that when include the constraints $m \le n-2$ and $Z \le a$, three type of solutions might be obtained:

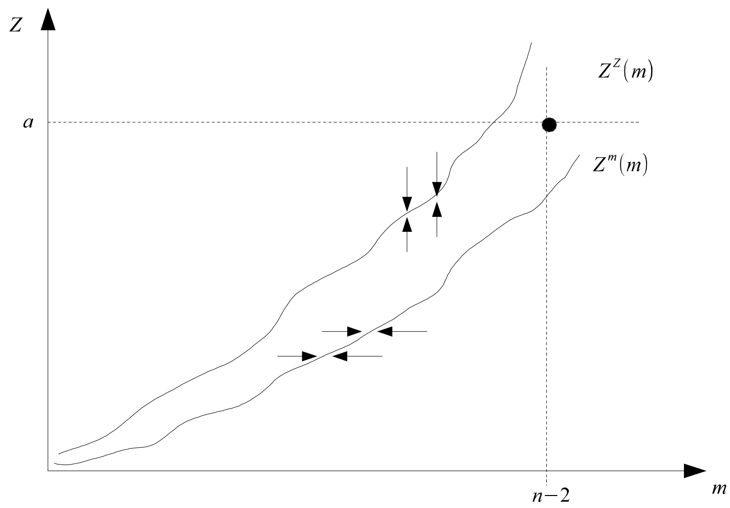

Figure 1: A geometric representation of the the optimal choice (dot). Arrows represent the direction in which costs are reduced. In this example $Z^m(n-2) < a < Z^Z(n-2)$.

1. $m = n-2, Z = a \Longleftrightarrow Z^Z(n-2) \geq a, Z^m(n-2) < a$. I.e. $Z^m(n-2) \leq a \leq Z^Z(n-2)$
2. $m = n-2, Z < a \Longleftrightarrow Z^Z(n-2) < a, Z^m(n-2) < a$
3. $m \leq n-2, Z = a \Longleftrightarrow Z^Z(n-2) > a, Z^m(n-2) > a$

where $m \leq n-2$ is used in point (3) instead of the strict inequality due to the discrete nature of the variable m. To be more specific, $m^* \in \{\lfloor m \rfloor, \lceil m \rceil\}$ where m solves $TC_m = 0$ at $Z = a$.

To finish the proof, we next find the thresholds for a for any of these situation to arise.

It is obvious that $Z^Z(n-2) \geq a \Longleftrightarrow TC_Z < 0$ when evaluated at TC_Z at $m = n-2$ and $Z = a$. This yields

$$TC_Z = \frac{1}{2}k\left(2\left(\frac{a}{n-2}\right)^{k-1} - (4-2k-n+kn)\left(\frac{1}{2}\right)^{k-1}(1-a)^{k-1}\right)$$

which is negative whenever

$$a < \overline{a}(k,n) = \frac{(n-2)(4-2k-n+kn)^{\frac{1}{k-1}} \cdot 2^{-\frac{k}{k-1}}}{1+(n-2)(4-2k-n+kn)^{\frac{1}{k-1}} \cdot 2^{-\frac{k}{k-1}}}$$

Similarly, $Z^m(n-2) \geq a \Longleftrightarrow TC_m > 0$ when evaluated at $m = n-2, Z = a$. We obtain,

$$TC_m = -\frac{1}{2}(k-1)\left(2\left(\frac{a}{n-2}\right)^{k} - (2-2k+kn)\left(\frac{1}{2}\right)^{k}(1-a)^{k}\right)$$

which is positive when

$$a < \underline{a}(k,n) = \frac{(n-2)(2-2k+kn)^{\frac{1}{k}} \cdot 2^{-\frac{k+1}{k}}}{1+(n-2)(2-2k+kn)^{\frac{1}{k}} \cdot 2^{-\frac{k+1}{k}}}$$

Therefore,

1. $m = n-2, Z = a \Longleftrightarrow Z^Z(n-2) \geq a, Z^m(n-2) < a \Longleftrightarrow a < \overline{a}(k,n)$ and $a > \underline{a}(k,n)$
2. $m = n-2, Z < a \Longleftrightarrow Z^Z(n-2) < a, Z^m(n-2) < a \Longleftrightarrow a > \overline{a}(k,n)$ and $a > \underline{a}(k,n)$
3. $m \leq n-2, Z = a \Longleftrightarrow Z^Z(n-2) > a, Z^m(n-2) > a \Longleftrightarrow a < \overline{a}(k,n)$ and $a < \underline{a}(k,n)$

As we know that $Z^m(m) < Z^Z(m)$ it is immediate that $\underline{a}(k,n) < \overline{a}(k,n)$ and the result follows.

C Proof of Proposition 4

In case that $m = n-1$, the size of the smallest lot $s \geq 1-Z$, would determine the "competitive" price. That is $p = c'(s)$. Moreover, other lots would have equal size $b = \frac{Z-s}{n-2} > s$, implying $s < \frac{Z}{n-1}$.

Now, total costs are

$$\begin{aligned}\overline{TC} &= (n-2)\left(c\left(\frac{Z-s}{n-2}\right) + c'(s)(1-Z) - c(1-Z)\right) + c(s) + c'(s)(1-Z) - c(1-Z) + c'(s)(a-Z) \\ &= (n-2)\,c\left(\frac{Z-s}{n-2}\right) + c(1-Z) + c(s) + n\left[c'(s)(1-Z) - c(1-Z)\right] - (1-a)\,c'(s)\end{aligned}$$

We can rewrite total costs in terms of b and s as

$$\overline{TC} = (n-2)\,c(b) + c(1-(n-2)b-s) + c(s) + n\left[c'(s)(1-(n-2)b-s) - c(1-(n-2)b-s)\right] - (1-a)\,c'(s)$$

Suppose there are no constraints to the buyer's minimization problem. The optimal policy would satisfy

$$\begin{aligned}\overline{TC}_b &= (n-2)\,c'(b) - (n-2)\,c'(d) + n\left[-(n-2)\,c'(s) + (n-2)\,c'(d)\right] = 0 \\ \overline{TC}_s &= -(n-1)\,c'(s) + (n-1)\,c'(d) + nc''(s)(d) - (1-a)\,c''(s) = 0 \\ \overline{TC}_{bb} &= (n-2)\,c''(b) + (n-2)^2 c'(d) - n(n-2)^2 c'(d) > 0 \\ \overline{TC}_{ss} &= c''(d) + c''(s) + n\left[c'''(s)(d) - 2c''(s) - c''(d)\right] - (1-a)\,c'''(s) > 0\end{aligned}$$

where $d = 1-(n-2)b-s$ denotes the total procurement requirements demanded by smallish units.

We next show that at any (b,s) satisfying $\overline{TC}_s = 0$ it is the case that $\overline{TC}_{ss} < 0$, which would imply that there is no solution where s is interior.

Under homogenous cost function we obtain

$$\overline{TC}_s = \left(-(n-1) + \left(\frac{k-1}{s}\right)(n(1-(n-2)b-s) - 1 + a)\right)s^{k-1} + (n-1)(1-(n-2)b-s)^{k-1}$$

and therefore, in order to have an interior solution we must have

$$-(n-1) + \left(\frac{k-1}{s}\right)(n(1-(n-2)b-s) - 1 + a) < 0 \tag{2}$$

For homogeneous costs, the second partial derivative with respect to s is

$$\overline{TC}_{ss} = k(k-1)\left\{\left[1-2n+\left[n(1-(n-2)b-s)-(1-a)\right]\left(\frac{k-2}{s}\right)\right]s^{k-2} - (n-1)(1-(n-2)b-s)^{k-2}\right\}.$$

Evaluation this expression at an interior solution, thus satisfying (2), we obtain

$$\begin{aligned}
& 1-2n+\left[n\left(1-(n-2)b-s\right)-(1-a)\right]\left(\frac{k-2}{s}\right) \\
= \;& 1-2n+\left[n\left(1-(n-2)b-s\right)-(1-a)\right]\left(\frac{k-1}{s}\right)-\left[n\left(1-(n-2)b-s\right)-(1-a)\right]\left(\frac{1}{s}\right) \\
< \;& 1-2n+(n-1)-\left[n\left(1-(n-2)b-s\right)-(1-a)\right]\left(\frac{1}{s}\right) \\
= \;& -n-\left[n\left(1-(n-2)b-s\right)-(1-a)\right]\left(\frac{1}{s}\right)<0
\end{aligned}$$

where the last inequality is derived from observing that $(n-2)\,b+s\leq a$.

Hence, if the optimal policy satisfies $m=n-1$, it must be that $s\in\{1-Z,a-(n-2)\,b\}$. Moreover, since $s=\frac{1-Z}{2}$ is equivalent to selecting only $n-2$ lots, it follows that s must be maximal. That is, either $s=a-(n-2)\,b$ or $s<a-(n-2)\,b$ and $s=b$. Since $b>1/n$ the latter cannot be since $\overline{TC}_b$ evaluated at $s=b$ yields

$$-(n-1)\,c'(b)+(n-1)\,c'(1-(n-1)\,b)<0,$$

implying that increasing b would reduce costs. Hence the optimal policy must satisfy $(n-2)\,b^*+s^*=a$.

Including the constraint $b=\frac{a-s}{n-2}$ into the minimization problem, we get

$$\overline{TC}=(n-2)\,c\left(\frac{a-s}{n-2}\right)+c\,(1-a)+c\,(s)+nc'\,(s)\,(1-a)-nc\,(1-a)-(1-a)\,c'\,(s)\,.$$

Therefore,

$$\begin{aligned}
\overline{TC}_s &= -c'\left(\frac{a-s}{n-2}\right)+c'\,(s)+nc''\,(s)\,(1-a)-(1-a)\,c''\,(s)=0, \\
\overline{TC}_{ss} &= \frac{1}{n-2}c''\left(\frac{a-s}{n-2}\right)+c''\,(s)+(n-1)\,c'''\,(s)\,(1-a)\,,
\end{aligned}$$

which, for homogeneous cost functions, gives the optimal size of the smallest lot implicitly as

$$\left(\frac{a-s}{n-2}\right)^{k-1}=s^{k-1}+(n-1)\,(k-1)\,s^{k-2}\,(1-a)\,.$$