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Testing for Deterministic Seasonality in Mixed-Frequency VARs*

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Abstract

This paper investigates the presence of deterministic seasonal features within a mixed frequency vector autoregressive model. A strategy based on Wald tests is proposed.

Key words: deterministic seasonal features, mixed frequency VARs
JEL: C32

1 Motivation

The mixed-frequency VAR (MF-VAR hereafter) proposed by Ghysels (2016) and Ghysels et al. (2016) is a multivariate modelling that stacks time series at different frequencies in order to jointly model the behavior of a low- and the high-frequency variables (respectively LF and HF hereafter). This approach complements the MIDAS regression (e.g., Ghysels et al., 2007) in which a single non-linear equation from the LF to HF series is estimated (see Foroni et al., 2015 for the reverse regression). The MF-VAR modelling, although to some extent inherited from the periodic autoregressive models (i.e., PAR(p)), is often estimated on seasonally adjusted data[1] or at least the consequences of such seasonality components is not really accounted for.

This note provides a strategy to estimate a full range of interesting hypotheses about deterministic seasonal features within raw data. We also examine the presence of common seasonal features (Engle and Hylleberg, 1996, Hecq, Laurent and Palm, 2012). We apply our testing framework on the relationship between quarterly employment and monthly tourist arrivals in the Balearic Islands.

2 Deterministic seasonality in MF-VARs

Let us assume a stationary structural mixed-frequency model of order 1 for a LF quarterly series y_t and one HF variables x_t with $m = 3$ HF monthly observations per LF t periods, $t = 1...T$. We can stack the

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[1] In Maravall (1993) and in Bell (2012) it is possible to find the pernitious effects of working on seasonally adjusted data when fitting VAR models.

variables in a vector $Z_t = (y_t, x_t^{(3)}, x_{t-1/3}^{(3)}, x_{t-2/3}^{(3)})'$ where e.g., y_t refers to a series for quarter 1, $x_t^{(3)}, x_{t-1/3}^{(3)}$ and $x_{t-2/3}^{(3)}$ denote respectively the monthly explanatory variable for March, February and January. One can then obtain a structural MF-VAR in which we add deterministic seasonality parameters in the 4×4 coefficient matrix $\tilde{\Theta}$,

$$A_c Z_t = \tilde{\Theta} D_t + A_1 Z_{t-1} ... + A_p Z_{t-p} + \varepsilon_t, \tag{1}$$

with $\varepsilon_t \sim NIID(0, \Omega_\varepsilon)$, Ω_ε being diagonal. We can use for D_t seasonal dummies such as $D_t = \iota_{T/4} \otimes I_4$ with $\iota_{T/4}$ a $T/4$ vector of ones (assuming that every year has four full quarters) or trigonometric components.[2] As an example, a structural MF-VAR of order 1 reads as

$$\begin{pmatrix} 1 & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \beta_1 & 1 & -\phi_1 & 0 \\ 0 & 0 & 1 & -\phi_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} y_t \\ x_t^{(3)} \\ x_{t-1/3}^{(3)} \\ x_{t-2/3}^{(3)} \end{bmatrix}$$

$$= \begin{pmatrix} \tilde{\alpha}_1 & \tilde{\alpha}_2 & \tilde{\alpha}_3 & \tilde{\alpha}_4 \\ \tilde{\mu}_{1,1} & \tilde{\mu}_{1,2} & \tilde{\mu}_{1,3} & \tilde{\mu}_{1,4} \\ \tilde{\mu}_{2,1} & \tilde{\mu}_{2,2} & \tilde{\mu}_{2,3} & \tilde{\mu}_{2,4} \\ \tilde{\mu}_{3,1} & \tilde{\mu}_{3,2} & \tilde{\mu}_{3,3} & \tilde{\mu}_{3,4} \end{pmatrix} D_t + \begin{pmatrix} \rho & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \pi_1 & 0 & 0 & 0 \\ \pi_2 & 0 & 0 & 0 \\ \pi_3 & \phi_1 & 0 & 0 \end{pmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1}^{(3)} \\ x_{t-(1+1/3)}^{(3)} \\ x_{t-(1+2/3)}^{(3)} \end{bmatrix} + \varepsilon_t. \tag{2}$$

System (2) implies: (*i*) nowcasting causality: $(\gamma_{0,1}, \gamma_{0,2}, \gamma_{0,3}) \neq 0$ and $\beta_1 \neq 0$ (see Götz and Hecq, 2014), (*ii*) Granger-causality $x \to y$:$(\gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}) \neq 0$ (Ghysels, 2016, Götz et al., 2016), (*iii*) Granger causality $y \to x$: $(\pi_1, \pi_2, \pi_3) \neq 0$ (Ghysels, 2016, Götz et al., 2016), (*iv*) AR(1) on x but not PAR(1) because ϕ_1 is the same for each months, ρ for each quarter,[3] (*v*) absence of noncausal relationships between the x (Götz et al., 2015), (*vi*) $m = 3$ months per quarter and we have annual data, (*vii*) deterministic seasonality in the 4×4 matrix coefficient $\tilde{\Theta}$ instead of the 4×1 vector in the usual MF-VAR literature.

From (1) we can get the reduced form MF-VAR using $Z_t = A_c^{-1} \tilde{\Theta} D_t + A_c^{-1} A_1 Z_{t-1} + A_c^{-1} \varepsilon_t$ that we simply denote $Z_t = \Theta D_t + \Phi_1 Z_{t-1} + u_t$. In terms of the parameters of the structural system (2) we get

$$\begin{pmatrix} y_t \\ x_t^{(3)} \\ x_{t-1/3}^{(3)} \\ x_{t-2/3}^{(3)} \end{pmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \mu_{1,1} & \mu_{1,2} & \mu_{1,3} & \mu_{1,4} \\ \mu_{2,1} & \mu_{2,2} & \mu_{2,3} & \mu_{2,4} \\ \mu_{3,1} & \mu_{3,2} & \mu_{3,3} & \mu_{3,4} \end{bmatrix} D_t + \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,3} & \phi_{1,4} \\ \phi_{2,1} & \phi_{2,2} & \phi_{2,3} & \phi_{2,4} \\ \phi_{3,1} & \phi_{3,2} & 0 & 0 \\ \phi_{4,1} & \phi_{4,1} & 0 & 0 \end{bmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1}^{(3)} \\ x_{t-(1+1/3)}^{(3)} \\ x_{t-(1+2/3)}^{(3)} \end{pmatrix} + u_t. \tag{3}$$

with $u_t \sim NIID(0, \Omega_u), \Omega_u$ positive definite, and $\mathbb{E}(||u_t||^4) < \infty$, where $|| \cdot ||$ is the Frobenius norm and the roots of $\det |I - \Phi_1 z| = 0$ are all outside the unit circle.

Although System (3) is explicitly written in terms of the low quarterly frequency, it also allows for

[2] Note $D_t = \left[D_t^1, D_t^2, D_t^3, D_t^4\right]$, where D_t^k are the quarterly dummies such that:

$$D_t^k = \begin{cases} 1 \; if \; k = 1 + int\left[(t-1) \bmod 4\right] \\ 0 \; if \; k \neq 1 + int\left[(t-1) \bmod 4\right] \end{cases},$$

where mod is the modulus such that $a \bmod b$ returns the remainder of a/b. As noted for example by Ghysels and Osborn (2001), there is one to one relationship between the seasonal dummies representation and the triginometric representation where $T_t = \left[\cos(0t), \cos\left(\frac{\pi}{2}t\right), \sin\left(\frac{\pi}{2}t\right), \cos(\pi t)\right]$.

[3] Note that this (strong) assumption is not often discussed in the literature although usually $\phi_i s$ are estimated unrestrictedly but in the Ghysels' ARX representation (mostly for Bayesian modelling).

testing monthly deterministic seasonality. For instance the parameter $\mu_{1,1}$ refers to as the last month of quarter one, namely march, $\mu_{1,2}$ is June, $\mu_{3,1}$ is January, etc. Finally note that (3) could be also specified replacing the seasonal dummies by the trigonometric representation of seasonality using $T_t = \left[\cos(0t), \cos\left(\frac{\pi}{2}t\right), \sin\left(\frac{\pi}{2}t\right), \cos(\pi t)\right]$, in $Z_t = \Psi T_t + \Phi_1 Z_{t-1} + u_t$, namely

$$\begin{pmatrix} y_t \\ x_t^{(3)} \\ x_{t-1/3}^{(3)} \\ x_{t-2/3}^{(3)} \end{pmatrix} = \begin{bmatrix} \delta_0^1 & \delta_{2c}^1 & \delta_{2s}^1 & \delta_1^1 \\ \delta_0^2 & \delta_{2c}^2 & \delta_{2s}^2 & \delta_1^2 \\ \delta_0^3 & \delta_{2c}^3 & \delta_{2s}^3 & \delta_1^3 \\ \delta_0^4 & \delta_{2c}^4 & \delta_{2s}^4 & \delta_1^4 \end{bmatrix} T_t + \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,3} & \phi_{1,4} \\ \phi_{2,1} & \phi_{2,2} & \phi_{2,3} & \phi_{2,4} \\ \phi_{3,1} & \phi_{3,2} & 0 & 0 \\ \phi_{4,1} & \phi_{4,1} & 0 & 0 \end{bmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1}^{(3)} \\ x_{t-(1+1/3)}^{(3)} \\ x_{t-(1+2/3)}^{(3)} \end{pmatrix} + u_t, \tag{4}$$

where the coefficients δ_0^i for $i = 1, 2, 3$ and 4 are associated to the zero frequency (oscillation that complete a full cycle every $2\pi/0 = \infty$ periods or quarters) , δ_{2c}^i and δ_{2s}^i for $i = 1, 2, 3$ and 4 are associated to the $\pi/2$ frequency (oscillation that complete a full cycle every $2\pi/(\pi/2) = 4$ periods or quarters) and finally δ_1^i for $i = 1, 2, 3$ and 4 are associated to the Nyquist (π) frequency (oscillation that complete a full cycle every $2\pi/\pi = 2$ periods or quarters). As shown in Ghysels and Osborn (2001) we could establish the following relationship between the coefficients associated to the deterministic seasonality of models (3) and (4):

$$\begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & -1/2 & 0 & 1/2 \\ 1/2 & 0 & -1/2 & 1/2 \\ -1/4 & 1/4 & -1/4 & 1/4 \end{bmatrix} \times \begin{bmatrix} \alpha_1 & \mu_{1,1} & \mu_{2,1} & \mu_{3,1} \\ \alpha_2 & \mu_{1,2} & \mu_{2,2} & \mu_{3,2} \\ \alpha_3 & \mu_{1,3} & \mu_{2,3} & \mu_{3,3} \\ \alpha_4 & \mu_{1,4} & \mu_{2,4} & \mu_{3,4} \end{bmatrix} = \begin{bmatrix} \delta_0^1 & \delta_0^2 & \delta_0^3 & \delta_0^4 \\ \delta_{2c}^1 & \delta_{2c}^2 & \delta_{2c}^3 & \delta_{2c}^4 \\ \delta_{2s}^1 & \delta_{2s}^2 & \delta_{2s}^3 & \delta_{2s}^4 \\ \delta_1^1 & \delta_1^2 & \delta_1^3 & \delta_1^4 \end{bmatrix}$$

hence for example in the case of the coefficient associated to y_t (the LF frequency variable) we have the following relationship between the elements of Θ and Ψ:

$$\delta_0^1 = \frac{1}{4}\sum_{s=1}^{4}\alpha_s;\ \delta_{2c}^1 = \frac{1}{2}(-\alpha_2 + \alpha_4);\ \delta_{2s}^1 = \frac{1}{2}(\alpha_1 - \alpha_3);\ \delta_1^1 = \frac{1}{4}(-\alpha_1 + \alpha_2 - \alpha_3 + \alpha_4).$$

3 Strategy for deterministic seasonality

We are interested in testing several restrictions on Θ (or Ψ) in a general MF-VAR of order p using the selection matrix R to construct the usual Wald test

$$\begin{aligned} \xi_W &= \left[\mathrm{R}vec(\hat{\Pi})\right]' (\mathrm{R}\hat{\Sigma}\mathrm{R}')^{-1} \left[\mathrm{R}vec(\hat{\Pi})\right], \\ \hat{\Sigma} &= (W'W)^{-1} \otimes \hat{\Omega}_u. \end{aligned} \tag{5}$$

We get the least square estimates $\hat{\Pi} := (\hat{\Theta} : \hat{\Phi}_1 : ...\hat{\Phi}_p)$ using $Z'W(W'W)^{-1}$. $W = (W_1, \ldots, W_T)'$ is a $T\times(4+np)$ matrix stacked over T observations of the regressor set consisting of $W_t = (D_t, Z_{t-1}, ..., Z_{t-p})'$; similarly $Z = (Z_1, \ldots, Z_T)'$ is $T \times n$. The operator vec stacks the column of a matrix; $\hat{\Omega}_u = \frac{1}{T}\hat{u}'\hat{u}$ is the empirical (asymptotic) covariance matrix of the disturbance terms. ξ_W follows asymptotically a $\chi^2_{rank(\mathrm{R})}$.

Note that we estimate the MF-VAR (3) or (4) without restrictions on $\Phi_i^{'}s$ because we do not impose at the estimation level that the HF follow an AR(1) and we allow for different autoregressive parameters for the HF variables.

3.1 Simple tests on seasonality

Here are the interesting null hypotheses about deterministic seasonality on Θ:

1. No deterministic seasonality for the quarterly series y_t

$$H_0^1 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4.$$

2. No deterministic seasonality for the monthly series x_t

$$H_0^2 : \mu_{1,1} = \mu_{1,2} = ... = \mu_{3,4}.$$

3. No deterministic seasonality at all

$$H_0^3 : H_0^1 \cup H_0^2.$$

4. Monthly seasonality but no differences by quarters

$$H_0^4 : \begin{cases} \mu_{1,1} = \mu_{1,2} = \mu_{1,3} = \mu_{1,4} \\ \mu_{2,1} = \mu_{2,2} = \mu_{2,3} = \mu_{2,4} \\ \mu_{3,1} = \mu_{3,2} = \mu_{3,3} = \mu_{3,4} \end{cases}.$$

5. The usual MF-VAR (Ghysels, 2015) where there might exist some monthly seasonality within each quarter that is similar to every quarter

$$H_0^5 : H_0^1 \cup H_0^4.$$

6. Quarterly differences but not monthly ones within each quarter

$$H_0^6 : \begin{cases} \mu_{1,1} = \mu_{2,1} = \mu_{3,1} \\ \mu_{1,2} = \mu_{2,2} = \mu_{3,2} \\ \mu_{1,3} = \mu_{2,3} = \mu_{3,3} \\ \mu_{1,4} = \mu_{2,4} = \mu_{3,4} \end{cases}.$$

3.2 Tests on slopes at different frequencies

Other relevant hypotheses about the deterministic seasonality that exploit the connection of the standard seasonal dummies and their trigonometric representation (see Ghysels and Osborn (2001) Section 2.2.2) are the following:

7. No slope associated to the zero frequency for the quarterly time series y_t

$$H_0^7 : \sum_{i=1}^{4} \alpha_i = 0.$$

Note that H_0^7 could also be expressed as $\delta_0^1 = 0$ in the trigonometric representation (4).

8. No slope associated to the zero frequency for the monthly time series x_t

$$H_0^8 : \begin{cases} \sum_{i=1}^4 \mu_{1,i} = 0 \\ \sum_{i=1}^4 \mu_{2,i} = 0 \\ \sum_{i=1}^4 \mu_{3,i} = 0 \end{cases} .$$

In the case of (4) H_0^8 becomes $\delta_0^2 = \delta_0^3 = \delta_0^4 = 0$.

9. No slope associated to the π frequency for the quarterly time series y_t

$$H_0^9 : -\alpha_1 + \alpha_2 - \alpha_3 + \alpha_4 = 0.$$

In the case of (4) H_0^9 becomes $\delta_1^1 = 0$.

10. No slope associated to the π frequency for the monthly time series x_t

$$H_0^{10} : \begin{cases} -\mu_{1,1} + \mu_{1,2} - \mu_{1,3} + \mu_{1,4} = 0 \\ -\mu_{2,1} + \mu_{2,2} - \mu_{2,3} + \mu_{2,4} = 0 \\ -\mu_{3,1} + \mu_{3,2} - \mu_{3,3} + \mu_{3,4} = 0 \end{cases} .$$

In the case of (4) H_0^{10} becomes $\delta_1^2 = \delta_1^3 = \delta_1^4 = 0$.

11. No slope associated to the $\pi/2$ frequency for the quarterly time series y_t

$$H_0^{11} : \begin{cases} -\alpha_2 + \alpha_4 = 0 \\ \alpha_1 - \alpha_3 = 0 \end{cases} .$$

In the case of (4) H_0^{11} becomes $\delta_{2c}^1 = \delta_{2s}^1 = 0$.

12. No slope associated to the $\pi/2$ frequency for the monthly time series x_t

$$H_0^{12} : \begin{cases} -\mu_{1,2} + \mu_{1,4} = 0 \\ \mu_{1,1} - \mu_{1,3} = 0 \\ -\mu_{2,2} + \mu_{2,4} = 0 \\ \mu_{2,1} - \mu_{2,3} = 0 \\ -\mu_{3,2} + \mu_{3,4} = 0 \\ \mu_{3,1} - \mu_{3,3} = 0 \end{cases} .$$

In the case of (4) H_0^{12} becomes $\delta_{2c}^2 = \delta_{2s}^2 = \delta_{2c}^3 = \delta_{2s}^3 = \delta_{2c}^4 = \delta_{2s}^4 = 0$.

13 Equal zero frequency slopes for y_t and x_t:

$$\begin{aligned} H_0^{13} \quad &: \quad \sum_{i=1}^4 \alpha_i = \sum_{i=1}^4 \mu_{1,i} = \sum_{i=1}^4 \mu_{2,i} = \sum_{i=1}^4 \mu_{3,i} \\ \text{or equivalently} \quad &: \\ H_0^{13} \quad &: \quad \delta_0^1 = \delta_0^2 = \delta_0^3 = \delta_0^4. \end{aligned}$$

14 Equal π frequency slopes for y_t and x_t:

$$\begin{aligned} H_0^{14} &: \sum_{i=1}^{4}(-1)^i\alpha_i = \sum_{i=1}^{4}(-1)^i\mu_{1,i} = \sum_{i=1}^{4}(-1)^i\mu_{2,i} = \sum_{i=1}^{4}(-1)^i\mu_{3,i} \\ \text{or equivalently} &: \\ H_0^{14} &: \delta_1^1 = \delta_1^2 = \delta_1^3 = \delta_1^4. \end{aligned}$$

15 Equal π frequency slopes for y_t and x_t:

$$\begin{aligned} H_0^{15} &: \left\{ \begin{array}{c} -\alpha_2+\alpha_4 = -\mu_{1,2}+\mu_{1,4} = -\mu_{2,2}+\mu_{2,4} = -\mu_{3,2}+\mu_{3,4} \\ \alpha_1-\alpha_3 = \mu_{1,1}-\mu_{1,3} = \mu_{2,1}-\mu_{2,3} = \mu_{3,1}-\mu_{3,3} \end{array} \right. \\ \text{or equivalently} &: \\ H_0^{15} &: \left\{ \begin{array}{l} \delta_{2c}^1 = \delta_{2c}^2 = \delta_{2c}^3 = \delta_{2c}^4 \\ \delta_{2s}^1 = \delta_{2s}^2 = \delta_{2s}^3 = \delta_{2s}^4 \end{array} \right. . \end{aligned}$$

Testing H_0^1 to H_0^{15} is straightforward as the selection matrix R only involves $0\prime s$ and $1\prime s$. For instance in case 1 for the MF-VAR(1), it is well known that

$$\mathrm{R} = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}.$$

3.3 Common seasonal features

Things are a bit more complicated when testing for common seasonal features (denoted H_0^{16}), namely whether there exists a matrix δ such as the linear combination $\delta'\Theta^*$ does not have the seasonal feature. With Θ^* the 4×3 matrix of the dummy coefficients in deviation from the first quarter benchmark, namely

$$\Theta^* = \begin{bmatrix} \alpha_2-\alpha_1 & \alpha_3-\alpha_1 & \alpha_4-\alpha_1 \\ \mu_{1,2}-\mu_{1,1} & \mu_{1,3}-\mu_{1,1} & \mu_{1,4}-\mu_{1,1} \\ \mu_{2,2}-\mu_{2,1} & \mu_{2,3}-\mu_{2,1} & \mu_{2,4}-\mu_{2,1} \\ \mu_{3,2}-\mu_{3,1} & \mu_{3,3}-\mu_{3,1} & \mu_{3,4}-\mu_{3,1} \end{bmatrix}.$$

Indeed testing for reduced rank restrictions in Θ amounts to find some combinations annihilating the whole set of deterministic terms, the intercept included. To test for the rank in Θ^* we estimate the eigenvectors and the eigenvalues corresponding to the symmetric matrix

$$\hat{\Sigma}_{Z_0Z_0}^{-1/2}\hat{\Sigma}_{Z_0D_0}\hat{\Sigma}_{D_0D_0}^{-1}\hat{\Sigma}_{D_0Z_0}\hat{\Sigma}_{Z_0Z_0}^{-1/2} \tag{6}$$

where $\hat{\Sigma}_{ij}$ represents the empirical covariance matrix of processes i and j (namely Z_t and the last three columns of D_t) to be concentrated out by the variables that do not enter in the reduced rank regression, i.e., $(Z_{t-1}...Z_{t-p})$ and an intercept. We then get $\hat{\delta} = \hat{\Sigma}_{Z_0Z_0}^{-1/2}v_1$, v_1 being the $n\times s$ eigenvector matrix corresponding to the smallest eigenvalues λ_s, $s = 2...n$, whose significance can be tested using the likelihood ratio statistics $\zeta_{LR} = -T\sum_{s=2}^{n}\ln(1-\lambda_s)$. Alternatively we can take R $=\hat{\delta}'$ and perform a Wald test.

Concerning the common feature space we must look at the first two eigenvectors normalized for instance such as

$$\hat{\delta}' = \begin{bmatrix} 1 & 0 & \hat{\delta}_{13} & \hat{\delta}_{14} \\ 0 & 1 & \hat{\delta}_{23} & \hat{\delta}_{23} \end{bmatrix},$$

and estimate by FIML a system

$$\begin{bmatrix} 1 & 0 & \hat{\delta}_{13} & \hat{\delta}_{14} \\ 0 & 1 & \hat{\delta}_{23} & \hat{\delta}_{23} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} y_t \\ x_t^{(3)} \\ x_{t-1/3}^{(3)} \\ x_{t-2/3}^{(3)} \end{pmatrix} = \begin{bmatrix} \alpha_1 & 0 & 0 & 0 \\ \mu_{1,1} & 0 & 0 & 0 \\ \mu_{2,1} & \mu_{2,3} - \mu_{2,1} & \mu_{2,3} - \mu_{2,1} & \mu_{2,4} - \mu_{2,1} \\ \mu_{3,1} & \mu_{3,2} - \mu_{3,1} & \mu_{3,3} - \mu_{3,1} & \mu_{3,4} - \mu_{3,1} \end{bmatrix} D_t^* + \hat{A}_1 \begin{pmatrix} y_{t-1} \\ x_{t-1}^{(3)} \\ x_{t-(1+1/3)}^{(3)} \\ x_{t-(1+2/3)}^{(3)} \end{pmatrix} + u_t,$$

with $\hat{A}_1$ collecting short run estimated coefficients. This shows that the asymptotic distribution is a χ^2 with 2 degrees of freedom: we have 6 restrictions in the deterministic matrix and we estimate 4 additional contemporaneous parameters. Similarly it is a $\chi^2(6)$ when $s = 3$ with

$$\hat{\delta}' = \begin{bmatrix} 1 & 0 & 0 & \hat{\delta}_{14} \\ 0 & 1 & 0 & \hat{\delta}_{24} \\ 0 & 0 & 1 & \hat{\delta}_{34} \end{bmatrix}.$$

4 Application

We consider the quarterly employment from the Balearic Islands (denoted *emp*) and a monthly airport passenger arrivals on the Balearic islands (*arriv*)[4]. The period ranges from 2002Q1 to 2015Q3, i.e., 55 quarterly observations. It is rather clear that there is a unit root at the zero frequency in $\ln(emp_t)$ whereas for the monthly variable we hesitate between three transformations. Consequently we estimate a MF-VAR for the following systems $Z_t^1 = (\Delta \ln(emp_t), \ln(arriv_t^{(3)}), \ln(arriv_{t-1/3}^{(3)}), \ln(arriv_{t-2/3}^{(3)}))$, $Z_t^2 = (\Delta \ln(emp_t), (1-L)\ln(arriv_t^{(3)}), (1-L)\ln(arriv_{t-1/3}^{(3)}), (1-L)\ln(arriv_{t-2/3}^{(3)}))$ and $Z_t^3 = (\Delta \ln(emp_t), \Delta_{\frac{\pi}{6}}(1-L)\ln(arriv_t^{(3)}), \Delta_{\frac{\pi}{6}}(1-L)\ln(arriv_{t-1/3}^{(3)}), \Delta_{\frac{\pi}{6}}(1-L)\ln(arriv_{t-2/3}^{(3)}))$, with $\Delta_{\frac{\pi}{6}} = \left(1 - 2\cos\left(\frac{\pi}{6}\right)L + L^2\right)$.[5] Note that we have to make the difference between the quarterly growth rate of the employment $\Delta \ln(emp_t)$ and the monthly growth rate of arrivals $(1-L)\ln(arriv_t^{(3)})$.

Using information criteria a MF-VARs of order 4 captures adequately the dynamics of the systems. Remains $T = 50$ observations. This specification do not show any sign of misspecification: we do reject the null of normality, heteroskedasticity and no autocorrelation using the multivariate versions of the tests. $R^2 = 0.97$ for the first equation and about for the last three. As an example we obtain for for Z_t^1

[4] The data is obtained from the web page of the IBESTAT (Regional Statistical office of the Balearic Islands), in the case of the quarterly employment data the source is the EPA (Encuesta de Población Activa) of the INE (National Statistical office of Spain). And in the case of the passenger arrivals the source is the IBESTAT based on the data provided by AENA.

[5] In order to check the presence of unit root in $\ln(emp_t)$ and $\ln(arriv_t)$ we use the information provided by the HEGY tests with OLS and GLS detrending where the order of augmentation it is based on the MAIC criteria (see Rodrigues and Taylor (2007), del Barrio Castro, Osborn and Taylor (2016)).

Table 1: Testing for deterministics

		Z_t^1		Z_t^2		Z_t^3	
Test	dist.	ξ_W	p-val	ξ_W	p-val	ξ_W	p-val
H_0^1	$\chi^2_{(3)}$	17.66	0.000	24.37	0.000	12.11	0.007
H_0^2	$\chi^2_{(11)}$	96.88	0.000	99.31	0.000	71.09	0.000
H_0^3	$\chi^2_{(14)}$	99.90	0.000	101.31	0.000	94.97	0.000
H_0^4	$\chi^2_{(9)}$	95.80	0.000	90.81	0.000	58.39	0.000
H_0^5	$\chi^2_{(12)}$	98.82	0.000	92.81	0.000	82.27	0.000
H_0^6	$\chi^2_{(8)}$	29.89	0.000	49.91	0.000	51.77	0.000
H_0^7	$\chi^2_{(1)}$	0.86	0.352	1.73	0.187	1.35	0.245
H_0^8	$\chi^2_{(3)}$	11.53	0.009	10.16	0.017	11.46	0.009
H_0^9	$\chi^2_{(1)}$	8.05	0.004	13.85	0.000	9.22	0.002
H_0^{10}	$\chi^2_{(3)}$	56.42	0.000	54.55	0.000	38.40	0.000
H_0^{11}	$\chi^2_{(2)}$	5.96	0.051	7.32	0.025	2.62	0.269
H_0^{12}	$\chi^2_{(6)}$	29.30	0.000	24.53	0.000	20.67	0.002
H_0^{13}	$\chi^2_{(3)}$	11.44	0.010	10.93	0.012	14.06	0.003
H_0^{14}	$\chi^2_{(6)}$	26.96	0.000	18.02	0.006	28.10	0.000
H_0^{15}	$\chi^2_{(3)}$	50.35	0.000	18.25	0.000	27.15	0.000
		ζ_{LR}	p-val	ζ_{LR}	p-val	ζ_{LR}	p-val
$H_{0,s=2}^{16}$	$\chi^2_{(2)}$	2.39	0.302	1.56	0.456	5.80	0.054
$H_{0,s=3}^{16}$	$\chi^2_{(6)}$	22.97	0.000	20.00	0.003	33.10	0.000
$H_{0,s=4}^{16}$	$\chi^2_{(12)}$	67.12	0.000	63.33	0.000	62.38	0.000

the deterministic estimated coefficient matrix with standard errors within brackets.

$$\hat{\Theta} = \begin{pmatrix} \underset{(0.610)}{0.711} & \underset{(0.617)}{0.457} & \underset{(0.602)}{0.547} & \underset{(0.609)}{0.546} \\ \underset{(1.637)}{3.360} & \underset{(1.655)}{2.694} & \underset{(1.616)}{3.088} & \underset{(1.636)}{2.832} \\ \underset{(1.080)}{3.980} & \underset{(1.092)}{3.202} & \underset{(1.066)}{3.767} & \underset{(1.079)}{3.354} \\ \underset{(1.926)}{3.439} & \underset{(1.947)}{3.022} & \underset{(1.901)}{4.094} & \underset{(1.924)}{3.675} \end{pmatrix}.$$

Table 1 summarizes the results for the three systems. It emerges that the only restrictions on deterministic seasonality that are convincing are H_0^7 and H_0^{16}. The only relevant common feature combination is $\hat{\delta}_{23} = 0.86$, the combination between $\ln(arriv_t^{(3)})$ and $\ln(arriv_{t-1/3}^{(3)})$. Note that for tests statistics associated with the hypothesis H_0^7 to H_0^{15} we obviously get the same results when computing them on (3) or on (4).

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