Group-contests with endogenous claims[☆]

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Abstract

Before group members individually decide their efforts in a contest to set a policy, groups are allowed to make some concessions to their opponent by choosing a less controversial policy to lobby for. When valuations over the set of policies follow a linear function, we show that concessions are never profitable when the contest success function is homogeneous of degree zero but they are when it is of difference form. Surprisingly, concessions might be detrimental for the members of the group that does not make them. Comparing this situation with another where efforts are decided collectively at a group level allows us to remark the effect of positive externalities of effort as the key cause of this damage.

Key words: Group contests; endogenous claims; conflict; rent-seeking
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1. Introduction

Contestants can often commit to altering their demands before making costly contributions that improve the chances to achieve their claims in a struggle. For example, lobbyists may change the policy proposal they will lobby for. In a conflict between an industry and an environmentalist interest group the parts may revise the pollution standards they will defend in the future. In a labor dispute on the minimum wage, the workers union and the capital owners may also adjust their claims before the confrontation. This strategic environment was analyzed by Epstein and Nitzan (2004) who show that in equilibrium contestants would strategically moderate their claims because this originates a positive effect by reducing

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the aggressiveness of the opposing group that dominates the negative effect on their own payoff from winning the conflict.[1]

As Epstein and Nitzan (2004) recognize *"what is needed for this result is that the rent-seeker's marginal cost from a moderation of his position is zero, because he starts his moderation from an ideal point at which the first-order condition holds"*. This study aims to reconsider strategic restraint in a contest when this marginal cost is different from zero, as if contestants would prefer more extreme positions if this were possible.[2] Moreover, we extent Epstein and Nitzan (2004) analysis by allowing the individuals of the groups in the confrontation to choose their effort levels independently. These efforts contribute to the aggregate group effort which can increase the probability that the policy they propose is implemented. This policy is a public good. This connects the present paper to a rapidly growing literature on group-contests.[3] This branch of the literature mainly focuses on the free-rider problem that normally arises when individuals must decide how much to contribute to a collective cause; this is a classical issue in Economics analyzed by Olson (1965), Bergstrom et al (1986), Gradstein et al. (1994), Varian (1994), or Vicary (1999), among others.

In our model two groups of (possibly) different size are involved in a contest to choose the policy that will be implemented. The group's effort is determined by an additively linear impact function, as in most of the papers in footnote 3 except Epstein and Mealem (2009), Lee (2012), Chowdhury et al (2013), and Kolmar and Rommeswinkel (2013). All members of a group are assumed to be identical with respect to their valuation of the prize and the cost of effort, as in Barbieri et al (2014) or Topolyan (2014). Unlike Epstein and Nitzan (2004), this valuation follows a linear function over all possible policies. Groups have opposite interests, so the most preferred policy for the members of a group is the least preferred for the members of the other. Moreover, individuals face strictly convex costs of effort.[4] Finally, this study considers alternative contest success functions (henceforth, CSFs) in order to analyze the impact of this functional form specification on the strategic restraint. In the literature there are two main families of CSFs: Those functions homogeneous of degree zero, *e.g.* Tullock (1980), and the difference-form CSFs, *e.g.* Hirshleifer (1989), Baik (1998), or Che and Gale (2000).[5] There is a branch

[1]See Epstein and Nitzan (2007).

[2]Münster (2006) also presents a reconsideration of this strategic restraint. His analysis focuses on the effects of a perfectly discriminating contest success function on the results and shows that, unlike Epstein and Nitzan (2004), moderation is extreme in this case, that is, policy proposals will coincide.

[3]See Baik et al (2001), Baik (2008), Epstein and Mealem (2009), Lee (2012), Chowdhury et al (2013), Kolmar and Rommeswinkel (2013), Barbieri et al (2014), or Topolyan (2014).

[4]Epstein and Mealem (2009) show that a model with linear impact functions and strictly convex costs is isomorphic to a model with decreasing returns to effort and linear costs.

[5]Beviá and Corchón (2015) present a CSF that is of difference-form and homogeneous of degree

of the contest literature which has provided a variety of foundations to the most frequently employed CSFs.[6] The results of this study are structured into two parts: First, we obtain results that hold for any CSF homogeneous of degree zero and, second, we focus on the specific CSF proposed by Che and Gale (2000) to show how a difference-form CSF shape the results.

We show that when the CSF is homogeneous of degree zero concessions are never profitable either if efforts are decided individually or collectively within each group. This illustrates that the assumptions of Epstein and Nitzan (2004) and Münster (2006) are key for strategic restraint. However, under the difference-form CSF the small group would be willing to make concessions under certain conditions when efforts are decided either individually or collectively. This benefits all players in the latter case but, surprisingly, damages the big-group members in the former case. Intuitively, a concession made by the small group has two opposite effects on the members of the big one: On the one hand, it reduces the effort of all individuals in the same quantity, so the winning probability of the big group is reduced. On the other hand, this concession increases the big-group members' payoff from losing the dispute. When efforts are decided individually the effect of positive externalities of effort adds to the first one and tips the balance against the interest of the big-group members, who do not make any concession.

The paper is organized as follows: Section 2 presents the model. The results are presented in section 3. Section 4 presents the results when agents are allowed to coordinate their choice of effort at a group level, and section 5 concludes.

2. The model

Consider two groups N and M consisting of n and m agents, respectively, with $n \geq m \geq 1$. These groups would compete in a contest for a public good, say a policy. The policy space is $X = [0, 1]$ and the preferences of agents are given by $u_i(x) = 1 - x$ for all $i \in N$ and $u_j(x) = x$ for all $j \in M$, for all $x \in X$.

Prior to the contest, the groups simultaneously select their target policy; that is, the policy they will implement in case of winning the subsequent contest. Let x and y denote the target policies of groups N and M, respectively. Once x and y have been settled, agents in N and M simultaneously choose effort in order to affect the probability of winning. Let a_i and b_j denote individual efforts of agents $i \in N$ and $j \in M$, respectively; and define $A = \sum_{i \in N} a_i$ and $B = \sum_{i \in M} b_i$ as the group efforts. These aggregate effort levels will determine the winning probability of each

zero.

[6]See Skaperdas (1996) and Münster (2009) for axiomatizations of the most relevant CSF for individual and group-contests, respectively. A part from axiomatizations, CSFs have been characterized from different approaches: stochastic, optimally-designed and micro-founded (see Jia, Skaperdas and Vaidya, 2013).

group, denoted by $p_N(A,B)$ and $p_M(A,B) = 1 - p_N(A,B)$. Efforts are costly, and we will consider strictly convex homogeneous (of degree k) cost functions; that is $c(z) = cz^k$ with $k > 1$. Accordingly, the preferences of agents are given by

$$\begin{aligned} u_i(a_i; A_{-i}, B, x, y) &= p_N(A_{-i} + a_i, B)(y - x) + 1 - y - ca_i^k, \forall i \in N \text{ and} \quad (1)\\ u_j(b_i; A, B_{-j}, x, y) &= p_M(A, B_{-j} + b_j)(y - x) + x - cb_j^k \text{ for all } j \in M, \quad (2) \end{aligned}$$

where $A_{-i} = \sum_{k \in N - \{i\}} a_k$ and $B_{-j} = \sum_{k \in M - \{j\}} b_k$.

Regarding how aggregate efforts affect the winning probabilities, we will consider two different specifications of the success function:

H Homogeneous (of degree zero) CSF.

L Linear difference-form CSF: $p_N(A,B) = 1/2 + s(A - B)$, for appropriate $s > 0$.

Remark 1. *In general, probabilities in the linear difference-form CSF are such that*

$$p_N(A,B) = \max\{0, \min[1/2 + s(A - B), 1]\}.$$

For some values of s, a pure strategy equilibrium fails to exist. Che and Gale (2000) characterize the set of mixed strategy equilibrium when $n = m = 1$ and costs are linear. To our knowledge there is no characterization of such equilibria in our environment. In this paper, we will focus on pure strategies equilibria. Hence, restricting the parameter set would be required to guarantee existence.

Although the target policy choice is not explicitly modeled, as the group members are homogeneous this choice might be interpreted as if it was made by one agent in each group, say $i \in N$ and $j \in M$. Adding this, the previous model defines a two-stage strategic game with complete information. A pure strategy of agent i would consist on a pair (x, a_i) where $x \in X$ and $a_i : X \times X \to \mathbb{R}_+$. Similarly, (y, b_j) denotes a strategy of player j. The strategy of any other k different from i or j would only consist on her effort. The equilibrium concept considered is subgame perfection. That is, a strategy profile $\left((x^*, a_i^*), \{a_k^*\}_{k \in N - \{i\}}, (y^*, b_j^*), \{b_k^*\}_{k \in M - \{j\}}\right)$ is a Subgame Perfect Equilibrium (SPE) whenever

1. $a_k^*(x,y) \in \arg\max_{a_k} u_k\left(a_k; A_{-k}^*, B^*, x, y\right)$ *and* $b_k^* \in \arg\max_{b_k} u_k\left(b_k; A_{-k}^*, B^*, x, y\right)$
2. $x^* \in \arg\max_x u_i\left(a_i^*, A_{-i}^*, B^*, x, y^*\right)$ *and* $y^* \in \arg\max_y u_i\left(b_i; A_{-i}^*, B^*, x^*, y\right)$.

3. The results

For any pair of policy proposals $x, y \in X$ with $x < y$,[7] any agent $i \in N$ and $j \in M$ choose a_i and b_j to maximize (1) and (2), respectively. Hence, any interior

[7]Obviously, $x \geq y$ would imply that the agents would prefer to lose the contest and therefore efforts will be zero.

solution $(a^*, b^* > 0)$ satisfies the first order conditions

$$\frac{\partial p_N(A_{-i}+a_i,B)}{\partial a_i}D - kc\left(a_i\right)^{k-1} = 0, \text{ for any } i \in N \text{ and} \tag{3a}$$

$$\frac{\partial p_M(A,B_{-j}+b_j)}{\partial b_j}D - kc\left(b_j\right)^{k-1} = 0, \text{ for any } j \in M. \tag{3b}$$

where $D = y - x$ is the endogenous prize premium. It is easy to see that the optimal choices within each group must be symmetric; that is, $a_i^* = a^*$ for any $i \in N$ and $b_j^* = b^*$ for any $j \in M$. Thus, $A^* = na^*$ and $B^* = mb^*$. Moreover, it is also immediate that $\partial a^*/\partial D > 0$ and $\partial b^*/\partial D > 0$; that is, efforts decrease as D is reduced.

Anticipating the bidding equilibrium for any pair (x, y), in the first stage of the game the groups N and M choose their target policies x and y, respectively. Due to the linearity of utilities, such choices can be summarized through $D = x - y$. That is, groups (or their representatives) would choose D to maximize:

$$v_i\left(D\right) = p_N(A^*,B^*)D + 1 - y - c\left(\frac{A^*}{n}\right)^k, \text{ and} \tag{3c}$$

$$v_j\left(D\right) = p_M(A^*,B^*)D + x - c\left(\frac{B^*}{m}\right)^k, \text{ respectively.} \tag{3d}$$

At this stage, agents/groups must account for three effects from moderating their target policies. First, a negative effect because their utility in case of winning the contest is reduced; second, a positive effect from the reduction of the effort levels of all agents; and third, these changes in the effort levels would affect the winning probability. The size of this impact on the winning probability will depend on the specific CSF and it results to be crucial for the existence of strategic restraint.

Note that FOCs (3a and 3b) imply

$$a_i^{k-1} = \frac{D}{kc}\frac{\partial p_N(A_{-i}+a_i,B)}{\partial a_i}, \text{ for any } i \in N \text{ and}$$

$$b_j^{k-1} = \frac{D}{kc}\frac{\partial p_M(A,B_{-j}+b_j)}{\partial b_j}, \text{ for any } j \in M.$$

In consequence,

$$\frac{b_j}{a_i} = \left(\frac{\partial p_M(A,B_{-j}+b_j)}{\partial b_j} / \frac{\partial p_N(A_{-i}+a_i,B)}{\partial a_i}\right)^{\frac{1}{k-1}};$$

meaning that, in equilibrium, the ratio B/A does depend on neither D nor c.

Since a homogeneous CSF involves $p_N(A,B) = p_N\left(1, \frac{B}{A}\right)$, we obtain that neither D nor c affect the equilibrium winning probabilities. As a consequence, conflict is never reduced, as the next result shows.

Proposition 2. *Under a CSF homogeneous of degree zero, moderating the target policy is never profitable.*

Proof. Eq. (3a) implies that in equilibrium

$$c\left(\frac{A^*}{n}\right)^k = \frac{D}{nk}\frac{\partial p_N(1, B^*/A^*)}{\partial a_i}.$$

Thus, (3c) can be written as

$$\begin{aligned} v_i(D) &= p_N\left(1, \frac{B^*}{A^*}\right) D + 1 - y - \frac{D}{nk}\frac{\partial p_N(1, B^*/A^*)}{\partial a_i} \\ &= \left[p\left(1, \frac{B^*}{A^*}\right) - \frac{1}{nk}\frac{\partial p_N(1, B^*/A^*)}{\partial a_i}\right] D + 1 - y. \end{aligned}$$

A positive effort is plausible in equilibrium whenever $v_i(D) > 1 - y$. Thus, the following must hold

$$p\left(1, \frac{B^*}{A^*}\right) > \frac{1}{nk}\frac{\partial p_N(1, B^*/A^*)}{\partial a_i}$$

which implies that $\frac{\partial v_i(D)}{\partial D} > 0$. This means that expected utility increases with the level of conflict. So, no concessions are made. A similar argument would prove $\frac{\partial v_{iM}(D)}{\partial D} > 0$. □

This result is not directly comparable to Epstein and Nitzan (2004) or Münster (2006) because agents choose their effort individually and independently; so our result is affected by the positive externalities of effort on the rest of group members. However, this result contrasts with Epstein and Nitzan (2004) in which a group is benefited by a marginal moderation of its target policy because *"this moderation has a first-order effect on the opponent's incentive to engage in rent-seeking efforts and, consequently, on the winning probability"* and *"reduces his gain from winning only by a second-order effect"* (Epstein and Nitzan, 2004). In line with this intuition, these two additional effects must be taken into account to interpret our result. On the one hand, when a group moderates its policy proposal its payoff from winning the contest is decreased. On the other hand, such a moderation cause an indirect effect on the incentives of all agents (the opposite group's members but also the members of the own group) to lobby for their own target-policy. In particular, these incentives are reduced, as the payoff difference between winning or losing the contest has decreased.[8] So, all agents will exert less effort as a consequence of a

[8]Note that $\frac{\partial a^*}{\partial D}, \frac{\partial b^*}{\partial D} > 0$.

target-policy moderation. However, since $p_l(A,B) = p_l\left(1, \frac{B}{A}\right)$ for $l = M, N$, and $\frac{b_j^*}{a_i^*}$ does not depend on D, it can be concluded that $\frac{\partial P_N^*}{\partial D} = \frac{\partial P_M^*}{\partial D} = 0$, so a moderation would not affect the equilibrium probabilities of winning the contest. The previous proposition shows that the cost-effort saving does not compensate the utility loss due to a target-policy moderation for none of the two groups. In fact, notice that $\frac{\partial v_i(D)}{\partial D}, \frac{\partial v_j(D)}{\partial D} > 0$ so both groups would like to choose more controversial target policies.

Example 3. *For Relative Difference CSF*[9] *($p_N(A,B) = \alpha + \beta\frac{A-sB}{A+B}$) and $c(z) = \frac{c}{2}z^2$ the equilibrium efforts and probabilities are as follows*

$$a^* = \frac{(m/n)^{1/4}\left(\beta(1+s)D/c\right)^{1/2}}{\sqrt{n}+\sqrt{m}}, b^* = \frac{(n/m)^{1/4}\left(\beta(1+s)D/c\right)^{1/2}}{\sqrt{n}+\sqrt{m}}$$

$$p_N(A^*, B^*) = \alpha - s\beta + \beta(1+s)\frac{1}{1+\sqrt{m/n}}$$

Substituting into (3c and 3d) it can be checked that $\frac{\partial v_i(D)}{\partial D}, \frac{\partial v_j(D)}{\partial D} > 0$, thus no group has incentives to make concessions.

Considering a non-homogeneous CSF will alter this result. Under the linear difference-form CSF specified above, conditions (3a and 3b) imply that any interior solution must be

$$a^* = b^* = \left(\frac{sD}{kc}\right)^{\frac{1}{k-1}} \text{ for any } i \in N \text{ and } j \in M,$$

and therefore,

$$p_N(A^*, B^*) = \frac{1}{2} + \left(\frac{s^k D}{kc}\right)^{\frac{1}{k-1}}(n-m).$$

Since $\partial a^*/\partial D = \partial b^*/\partial D > 0$, all players reduce their individual effort in the same amount when D decreases. As a consequence, $\partial P_N/\partial D > 0 > \partial P_M/\partial D$ whenever $n > m$. Therefore, the equilibrium winning probability of M is increased when some group moderates its target policy. This increase can be sufficiently important to overcome the direct payoff decrease due to a target-policy moderation. This will happen when the winning probability is sufficiently responsive to the differences in effort. Consequently, in these cases group M will have incentives to moderate its target policy when s is sufficiently high, as showed in the next proposition.[10]

[9] This CSF was introduced by Beviá and Corchón (2015). As they show, this is a twin CSF to $p_N(A,B) = \gamma + \delta\frac{A}{A+B}$ if $\gamma = \alpha - s\beta$ and $\delta = \beta(1+s)$. Notice that when $\alpha = s = 0$ and $\beta = 1$ we obtain the Tullock CSF in ratio form.

[10] Moreover, s must be sufficiently low to get a pure strategy SPE as the proof shows.

Proposition 4. *Under the linear difference-form CSF specified above, there exist an interval (s_l, s_h) such that, for all $s \in (s_l, s_h)$ only the small group moderates its target policy when $n > m$ and both groups might do it when $n = m$.*

Proof. Notice that $p_N(A^*, B^*) \leq 1$ for any $D \in [0,1]$, whenever

$$s \leq \left(\frac{1}{2(n-m)}\right)^{\frac{k-1}{k}} (kc)^{\frac{1}{k}}. \tag{C1}$$

In these cases, according to (3c and 3d) utilities can be written as

$$\begin{aligned} v_i(D) &= \left[\frac{1}{2} + \left(\frac{s^k D}{kc}\right)^{\frac{1}{k-1}} (n-m)\right] D + 1 - y - c\left(\frac{sD}{kc}\right)^{\frac{k}{k-1}} \\ v_j(D) &= \left[\frac{1}{2} - \left(\frac{s^k D}{kc}\right)^{\frac{1}{k-1}} (n-m)\right] D + x - c\left(\frac{sD}{kc}\right)^{\frac{k}{k-1}} \end{aligned}$$

where $v_i(D) > 1 - y$ for any D, when $n > m$ and $k > 1$. Moreover, $v_j(D) > x$ whenever

$$s < s_h \equiv \left(\frac{k}{2(k(n-m)+1)}\right)^{\frac{k-1}{k}} (kc)^{\frac{1}{k}}. \tag{C2}$$

Thus, if $n > m$ agents in N will always exert a positive effort whereas the members of M will only exert some positive effort in the contest when condition C2 is satisfied. When $n = m$, both $v_i(D) > 1 - y$ and $v_j(D) > x$ whenever $s < s_h$. Hence, in order to guarantee a positive effort condition C2 is required. It is immediate that this condition implies C1.

Now consider the first stage of the game. Taking partial derivatives of the previous utilities, we obtain

$$\begin{aligned} \frac{\partial v_i(D)}{\partial D} &= \frac{1}{2} + \left(\frac{s^k D}{kc}\right)^{\frac{1}{k-1}} \left(\frac{k(n-m)-1}{k-1}\right) \\ \frac{\partial v_j(D)}{\partial D} &= \frac{1}{2} - \left(\frac{s^k D}{kc}\right)^{\frac{1}{k-1}} \left(\frac{k(n-m)+1}{k-1}\right). \end{aligned}$$

Suppose first that $n > m$. In these cases, since $\frac{\partial v_i(D)}{\partial D} > 0$ for any $D \in [0,1]$ and $k > 1$, group N would never moderate its target policy. Hence, $x = 0$. However, group M would strategically claim $y = D$, satisfying

$$D = \left(\frac{k-1}{2(k(n-m)+1)}\right)^{k-1} \frac{kc}{s^k}. \tag{4}$$

Hence, $D < 1$ whenever $\frac{\partial v_j(D)}{\partial D} |_{D=1} < 0$; i.e., when the following condition is satisfied

$$s > s_l \equiv \left(\frac{k-1}{2(k(n-m)+1)}\right)^{\frac{k-1}{k}} (kc)^{\frac{1}{k}}. \tag{C3}$$

Thus, the equilibrium of this two-stage game yields

$$x^* = 0 \text{ and } y^* = \min\left\{\left(\tfrac{k-1}{2(k(n-m)+1)}\right)^{k-1} \tfrac{kc}{s^k}, 1\right\}.$$

Similarly, when $n = m$, we obtain that both groups N and M claim

$$D = \left(\frac{k-1}{2}\right)^{k-1} \frac{kc}{s^k}$$

whenever $s > s_l$. Hence, in this case, there are multiple combinations of x and y that satisfy this condition. □

An interior solution requires $s < s_h$. If $s > s_h$, the combination of aggregate efforts (A^*, B^*) cannot be sustained in equilibrium because $1/2 + s(A^* - B^*) > 1$, so the members of the small group would benefit from reducing their effort without altering their winning probability. As advanced above, only when $s > s_l$ the group M has incentives to moderate its target policy. An alternative explanation can be formulated in terms of group sizes: Since all individuals' efforts respond equally to changes in D, only when N is sufficiently bigger than M the increase of the winning probability of M due to a target-policy moderation overcomes the negative payoff consequences of this moderation.

It is worth to note that when $n > m$ and $s \in (s_l, s_h)$, agents in N would like to select a more controversial target policy, but this is not possible because $x^* = 0$. Therefore, the prize premium is suboptimal for these agents. Hence, concessions by the opponents might have negative consequences on the payoffs of the members of the largest group. Regarding the payoff consequences of concessions, several forces come into play: First, it is obvious that selecting a more moderate policy should benefit the opposite group when the policy is implemented. Second, the equilibrium efforts are lower under concessions. Finally, and as a consequence of the previous effect, the equilibrium probabilities of winning the contest are affected by the prize premium D. At this point, it is important to note that the (non-internalized) positive externalities of individuals' effort on the rest of group members will also play a role. The next proposition summarizes the result of the interaction among all these forces.

Proposition 5. *With a linear CSF, if*

$$n - m > 2\left(\left(\frac{k}{k-1}\right)^{k-1} - 1\right) + \frac{1}{k}$$

there exists a threshold $\overline{s} \in (s_l, s_h)$ such that members of N are damaged by the target-policy moderation of the other group when $s \in (\overline{s}, s_h)$.

Proof. When $s > s_l$, there is a target-policy moderation since

$$y^* = D = \left(\frac{k-1}{2\left(k\left(n-m\right)+1\right)}\right)^{k-1}\frac{kc}{s^k} < 1.$$

Then,

$$\begin{aligned} v_i\left(D\right) - v_i\left(1\right) &= \\ P\left(s\right) &\equiv -2\left(n-m-1/k\right) + \left(\frac{ck}{s^k}\right)^{\frac{1}{k-1}} \\ & \quad -\left(\frac{ck}{s^k}\right)^{\frac{k}{k-1}}\left(1-2\left(n-m-1/k\right)\frac{k-1}{2\left(k\left(n-m\right)+1\right)}\right) \\ & \quad \left(\frac{k-1}{2\left(k\left(n-m\right)+1\right)}\right)^{k-1}. \end{aligned}$$

Notice that

$$\begin{aligned} \frac{\partial P\left(s\right)}{\partial s} &= -\left(ck\right)^{\frac{1}{k-1}}\frac{k}{k-1}s^{-\frac{2k-1}{k-1}} \\ & \quad +\left(1-2\left(n-m-1/k\right)\frac{k-1}{2\left(k\left(n-m\right)+1\right)}\right)\left(\frac{k-1}{2\left(k\left(n-m\right)+1\right)}\right)^{k-1} \\ & \quad \left(ck\right)^{\frac{k}{k-1}}\frac{k^2}{k-1}s^{-\frac{k^2+k-1}{k-1}}. \end{aligned}$$

Hence, $\frac{\partial P(s)}{\partial s} = 0$ holds for a unique value of s. Moreover, by definition $P\left(s_l\right) = 0$ because $D = 1$ when $s = s_l$. After some algebra it can be checked that $\frac{\partial P(s)}{\partial s} \mid_{s=s_l} > 0$. Finally, $P\left(s_h\right) < 0$ only when

$$n - m > 2\left(\left(\frac{k}{k-1}\right)^{k-1} - 1\right) + \frac{1}{k}.$$

The statement of the proposition follows consequently. □

Surprisingly, the concessions of group M might harm the members of N. When the relative size of N is sufficiently large and the winning probabilities are sufficiently sensitive to effort differences, the reduction of the winning probability of group N obtained from this moderation. This is due to the inefficient (individual) choices of efforts within each group, which is more pronounced when the size of the group is large.[11]

The following example illustrates the effects of concessions on the payoffs of the members of both groups.

[11]The payoff consequences of this effect will become clear in the next session where agents coordinate efforts.

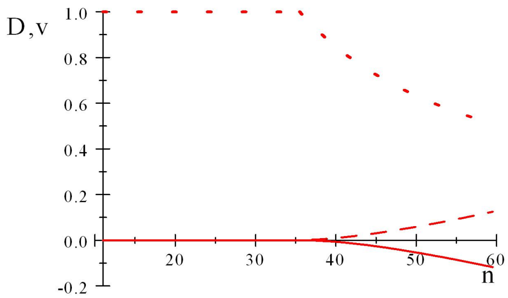

Figure 1: D (dots), $v_i(D) - v_i(1)$ (solid line) and $v_j(D) - v_j(1)$ (dashed line).

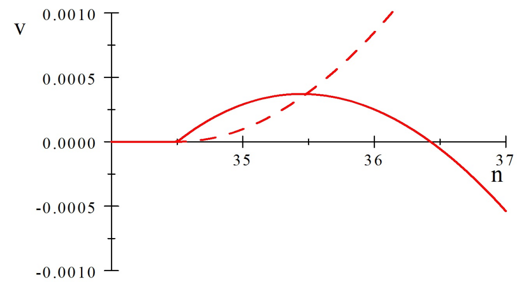

Figure 2: Zoom on the previous figure.

Example 6. *Let $c = 1/2$, $s = 0.1$, $k = 2$ and $m = 10$. Substituting into (4) we obtain $y^* = y(n) = D = \frac{100}{4(n-10)+2}$. Figure 1 depicts $D(n)$, $v_i(D) - v_i(1)$ (solid line), and $v_j(D) - v_j(1)$ (dashed line) in pure-strategies equilibria, where all agents exert a positive effort; these equilibria exist whenever $s < s_h$ or equivalently when $n < m + \left(\frac{kc}{s^k}\right)^{\frac{1}{k-1}} \frac{1}{2} - \frac{1}{k} = 59.5$. Notice that group M will make concessions whenever $n > \left(\left(\frac{kc}{s^k}\right)^{\frac{1}{k-1}}(k-1) - 2\right)\frac{1}{2k} + m = 34.5$ (that corresponds to the condition $s > s_l$) and this will damage the big group as long as $n > 36.428$, as illustrated by Figure 2.*

4. Coordinate bidding

Consider now that the agents in each group decide their aggregate effort collectively. Under this new setting (which obviously requires individual efforts being contractible), the inefficiency arising from the positive externalities of individual efforts within each group is eliminated. Hence, a comparison with the results of the previous section will allow us to analyze the role of these externalities on both the strategic restraint and their consequences on agents' payoffs.

In the second stage of the game, agents will choose $A = na$ and B to maximize

$$U_N = n\left[p_N(A,B)D + 1 - y - c\,(A/n)^k\right] \text{ and}$$
$$U_M = m\left[p_M(A,B)D + x - c\,(B/n)^k\right], \text{ respectively.}$$

The FOCs of this problem are

$$n\left(\frac{\partial p_N(A,B)}{\partial A}D - \frac{kc}{n}\left(\frac{A}{n}\right)^{k-1}\right) = 0, \text{ for group } N \text{ and} \quad (5)$$
$$m\left(\frac{\partial p_M(A,B)}{\partial B}D - \frac{kc}{m}\left(\frac{B}{m}\right)^{k-1}\right) = 0, \text{ for group } M. \quad (6)$$

Let A^* and B^* denote the solution of this system of equations.

Then, in the first stage of the game, members of groups N and M choose D to maximize:

$$V_N(D) = n\left[p_N(A^*,B^*)D + 1 - y - c\left(\frac{A^*}{n}\right)^k\right], \text{ and} \quad (7)$$

$$V_M(D) = m\left[p_M(A^*,B^*)D + x - c\left(\frac{B^*}{m}\right)^k\right], \text{ respectively.} \quad (8)$$

Under a homogeneous CSF, results are qualitatively similar to those obtained in the previous section, as the next proposition states.

Proposition 7. *Under a CSF homogeneous of degree zero, moderating the target policy is never profitable when efforts are decided collectively by each group.*

Proof. We follow the same steps as in the non-coordination case. Notice that FOCs (5 and 6) imply that

$$A^{k-1} = \frac{Dn^k}{kc}\frac{\partial p_N(A,B)}{\partial A}, \text{ for group } N \text{ and}$$

$$B^{k-1} = \frac{Dm^k}{kc}\frac{\partial p_M(A,B)}{\partial B}, \text{ for group } M.$$

In consequence,

$$\frac{B}{A} = \left(\frac{m^k}{n^k}\frac{\partial p_M(A,B)}{\partial B}/\frac{\partial p_N(A,B)}{\partial A}\right)^{\frac{1}{k-1}}.$$

Therefore, in equilibrium B/A does not depend on D and c. Applying the properties of the functions homogeneous of degree zero introduced previously, we obtain

$$c\left(\frac{A^*}{n}\right)^k = \frac{D}{k}\frac{\partial p_N(1,B/A)}{\partial A}.$$

Thus, (7) can be written as

$$\begin{aligned} V_N(D) &= n\left[p_N(1,\frac{B^*}{A^*})D + 1 - y - \frac{D}{k}\frac{\partial p_N(1,B^*/A^*)}{\partial A}\right] \\ &= n\left[\left[p\left(1,\frac{B^*}{A^*}\right) - \frac{1}{k}\frac{\partial p_N(1,B^*/A^*)}{\partial A}\right]D + 1 - y\right] \end{aligned}$$

A positive aggregate effort of group N is plausible in equilibrium whenever $V_N(D) > n(1-y)$, hence the following must hold

$$p\left(1,\frac{B^*}{A^*}\right) > \frac{1}{k}\frac{\partial p_N(1,B^*/A^*)}{\partial A}$$

which implies that $\frac{\partial V_N(D)}{\partial D} > 0$. Similarly, $\frac{\partial V_M(D)}{\partial D} > 0$. □

Despite efforts are decided collectively, as in Epstein and Nitzan (2004) or Münster (2006), there is no strategic restraint in equilibrium. With linear preferences, the negative effect of a target-policy moderation on the gain from winning does no longer have a second-order effect as in their setting, but a first-order one. Moreover, a CSF homogeneous of degree zero involves that $\frac{\partial P_N^*}{\partial D} = \frac{\partial P_M^*}{\partial D} = 0$ (as argued in the previous section), so the first-order effect of moderation on winning probabilities remarked by Epstein and Nitzan (2004) is completely mitigated. Consequently, a target-policy moderation is not profitable.

In case of the linear CSF specified above, the result is also analogous to the result in the previous section. They only differ in the thresholds on s that delimit the existence of equilibria in pure strategies and the existence of concessions in equilibrium.

Proposition 8. *With a linear CSF, coordinate bidding, and $s \in (\widehat{s}_l, \widehat{s}_h)$, only the small group moderates its target policy when $n > m$ and both groups might do it when $n = m$.*

Proof. FOCs (5 and 6) imply

$$A^* = \left(\frac{n^k sD}{kc}\right)^{\frac{1}{k-1}} \text{ and } B^* = \left(\frac{m^k sD}{kc}\right)^{\frac{1}{k-1}}$$

and

$$p_N(A^*, B^*) = \frac{1}{2} + \left(\frac{s^k D}{kc}\right)^{\frac{1}{k-1}} (n^{\frac{k}{k-1}} - m^{\frac{k}{k-1}}).$$

Notice that $p_N(A^*, B^*) \leq 1$, for any D, whenever

$$s \leq \left(\frac{1}{2(n^{\frac{k}{k-1}} - m^{\frac{k}{k-1}})}\right)^{\frac{k-1}{k}} (kc)^{\frac{1}{k}}. \tag{D1}$$

Thus, according to (7 and 8) utilities can be written as

$$\begin{aligned} V_N(D) &= n\left[\frac{D}{2} + \left(\frac{s^k D^k}{kc}\right)^{\frac{1}{k-1}} \left(n^{\frac{k}{k-1}} - m^{\frac{k}{k-1}}\right) + 1 - y - c\left(\frac{nsD}{kc}\right)^{\frac{k}{k-1}}\right] \\ V_M(D) &= m\left[\frac{D}{2} - \left(\frac{s^k D^k}{kc}\right)^{\frac{1}{k-1}} \left(n^{\frac{k}{k-1}} - m^{\frac{k}{k-1}}\right) + x - c\left(\frac{msD}{kc}\right)^{\frac{k}{k-1}}\right] \end{aligned}$$

where $V_N(D) > n(1-y)$ for any D, whenever

$$\begin{aligned} s &> \left(-\frac{k}{2\left((k-1)\,n^{\frac{k}{k-1}} - km^{\frac{k}{k-1}}\right)}\right)^{\frac{k-1}{k}} (kc)^{\frac{1}{k}} \text{ when } (k-1)\,n^{\frac{k}{k-1}} > km^{\frac{k}{k-1}}, \text{ and} \\ s &< \left(\frac{k}{2\left(km^{\frac{k}{k-1}} - (k-1)\,n^{\frac{k}{k-1}}\right)}\right)^{\frac{k-1}{k}} (kc)^{\frac{1}{k}} \text{ when } (k-1)\,n^{\frac{k}{k-1}} < km^{\frac{k}{k-1}} \end{aligned}$$

Moreover, $V_M(D) > mx$ for any D whenever

$$s < \widehat{s_h} \equiv \left(\frac{k}{2\left(kn^{\frac{k}{k-1}} - (k-1)\,m^{\frac{k}{k-1}}\right)}\right)^{\frac{k-1}{k}} (kc)^{\frac{1}{k}}. \tag{D2}$$

Notice that Condition D1 is implied by Condition D2. Note also that Condition D1 is implied by Condition D2 when $(k-1)\,n^{\frac{k}{k-1}} < km^{\frac{k}{k-1}}$, whereas condition D2 does never bind when $(k-1)\,n^{\frac{k}{k-1}} > km^{\frac{k}{k-1}}$ because $s > 0$. Thus, agents in both groups will only exert some positive effort in the contest when $s < \widehat{s_h}$. This condition also implies that $p_N(A^*, B^*) < 1$.

Taking partial derivatives of the utilities written above we obtain

$$\begin{aligned} \frac{\partial V_N(D)}{\partial D} &= n\left[\frac{1}{2} + \left(\frac{s^k D}{kc}\right)^{\frac{1}{k-1}} \left(n^{\frac{k}{k-1}} - \frac{k}{k-1} m^{\frac{k}{k-1}}\right)\right] \\ \frac{\partial V_M(D)}{\partial D} &= m\left[\frac{1}{2} - \left(\frac{s^k D}{kc}\right)^{\frac{1}{k-1}} \left(\frac{k}{k-1} n^{\frac{k}{k-1}} - m^{\frac{k}{k-1}}\right)\right]. \end{aligned}$$

When $n > m$, since $\frac{\partial V_N(D)}{\partial D} > \frac{\partial V_M(D)}{\partial D}$ $\forall D$ and $k > 1$, group N would never make concessions. However, group M would strategically claim

$$D = \left(\frac{k-1}{2\left(kn^{\frac{k}{k-1}} - (k-1)\,m^{\frac{k}{k-1}}\right)}\right)^{k-1} \frac{kc}{s^k} \tag{9}$$

which is smaller that 1 whenever $\frac{\partial V_M(D)}{\partial D}\mid_{D=1} < 0$. That is, when

$$s > \widehat{s_l} \equiv \left(\frac{k-1}{2(kn^{\frac{k}{k-1}} - (k-1)\,m^{\frac{k}{k-1}})}\right)^{\frac{k-1}{k}} (kc)^{\frac{1}{k}}. \tag{D3}$$

Thus, when agents can coordinate with the members of their group to choose the aggregate effort, the equilibrium of this two-stage game yields

$$x^* = 0 \text{ and } y^* = \min \left\{ \left(\frac{k-1}{2\left(kn^{\frac{k}{k-1}} - (k-1)\, m^{\frac{k}{k-1}}\right)} \right)^{k-1} \frac{kc}{s^k}, 1 \right\}.$$

When $n = m$, both groups N and M would strategically claim

$$D = \left(\frac{k-1}{2} \right)^{k-1} \frac{kc}{(sn)^k}$$

whenever $s > \widehat{s_l}$. Notice that in this case, there are multiple combinations of x and y that satisfy this condition. □

Coordinating the choice of efforts does not alter this result qualitatively.[12] Therefore when $n > m$, moderating the target policy is only profitable for group M because N reduces aggregate effort more than M,[13] and consequently M increases its winning probability. As in the previous section, this positive effect on M overcomes the decrease of its gain from winning when the probability of winning is sufficiently responsive to the differences in effort (i.e. $s > \widehat{s_l}$). However, unlike the non-coordination case, the following proposition shows that the concessions of the group M always benefit the members of both groups.

Proposition 9. *With a linear CSF and coordinate bidding, the concession of the smallest group is beneficial for all members of the largest group.*

Proof. When $s > \widehat{s_l}$, conflict is reduced since

$$y^* = D = \left(\frac{k-1}{2\left(kn^{\frac{k}{k-1}} - (k-1)\, m^{\frac{k}{k-1}}\right)} \right)^{k-1} \frac{kc}{s^k} < 1.$$

[12] As said, the small group will moderate its target policy when the probability of winning is sufficiently sensitive to the differences in effort. When efforts are chosen coordinately, the effect of positive externalities is eliminated so the equilibrium efforts are higher. Consequently, reducing these efforts by means of a target-policy moderation generates a higher positive impact on the payoff due to the convexity of the effort cost function. Thus, in the coordinated case a target-policy moderation will take place under less sensitive CSF. That is, $\widehat{s_l} < s_l$. As also said, both groups will exert a positive effort in equilibrium when the sensitivity of the winning probability to the differences in effort is sufficiently low. When the positive externalities of effort are eliminated in the coordinated case, the equilibrium effort of the big group should be relatively increased because the effect of externalities was higher in this group. Consequently, the upper-bound of s that delimits when the small group members will exert a positive effort is lower in the coordinate case, that is $\widehat{s_h} < s_h$.

[13] Note that $\frac{\partial A^*}{\partial D} > \frac{\partial B^*}{\partial D} > 0$.

Then,

$$\frac{V_N(D)}{n} - \frac{V_N(1)}{n} = Q(s) \equiv \frac{1}{2} - \left(\frac{ck}{s^k}\right)^{-\frac{1}{k-1}} \left(\frac{k-1}{k} n^{\frac{k}{k-1}} - m^{\frac{k}{k-1}}\right)$$
$$-\frac{ck}{s^k}\left(\frac{k-1}{2\left(kn^{\frac{k}{k-1}} - (k-1)\, m^{\frac{k}{k-1}}\right)}\right)^{k-1}\left(\frac{1}{2} - \frac{\frac{k-1}{k} n^{\frac{k}{k-1}} - m^{\frac{k}{k-1}}}{2\left(\frac{k}{k-1} n^{\frac{k}{k-1}} - m^{\frac{k}{k-1}}\right)}\right)^{k-1}.$$

Notice that $\frac{\partial Q(s)}{\partial s} = 0$ holds for a unique value of s. Moreover, by definition $Q(\widehat{s_l}) = 0$ because $D = 1$ when $s = \widehat{s_l}$. After some algebra it can be checked that $\frac{\partial Q(s)}{\partial s} |_{s=\widehat{s_l}}, \frac{\partial Q(s)}{\partial s} |_{s=\widehat{s_h}} > 0$. Thus, the statement of the Proposition follows consequently. □

Now a target-policy moderation of group M do not harm the members of N. The difference must be attributed to the effect of eliminating the inefficiencies generated by the positive externalities of effort. When efforts are chosen collectively this effect does not come into play, so efforts maximize the aggregate utility of the group. This effect adds to the forces explained in the previous section and benefits the big-group members, as illustrated by the following example.

Example 10. *Let* $c = 1/2$*,* $m = 10$*,* $k = 2$*, and* $s = 0.01$*.Substituting into (9) we obtain* $y^* = D = \frac{100}{0.04n^2-2}$*. Figure 3 plots* $D(n)$ *(dots),* $(1/n)(V_N(D) - V_N(1))$ *(solid line) and* $(1/m)(V_M(D) - V_M(1))$ *(dashed line) when there is a pure strategy equilibrium of the contest game (i.e. when* $s < \widehat{s_h}$*, which corresponds to* $n < 71.063$*) and group* M *makes concessions (i.e. when* $s > \widehat{s_l}$*, which corresponds to* $n > 50.498$*). Remember that when* $n < 50.498$*, there is a pure strategy equilibrium where both groups exert a positive effort but none of them make concessions.*

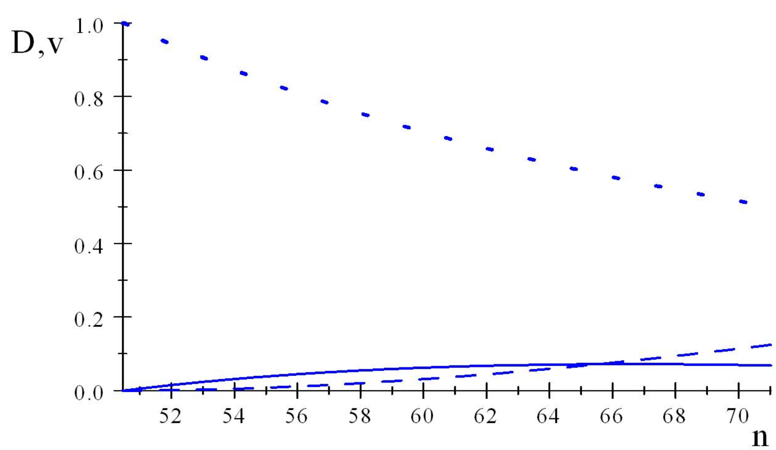

Figure 3: D (dots), $V_N(D) - V_N(1)$ (solid line) and $V_M(D) - V_M(1)$ (dashed line).

5. Conclusions

We studied a public good contest game between two groups of agents with opposite preferences, in which each group choose the policy to bid for previously to the contest stage. Our model builds on two assumptions: first, the utilities of the agents over the public good are linear; and second, cost are strictly convex. We characterized the equilibria under two alternative specifications of the contest success function: either homogeneous of degree zero or the difference-form function specified by Che and Gale (2000). In this framework, we showed that agents do never reduce their claims when the CSF is homogeneous of degree zero, which contrasts with previous results. However, under the difference-form contest success function, at least one of the groups might renounce to part of the prize in order to reduce the conflict. Specifically, the smallest group might prefer to fix a less controversial target policy. This would reduce the efforts of all agents. Obviously, the members of the largest group obtain a direct benefit from this concession because their payoff in case of losing the contest increase and their equilibrium efforts decrease. But this alteration of the equilibrium efforts also causes an indirect effect: A reduction of the winning probability of the bigger group (the small group reduction of the aggregate equilibrium effort is smaller). When efforts are settled individually, the results show that this negative indirect effect from concessions of the smallest group might overcome the positive effects, so that a target-policy moderation of the small group might harm the big one. Instead when efforts are selected cooperatively within groups, so that the positive externalities of individual efforts among group members are internalized, all agents benefit from a target-policy moderation of the small group.

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