Tax evasion and the optimal non-linear labour income taxation

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Abstract

The present work studies optimal taxation of labour income when taxpayers are allowed to evade taxes. The analysis is conducted within a general non-linear tax framework, providing a characterisation of the solution for risk-neutral and risk-averse agents. For risk-neutral agents the optimal government choice is to enforce no evasion and to apply the original Mirrlees' rule for the optimal tax schedule. The no evasion condition is precisely determined by a combination of a sufficiently large penalty and a constant auditing probability. Similar results hold for risk-averse agents. Our findings imply that a government aiming at maximizing social welfare should always enforce no evasion and provide simple rules to pursue this objective.

Keywords: Tax Evasion, Optimal Taxation, Social Welfare

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1 Introduction

The modern approach to modelling tax evasion within the optimal tax theory basically starts with Allingham and Sandmo (1972). Almost at the same time a seminal work by Mirrlees (1971) revolutionized previous research on the optimal taxation theory by introducing a framework for the optimal design of non-linear income taxation schedules. However, it was up to relatively recent times that the two approaches 'got married' (Cremer and Gahvari, 1995). Since then, however, very few works have further investigated this topic, shifting the attention towards tax avoidance, with a special interest on the optimal marginal tax rate of top tail of the income distribution.[1] Chetty (2009) generalizes Feldstein's 1999 formula for computing the deadweight loss in presence of avoidance by separating taxable income and total earned income elasticities, and show how the efficiency cost of taxing high income individuals may not be large. Piketty et al. (2014) develop a model where the top incomes respond to marginal tax rates through the labour supply elasticity, the avoidance elasticity and the bargaining elasticity. They argue that the third elasticity is the main source of response. Labour supply elasticity is generally small and is the sole real factor limiting the top tax rate. Finally, tax avoidance elasticity is the result of a poorly designed tax system, and can be confined to be close to zero[2] mostly by costless tax design reforms.

The issue of tax evasion is different in nature and has still to be explored within a fairly general formulation of the optimal taxation problem. Would results similar to tax avoidance still hold under a model of non-linear optimal labour income taxation *a la* Mirrlees with tax evasion? While the original goal of the present study was to completely characterize the non-linear optimal taxation schedule of labour income taxation when the consumer had the opportunity to evade taxes, our main result makes it an easy task. Indeed, we find that the optimal behaviour of the social planner is to enforce no evasion by setting the expected penalty incurred when concealing income larger than the expected benefit. As a consequence, the optimal tax system is the same as in Mirrlees' original problem.

The behavioural modelling of evasion started with Allingham and Sandmo (1972),[3] in which individuals are deterred from evasion by a fixed probability of auditing and a proportional penalty to be applied over and above the payment of the true liability. Within a linear tax system, taxpayers are risk averse and taxable income is exogenous. Yitzhaki (1987) extended the original model by assuming the probability of auditing to be an increasing function of evaded income, but with risk-neutral agents. Several authors will relax the risk-neutrality assumption and introduced expenditure on concealment (see Cowell, 1990; Kaplow, 1990; Cremer and Gahvari, 1994, among others).

A common weakness of these models is that they consider taxable income as exogenous, while it ap-

[1] Tax evasion is conceptually different from avoidance since it always imply an illegal action.
[2] Their empirical results confirm a small value of the avoidance elasticity for the US.
[3] For a more comprehensive review of tax evasion literature see Sandmo (2012), Sandmo (2005) and Slemrod and Yitzhaki (2002).

pears more reasonable to assume that income is generated by labour supply decisions given an exogenous earning capacity. This is the direction taken by Sandmo (1981), who argue that although results are more complex due to the increased number of choice variable in the model, the effects of the penalty and probability of detection are similar to previous models. This may be the reason why several years passed before further contributions appeared within this line of research. Cremer and Gahvari (1994) introduced labour supply decision within a model of evasion, but remained within the linear tax model. It is only with Cremer and Gahvari (1995) and Schroyen (1997) that the endogenous labour supply decision has been embedded within a non-linear optimal income tax framework *à la* Mirrlees with evasion. The two articles share a two-skills economy, characterized by high- and low-wage individuals, but differ on the penalty mechanism assumptions. Although the two-skills assumption greatly simplifies the mathematical tractability of the problem, it also implicitly generates some the most relevant results of the papers, such as the optimality of the zero marginal tax rate for high-skill workers. As clearly explained by Piketty and Saez (2013) and Piketty et al. (2014), this famous result, already present in the original Mirrlees paper, is valid only for the unique richest individual in the economy, while progressive marginal tax rates at the top of the income distribution can still be optimal. This kind of reasoning can also be applied to Cremer and Gahvari (1995) and Schroyen (1997), since their high-wage individuals are also the richest individuals in their setting.

This weakness motivated us to build a more general non-linear optimal income tax model with evasion. We relax the two-skills economy assumption, and adopt a labour supply model for both risk-neutral and risk-averse agents. Respect to the previous literature, our study suggest that ethically desirable governmental behaviours –enforcing no evasion and choosing audited individual randomly– are optimal, while optimal taxation rules are the same of the original Mirrlees' model, and thus unaffected by tax evasion.

The remainder of the paper is organized as follows. Section 2 describes the main model and its assumptions. Section 3 discusses the incentive-compatibility constraints and the no evasion conditions. Section 4 presents the social planner's problem and its solutions for risk-neutral and risk-averse agents. Section 5 concludes the work.

2 General model and assumptions

We consider a society composed of rational individuals (agents) and a social planner (principal). The social planner seeks to reduce inequality. In order to do so, it is endowed with the right of collecting taxes from the individuals and redistributing a part of the tax collected. In addition, it has the power to establish an auditing system for controlling that individuals do not evade, and a penalty for evaders.

All individuals have the same utility function $U(C, L)$ that determines individual welfare as a function of the individual's consumption, C, and labour supplied, L. L and C can have any non-negative value, while U is continuous and sufficiently differentiable, increasing in C, decreasing in L and concave in C and L simultaneously.

Every individual is endowed with earning ability $w > 0$, which allows him to earn an income $Y = wL$, where L is the labour supplied by the individual. The earning ability w of each individual is private information.

Individuals consume all their net income and they behave rationally, i. e., in every instance they choose the consumption level and labour supply that maximize their utility function.

The social planner seeks to maximize social welfare by redistributing individuals' income. To this end, the social planner establishes an income tax system, $T(Y)$.

However, the social planner has no information about the real income Y earned by each individual. The real income Y of an individual can be observed by the social planner only through an auditing mechanism, which has an exogenous fixed cost κ.

In these conditions, individuals can evade taxes by reporting an income R different from their real income Y. In order to avoid it, the social planner also establishes a random auditing process, with auditing probability $p(R)\epsilon[0,1]$. An individual does not know in advance whether or not he will be audited.

Audited individuals will pay the tax corresponding to their real income, plus a penalty $F(Y,R)$, that can depend, for example, on the evaded income $(Y - R)$ or the benefit of evasion $(T(Y) - T(R))$. The penalty $F(Y,R)$ expresses the disutility incurred by audited cheaters, and includes e.g. the economic cost of imprisoning, social repudiation, etc. Clearly, when the declared income is equal to the real income the fine is $F(Y,R) = 0$.

The social planner has to determine the tax and penalty systems and the auditing probability that maximize social welfare subject to a budgetary restriction and to the limitations imposed by the behavior of the agents.

3 Incentive-compatibility constraints

Through the tax schedule and the auditing and penalty mechanism, the social planner seeks to assign to each individual —characterized by its earning ability w— a specified income level $Y(w)$ and a reported income level $R(w)$. The tax system, the penalty and the auditing probability fix a functional relationship between income, reported income and consumption for both audited non-audited individuals, $C^A(Y,R)$ and $C^{NA}(Y,R)$, which we assume to be continuous and at least twice differentiable.

The consumption of audited and non-audited individuals are related to the tax schedule and the evasion penalty by

$$
\begin{aligned}
C^{NA}(Y,R) &= Y - T(R) \\
C^{A}(Y,R) &= Y - T(Y) - F(Y,R)
\end{aligned}
$$

Individuals behave rationally and they pick the income and reported income levels that maximize their utility. However, individuals do not know whether or not they are going to be audited by the social planner, hence they must decide their behavior based upon their expected utility. When individual w earns an income Y and reports an income R, his expected utility reads

$$
\mathcal{U}(Y,R,w) = [1-p(R)]\,U\left(Y - T(R), \frac{Y}{w}\right) + p(R)U\left(Y - T(Y) - F(Y,R), \frac{Y}{w}\right) \ . \tag{1}
$$

The social planner assigns to each individual w a given income $Y(w)$ and a reported income $R(w)$, hereby assigning to each an expected utility

$$
V(w) = \mathcal{U}(Y(w), R(w), w) \ . \tag{2}
$$

However, the agent —whose earning ability is private information and whose real income is unknown if not audited— may decide to earn the income of another type (ρ, say) while reporting the income of a third type, σ, if such a choice reports him a larger expected utility. Thus, the income and the reported income levels that the social planner assigns to individual w ought to verify the incentive-compatibility constraint (ICC)

$$
V(w) \equiv \mathcal{U}\left(Y(w), R(w), w\right) \geq \mathcal{U}\left(Y(\rho), R(\sigma), w\right) \quad \forall w, \rho, \sigma \ . \tag{3}
$$

In order to analyze the implications of (3), it is convenient to define the utility difference function $D(\rho,\sigma,w) = V(w) - \mathcal{U}(Y(\rho), R(\sigma), w)$, which must be non-negative everywhere while being zero along the diagonal $w = \rho = \sigma$. Hence, when one of the variables is fixed, D has to have a minimum on the diagonal with respect to the other two variables.

This imposes the First-Order Incentive-Compatibility Conditions (FO-ICC)

$$
[D_\rho] = 0 = [D_\sigma] = [D_w] \ , \tag{4}
$$

where subscripts indicate partial derivation (i.e., $D_w \equiv \partial D/\partial w$) and the square brackets indicate that the magnitude is to be evaluated on the diagonal $\rho = \sigma = w$. It also imposes the Second-Order Incentive-Compatibility Conditions (SO-ICC) that the Hessian of D, $H(D)$ is positive semi-definite on the diagonal, where

$$H(D) = \begin{pmatrix} [D_{\rho\rho}] & [D_{\rho\sigma}] & [D_{\rho w}] \\ [D_{\sigma\rho}] & [D_{\sigma\sigma}] & [D_{\sigma w}] \\ [D_{w\rho}] & [D_{w\sigma}] & [D_{ww}] \end{pmatrix} , \tag{5}$$

which implies that

$$[D_{\rho\rho}] \geq 0 \, , \left[D_{\rho\rho} D_{\sigma\sigma} - D^2_{\rho\sigma}\right] \geq 0 \, , \det H(D) \geq 0 \, . \tag{6}$$

The third SO-ICC is identically fulfilled, as can be easily demonstrated by deriving the FO-ICC in (4) with respect to w, which imposes that $[D_{w\sigma}] = \frac{1}{2}[D_{\rho\rho} - D_{ww} - D_{\sigma\sigma}]$, $[D_{\rho\sigma}] = \frac{1}{2}[D_{ww} - D_{\rho\rho} - D_{\sigma\sigma}]$, $[D_{w\rho}] = \frac{1}{2}[D_{\sigma\sigma} - D_{\rho\rho} - D_{ww}]$. Substitution into (5) gives that $detH(D) \equiv 0$, which is to be expected from the constant zero value of $[D]$: the null curvature of $[D]$ must correspond to a zero eigenvalue.

3.1 No evasion condition

In the former discussion about the incentive-compatibility conditions, the income R that each agent reports is not necessarily equal to his/her real income Y. If the social planner wants the agents reporting their true income, i. e., $R = Y$ for all agents independently of their earning ability w, this option must be incentive-compatible. In this case, we must have that

$$\mathcal{U}\left(Y(w), Y(w), w\right) \geq \mathcal{U}\left(Y(w), R(\sigma), w\right) \quad \forall w, \sigma \, . \tag{7}$$

From eq.(1), we have that

$$\begin{aligned} \mathcal{U}(Y, R, w) &= [1 - p(R)]\, U\left(Y - T(R), \frac{Y}{w}\right) + p(R) U\left(Y - T(Y) - F(Y, R), \frac{Y}{w}\right) \\ &\leq U\left(Y - p(R)T(Y) - [1 - p(R)]\, T(R) - p(R)F(Y, R), \frac{Y}{w}\right) \end{aligned} \tag{8}$$

due to the concavity of $U(C, L)$.[4]

In addition, recalling that $F(Y, R) = 0$ if $Y = R$, we have that

$$\mathcal{U}\left(Y(w), Y(w), w\right) = U\left(Y(w) - T(Y(w)), \frac{Y(w)}{w}\right) \, . \tag{9}$$

Thus, since U is increasing on consumption, a sufficient condition for the no evasion condition (7) to

[4]Equality in (8) holds when the utility function is lineal in consumption, and thus the higher bound for any concave utility function.

be fulfilled is that

$$\begin{aligned} Y - T(Y) &\geq Y - p(R)T(Y) - [1 - p(R)]\,T(R) - p(R)F(Y,R) \\ &\Rightarrow p(R)F(Y,R) \geq [1 - p(R)]\,[T(Y) - T(R)] \ , \end{aligned} \tag{10}$$

which has a clear economic interpretation: the expected penalty incurred by reporting a fake income has to be larger than the expected benefit. In these conditions, agents will report their true income regardless of their earning ability.

Equation (10) highlights two important features: first, the penalty is inversely proportional to the auditing probability; second the penalty is directly proportional to the avoided tax payments $(T(Y) - T(R))$ rather than to the sheltered income $(Y - R)$.

The implication of a penalty inversely proportional to the auditing probability are known since (Becker, 1968) and are relevant for the optimal choice of the government, since a lower auditing probability reduces the monitoring costs and thus the overall governmental budget constraint. On the other hand, imposing disproportionally large penalties imply that for relatively small errors in the revenues declaration could imply dramatic consequences on the taxpayer. Although this is a sensible reasoning for the real world implementation of the penalty system, this class of models work with rational individuals under perfect information, hence errors in the revenues declaration is not an option.

As shown in Section 4, the no evasion condition (10) together with the usual Mirrlees rules for the optimal marginal tax rates is an optimal strategy for a government aiming at maximizing social welfare.

3.2 Separable utility function

The simpler case of separable utility functions is widely used in the literature in order to describe the preferences of agents. In this case,

$$U(C, L) = K(C) - A(L) \ , \tag{11}$$

where $K(C)$ expresses the utility associated with consumption C, while $A(L)$ describes the disutility associated with labour. The general assumptions on U imply that in the present case, $K' > 0$, $K'' \leq 0$, $A' \geq 0$ and $A'' \geq 0$.

In this case, the expected utility function reads

$$\begin{aligned} \mathcal{U}(Y, R, w) &= [1 - p(R)]\,K(Y - T(R)) + p(R)K(Y - T(Y) - F(Y,R)) - A\left(\frac{Y}{w}\right) \\ &\equiv Q(Y,R) - A\left(\frac{Y}{w}\right) \end{aligned} \tag{12}$$

and the FO-ICC, eq. (4), impose that

$$
\begin{aligned}
[D_w] &= \left[V'(w) - A'\left(\frac{Y(\rho)}{w}\right)\frac{Y(\rho)}{w^2}\right] = 0 \\
&\Rightarrow \; V'(w) = A'\left(\frac{Y(w)}{w}\right)\frac{Y(w)}{w^2} \; , \qquad (13) \\
[D_\rho] &= -\left[Q_Y(Y(\rho), R(\sigma))Y'(\rho) - A'\left(\frac{Y(\rho)}{w}\right)\frac{Y'(\rho)}{w}\right] = 0 \\
&\Rightarrow \; Q_Y(Y(w), R(w)) = \frac{1}{w}A'\left(\frac{Y(w)}{w}\right) \; , \qquad (14) \\
[D_\sigma] &= -\left[Q_R(Y(\rho), R(\sigma))R'(\sigma)\right] = 0 \\
&\Rightarrow \; Q_R(Y(w), R(w)) = 0 \; , \qquad (15)
\end{aligned}
$$

A very important consequence of the separability of the utility function is that $D_{w\sigma} \equiv 0$, because D_w does not depend on R hence on σ. This implies, by deriving the FO-ICC with respect to w along the diagonal, that

$$
\begin{aligned}
[D_{\rho\rho} + D_{\sigma\rho} + D_{w\rho}] &= 0 \; , \qquad (16) \\
[D_{\rho\sigma} + D_{\sigma\sigma}] &= 0 \Rightarrow [D_{\rho\sigma}] = [-D_{\sigma\sigma}] \; , \qquad (17) \\
[D_{\rho w} + D_{ww}] &= 0 \Rightarrow [D_{\rho w}] = -[D_{ww}] \; , \qquad (18)
\end{aligned}
$$

whence

$$
[D_{\rho\rho}] = [D_{\sigma\sigma} + D_{ww}] \; . \qquad (19)
$$

As in the general case, $\det H(D) \equiv 0$ along the diagonal. In addition, using (17)-(19) into (6), we see that the other SO-ICC can be recast as

$$
[D_{ww}] \geq 0 \; , \; [D_{\sigma\sigma}] \geq 0 \; . \qquad (20)
$$

Explicitly,

$$
\begin{aligned}
[D_{ww}] &= V''(w) + \frac{Y(w)}{w^3}\left[2A'\left(\frac{Y(w)}{w}\right) + \frac{Y(w)}{w}A''\left(\frac{Y(w)}{w}\right)\right] \geq 0 \; , \qquad (21) \\
[D_{\sigma\sigma}] &= -Q_{RR}(Y(w), R(w))R'(w) \geq 0 \\
&\Rightarrow \; Q_{RR}(Y(w), R(w)) \leq 0 \; . \qquad (22)
\end{aligned}
$$

Conditions (15) and (22) reveal the behaviour of the agents. As a first step, agents choose —through (15)— the reported income level R that maximizes their expected utility as a function of their income level Y. For separable utility functions, this is independent of the agent's learning ability, w, thus the dependence of R on w arises only through the dependence of Y on w. In a second step, the income level

Y of each agent is determined by (14).

Proposition 1 : The Spence-Mirrlees condition $Y'(w) \geq 0$ is a necessary and sufficient condition for (21).

Proof. Deriving (13) with respect to w, we have that

$$V''(w) = \frac{Y'(w)}{w^2}\left[A'\left(\frac{Y}{w}\right) + \frac{Y(w)}{w}A''(\frac{Y}{w})\right] - \frac{Y(w)}{w^3}\left[2A'(\frac{Y}{w}) + \frac{Y(w)}{w}A''(\frac{Y}{w})\right] , \tag{23}$$

whence condition (21) becomes

$$\frac{Y'(w)}{w^2}\left[A'\left(\frac{Y}{w}\right) + \frac{Y(w)}{w}A''(\frac{Y}{w})\right] \geq 0 , \tag{24}$$

which is identically satisfied if $Y'(w) \geq 0$. □

Conversely, if (21) is satisfied, it implies that $Y'(w) \geq 0$.

3.3 Quasi-linear utility function

A further simplification often studied in the literature is that of utility functions which are linear on consumption, i. e., $K(C) = C$. This implies risk neutral consumers, in the sense that the utility generated by an uncertain expected consumption level is the same of its certain equivalent. In this case,

$$Q(Y, R) = Y - [1 - p(R)]\,T(R) - p(R)\,[T(Y) + F(Y, R)] \equiv \mathcal{C}(Y, R) , \tag{25}$$

which corresponds to the expected consumption, and

$$\mathcal{U}(Y, R, w) = Y - [1 - p(R)]\,T(R) - p(R)\,[T(Y) + F(Y, R)] - A\left(\frac{Y}{w}\right) \equiv \mathcal{C}(Y, R) - A\left(\frac{Y}{w}\right) . \tag{26}$$

The FO-ICC then read

$$V'(w) = A'\left(\frac{Y(w)}{w}\right)\frac{Y(w)}{w^2} , \tag{27}$$

$$\mathcal{C}_Y(Y(w), R(w)) = \frac{1}{w}A'\left(\frac{Y(w)}{w}\right) , \tag{28}$$

$$\mathcal{C}_R(Y(w), R(w)) = 0 , \tag{29}$$

while the SO-ICC reduce to

$$Y'(w) \geq 0 \text{ and } \mathcal{C}_{RR}(Y(w), R(w)) \leq 0 . \tag{30}$$

4 Social planner's problem

The social planner is aware of the rational behaviour of individuals, and it knows the distribution of earning abilities, $f(w)$. The social planner seeks to establish the tax schedule, $T(R)$, the auditing probability, $p(R)\epsilon[0,1]$ and the penalty system, $F(Y,R)$ that lead to an income and reported income structures that maximize the social welfare

$$S = \int_0^\infty f(w)G(V(w))dw \ , \tag{31}$$

where G is the weight function that the social planner gives to individuals' welfare. A social planner who is adverse to inequality is characterized by $G' > 0$ and $G'' < 0$. The maximization is subject to a budgetary restriction that has to be verified by the taxes collected and includes the cost of the auditing,

$$\mathcal{B} = \int_0^\infty f(w)\left\{[1-p(R)]\,T(R) + p(R)\,[T(Y) + F(Y,R) - \kappa]\right\} dw \geq T_0 \ , \tag{32}$$

where T_0 is an exogenous revenue requirement for the government and κ is the (exogenous) cost of auditing an individual which we assume to be constant.

The cost of auditing can be assumed to be endogenous[5] and the analysis would be basically unchanged. To ensure the concavity of the Hamiltonian the endogenous cost of auditing $\kappa(Y)$ must be increasing and convex in income and this would lead to the same conclusions of the constant case, except that the optimal marginal tax rate should be increased by the expected marginal cost of auditing $p_0\kappa'(Y)$. Given that the additional contribution of having an endogenous cost of auditing is small and that the assumption of a constant cost of auditing is safe –at least for the analysis of personal labour income–, in what follows we maintain the hypothesis of constant cost of auditing.

4.1 Risk-neutral agents

The choice of $T(R)$, $p(R)$ and $F(Y,R)$ is constrained by the behavior of the individuals, which under the assumption of separable, quasi-linear preferences is described by the incentive-compatibility conditions (27)-(30). Using (25) and (2) into (32), the problem can be recast as

$$\max_{Y,R} S = \int_0^\infty f(w)G(V(w))dw \tag{33}$$

[5] For example Schroyen (1997) assumes κ to be an increasing and convex function of the probability of being audited conditional on income level $p(Y)$. In the two-skills economy this assumption ensures that auditing an entire income class is prohibitively costly. Adopting a similar assumption for an economy with a continuous income distribution makes no sense.

subject to

$$\mathcal{B} = \int_0^\infty f(w)\left[Y(w) - V(w) - A\left(\frac{Y(w)}{w}\right) - \kappa p(R(w))\right] dw \geq T_0 \tag{34}$$

$$V' = \frac{Y}{w^2} A'\left(\frac{Y}{w}\right) \tag{35}$$

$$Y' \geq 0\,, \quad \mathcal{C}_{RR} \leq 0\,. \tag{36}$$

Introducing the (constant) Lagrange multiplier ν associated to the budgetary restriction and $\mu(w)$ as the costate variable associated to V, the Hamiltonian (assuming non-binding SO-ICC) reads

$$H = f(w)G(V) + \mu\frac{Y}{w^2}A'\left(\frac{Y}{w}\right) + \nu f(w)\left[Y - V - A\left(\frac{Y}{w}\right) - \kappa p(R)\right]\,. \tag{37}$$

Clearly, ν can be set to one without loss of generality by simply scaling $G(V)$ and μ.

Maximization with respect to R immediately yields that $p(R) = p_0$, constant. Maximization with respect to Y yields

$$\frac{\partial H}{\partial Y} = 0 \Rightarrow \frac{\mu}{w^2}\left[A'(\frac{Y}{w}) + \frac{Y}{w}A''(\frac{Y}{w})\right] = -f(w)\left[1 - \frac{1}{w}A'(\frac{Y}{w})\right]\,, \tag{38}$$

with the second order condition

$$\frac{\partial^2 H}{\partial Y^2} = \frac{\mu}{w^3}\left[2A''(\frac{Y}{w}) + \frac{Y}{w}A''(\frac{Y}{w})\right] - \frac{f(w)}{w^2}A''(\frac{Y}{w}) \leq 0\,. \tag{39}$$

In order to verify (39) it is sufficient that $\mu \leq 0$. Then, from (38) we have that

$$1 - \frac{1}{w}A'(\frac{Y}{w}) \geq 0 \tag{40}$$

guarantees $\mu \leq 0$.

On the other hand, we have that

$$\mu' = -\frac{\partial H}{\partial V} = -f(w)\left[G'(V) - 1\right] \quad \text{with } \mu(\infty) = 0\,, \tag{41}$$

hence

$$\mu = -\int_w^\infty f(x)\left[1 - G'(V(x))\right] dx\,. \tag{42}$$

Therefore, the requirement $\mu \leq 0$ implies that $G'(V) \leq 1$.

Thus, the income and expected utility of the agents are finally determined by equations (35) and

(38), and read

$$V' = \frac{Y}{w^2} A'(\frac{Y}{w}) \text{ with } V(0) = V_0 \,, V(\infty) \text{ free ,} \tag{43}$$

$$1 = \frac{1}{w} A'(\frac{Y}{w}) + \frac{A'(\frac{Y}{w}) + \frac{Y}{w} A''(\frac{Y}{w})}{w^2 f(w)} \int_w^\infty f(x) \left[1 - G'(V(x))\right] dx \,. \tag{44}$$

We thus see that the income assigned to every individual is given by exactly the same rule as in Mirrlees' original problem.

The penalty and optimal tax can then be determined from (28) and (29).

Proposition 2 : For a government aiming at maximizing social welfare of risk-neutral agents it is optimal to enforce no evasion. The penalty to apply should be proportional to the tax evasion $T(Y)-T(R)$ and inversely proportional to the auditing probability p_0.

Proof. From (29), we have that

$$\mathcal{C}_R = 0 \;\Rightarrow F_R(Y, R) = -\frac{1 - p_0}{p_0} T'(R) \,, \tag{45}$$

hence one immediately finds that

$$F(Y, R) = \gamma(Y) - \frac{1 - p_0}{p_0} T(R) \,. \tag{46}$$

The requirement that $F(Y, Y) \equiv 0 \; \forall Y$ imposes that

$$F(Y, R) = \frac{1 - p_0}{p_0} \left[T(Y) - T(R)\right] \,, \tag{47}$$

□

Proposition 2 implies that $\mathcal{C} = Y - T(Y)$, hence $\mathcal{C}_{RR} \equiv 0$ and in addition it ensures that agents will report their true income (see eq. 10). Therefore, the penalty is proportional to the tax evasion, and the proportionality constant is $p_0^{-1} - 1$; this means that the penalty has to be very high if the fraction p_0 of audited individuals is low. It should also be noted that the constancy of $p(R)$ implies randomness in the auditing system. This conveys a sense of equality for all agents, which in democratic societies can help to enforce the tax and penalty schedule proposed by the social planner. This is in contrast with the previous literature, which under the simplifying assumption of a two-skills economy found that the richest should never be audited (Cremer and Gahvari, 1994; Schroyen, 1997) –an ethically controversial criterion.

Given the optimality of the no evasion condition defined by equation (47), the optimal marginal tax rate can be determined from equation (28).

Proposition 3 : Given Proposition 2, the optimal marginal tax rate in the case of separable quasi-linear utility functions is determined by the same rule of Mirrlees (1971).

Proof. Under enforcement of no evasion, as implied by equation (47), $\mathcal{C}_Y = 1 - T'(Y)$, thus by equation (28), the optimal marginal tax rate is defined by

$$1 - T'(Y(w)) = \frac{1}{w} A' \left(\frac{Y(w)}{w} \right) . \tag{48}$$

By equation (44) we obtain

$$\frac{1}{w} A' \left(\frac{Y(w)}{w} \right) = 1 - \frac{A' \left(\frac{Y(w)}{w} \right) + \frac{Y}{w} A'' \left(\frac{Y(w)}{w} \right)}{w^2 f(w)} \int_w^\infty f(x) \left[1 - G'(V(x)) \right] dx , \tag{49}$$

and the optimal marginal tax rate reads

$$T'(Y(w)) = 1 - \frac{1}{w} A' \left(\frac{Y(w)}{w} \right) = \frac{A' \left(\frac{Y(w)}{w} \right) + \frac{Y(w)}{w} A'' \left(\frac{Y(w)}{w} \right)}{w^2 f(w)} \int_w^\infty f(x) \left[1 - G'(V(x)) \right] dx . \tag{50}$$

□

Thus, allowing for tax evasion will not change the Mirrlees' optimal marginal taxation rule in the case of separable quasi-linear utility functions.

Propositions 2 and 3 have important implications respect to the previous literature on income tax evasion, suggesting that avoiding the auditing of the richest individuals is not optimal, nor it is to apply lower marginal tax rates to the top of the distribution because of the evasion opportunity.

4.2 Risk-averse agents

The results obtained for the quasi-linear separable utility function discussed above can be generalized to non-linear utility functions that correspond to the case of risk-averse agents.

Although the mathematical demonstration is somewhat cumbersome, the economic intuition behind it is relatively straightforward. Risk averse agents have a concave utility function and will always prefer a certain outcome over its uncertain equivalent, *ceteris paribus.* Assuming income tax evasion, the utility derived from the certain equivalent of the expected income is always higher, as formalized by equation (8).

Having the possibility to evade taxes, makes the individuals who decide to evade wealthier. Hence, if the government 'permits' some level of evasion, this implicitly implies that the government has some budget margin to reduce tax revenues, which could be equally done by directly reducing tax rates. The latter case, however would imply a larger welfare increase because of the absence of uncertainty and the concavity of the utility function. As a consequence, enforcing no evasion is an optimal strategy for the government, and, since nobody would evade, the optimal marginal tax rate will be the classical Mirrlees' one.

Proposition 4 : For risk-averse agents enforcing no evasion is optimal and the rule that determines the optimal marginal tax rate is the same Mirrlees (1971).

Proof. For the proof of the proposition let restate the social planner problem starting. Recalling equation (12), for the risk-averse agent with additively separable utility function $U(C,L) = K(C) - A(L)$, the expected utility function can be written as

$$\mathcal{U}(Y,R,w) = [1 - p(R)]\, K(n(Y,R)) + p(R)K(a(Y,R)) - A\left(\frac{Y}{w}\right)\ , \tag{51}$$

where $n(Y,R) = Y - T(R)$ is consumption if not audited and $a(Y, R = Y - T(Y) - F(Y,R)$ is consumption if audited.

Given any tax structure, auditing probability and penalty, the agent maximizes $\mathcal{U}(Y,R,w)$ respect to Y and R with the following first order conditions

$$\mathcal{U}_Y = 0\ , \tag{52}$$

$$\mathcal{U}_R = 0\ . \tag{53}$$

The optimal choice of Y and R will depend on the given scenario, and can be written as $Y^*(w; p(R), T(Y), T(R), F(Y,R), ...)$ and $R^*(w; p(R), T(Y), T(R), F(Y,R), ...)$.

On the other hand, the social planner chooses $p(R)$, $T(R)$ and $F(Y,R)$ in order to maximize social welfare

$$S = \int_0^\infty f(w)G(\mathcal{U}^*(Y,R,w))dw \tag{54}$$

given the budgetary restriction

$$B = \int_0^\infty f(w)\{\,[1 - p(R^*)]T(R^*) + p(R^*)[T(Y^*) + F(Y^*,R^*) - \kappa\,\}dw\ . \tag{55}$$

Given the difficult direct mathematical treatment of the problem, it is convenient to restate it as an optimal control problem, where the social planner maximizes

$$S = \int_0^\infty f(w)G(\mathcal{U}(Y,R,w))dw\ , \tag{56}$$

with

$$\mathcal{U}(Y,R,w) = [1-p(R)]\,K(n(Y,R)) + p(R)K(a(Y,R)) - A\left(\frac{Y}{w}\right) \tag{57}$$

under the following restrictions:

$$\mathcal{U}_Y = 0 \tag{58}$$

$$\mathcal{U}_R = 0 \tag{59}$$

$$B = \int_0^\infty f(w)\left\{Y - [1-p(R)]n(Y,R) - p(R)[a(Y,R)+\kappa]\right\}dw\ . \tag{60}$$

To solve the problem, the social planner has to choose Y, R, n, a and $p(R)$, taking into account that (58) and (59) involve $n(Y,R)$ y $a(Y,R)$, which in turn determine the optimal tax schedule and the penalty.

Defining the expected utility as

$$V(w) = \mathcal{U}(Y(w), R(w), w)\ , \tag{61}$$

we have that

$$V'(w) = \frac{Y}{w^2}A'\left(\frac{Y}{w}\right)\ , \tag{62}$$

and solving the expected utility function for $a(Y,R)$ gives

$$a(Y,R) = K^{-1}\left(\frac{V + A\left(\frac{Y}{w}\right) - (1-p)K(n(Y,R))}{p}\right)\ . \tag{63}$$

With this change equations (58) and (59) are identically verified, such that the problem can be written as

$$S = \int_0^\infty f(w)G(V(w))dw \tag{64}$$

subject to the restrictions

$$V'(w) = \frac{Y}{w^2}A'\left(\frac{Y}{w}\right) \tag{65}$$

$$B = \int_0^\infty f(w)\left\{Y - [1-p(R)]n(Y,R) - p(R)[a+\kappa]\right\}dw\ , \tag{66}$$

with a given by (63).

The Hamiltonian for this problem is

$$H = f(w)G(V) + \mu \frac{Y}{w^2} A'\left(\frac{Y}{w}\right) + \lambda f(w)\left\{Y - [1-p(R)]n(Y,R) - p(R)[a+\kappa]\right\} \ , \tag{67}$$

which needs being maximized over the control variables. It should be noted that the dependency of the Hamiltonian on R comes only through $p(R)$ and $n(Y,R)$.

Now note that the expected consumption can be written as a function of $n(Y,R)$

$$\begin{aligned} \Phi(n) &= (1-p(R))n(Y,R) + p(R)a(Y,R) && (68)\\ &= (1-p(R))n(Y,R) + p(R)K^{-1}\left(\frac{V + A\left(\frac{Y}{w}\right) - (1-p(R))K(n(Y,R))}{p(R)}\right) && (69) \end{aligned}$$

which has

$$\begin{aligned} \frac{d\Phi(n)}{dn} &= (1-p(R))\left[1 - \frac{K'(n(Y,R))}{K'\left(K^{-1}\left(\frac{V+A\left(\frac{Y}{w}\right)-(1-p(R))K(n(Y,R))}{p(R)}\right)\right)}\right] && (70)\\ &= (1-p(R))\left[1 - \frac{K'(n(Y,R))}{K'(a(Y,R))}\right] \ , && (71) \end{aligned}$$

which is zero if and only if $a(Y,R) = n(Y,R)$. Moreover this stationary point is a minimum since

$$\frac{d^2\Phi(n)}{dn^2} = -(1-p(R))\frac{K''(n(Y,R))}{K'(a(Y,R))} \geq 0 \ . \tag{72}$$

This implies that the maximization of H can be done in two steps: first minimize $\phi(n(Y,R))$ for any given Y, R and w, that is $a(Y,R) = n(Y,R)$, and then maximize H for Y and R.

Note that setting $a(Y,R) = n(Y,R)$ corresponds to the no evasion condition 10, and the Hamiltonian becomes

$$H = f(w)G(V) + \mu \frac{Y}{w^2} A'\left(\frac{Y}{w}\right) + \lambda f(w)\left[Y - n(Y,R) - p(R)\kappa\right] \ , \tag{73}$$

with

$$V = K(n(Y,R)) - A\left(\frac{Y}{w}\right) \ , \tag{74}$$

such that the problem is non-linear utility equivalent of equation (37), and correspond to the classic Mirrlees' case

$$H = f(w)G(V) + \mu \frac{Y}{w^2} A'\left(\frac{Y}{w}\right) + \lambda f(w)\left[Y - K^{-1}\left(V + A\left(\frac{Y}{w}\right)\right) - p(R)\kappa\right] \ , \tag{75}$$

except for the auditing cost $p(R)\kappa$.

Maximizing with immediately leads to $p(R) = p_0$, constant, and the optimal marginal tax rate follows Mirrlees' rule, provided that the utility function can be inverted. □

This proves that the social welfare is maximized when there is no evasion, i.e. $Y = R$, and that this condition can be enforced *ex ante* by a random auditing process where the auditing probability is independent of the declared income. Under no evasion the optimal tax schedule can be determined and the penalty that satisfies the no evasion condition (10) can be chosen *ex post* according to the auditing probability p_0.

5 Conclusions

Traditionally, the issue of optimal labour income taxation in presence of evasion has been studied within a linear tax framework and only relatively recent articles tried to address the non-linear income taxation case, with the simplifying assumption that individuals may be either low- or high-skill workers. Driven by this simplifying assumption, this stream of literature has confirmed the zero top income marginal tax rate result obtained by Mirrlees (1971). Indeed, in a two-skills economy, it is natural that the whole share of population with high skill is the richest by design. Moreover, only the rich have incentive to evade, and thus the zero marginal tax rate also minimizes the incentive to evade for this class of individuals.

Recent studies, however, have shown that the original result have been misinterpreted (i.e. Piketty and Saez, 2013), in the sense that it refers to the single richest individual in the income distribution, and have shown how progressive taxation can be optimal at the higher tail of the income distribution as well. Moreover, analysing tax avoidance for the top income distribution Piketty et al. (2014) found that avoidance had basically no impact on the optimal taxation design.

Respect to the previous literature, our study suggest that ethically desirable governmental behaviours –enforcing no evasion and choosing audited individual randomly– are optimal. Moreover, under a general implementation of the optimal non-linear income tax framework with evasion, the optimal marginal tax schedule is the same as in the original Mirrlees problem and the government should enforce no evasion by imposing a sufficiently large penalty under a constant auditing probability. This implies that the results obtained using non-linear optimal taxation models, such as those presented in Piketty and Saez (2013) are valid also in presence of tax evasion.

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