

Divided Majority and Information Aggregation: Theory and Experiment*

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Abstract

This paper studies theoretically and experimentally the properties of plurality and approval voting when a majority gets divided by information imperfections. The majority faces two challenges: aggregating information to select the best majority candidate and coordinating to defeat the minority candidate. Under plurality, the majority cannot achieve both goals at once. Under approval voting, it can: welfare is strictly higher because some voters approve of both majority alternatives. In the laboratory, we find (i) strong evidence of strategic voting under both voting rules, and (ii) superiority of approval voting over plurality. Finally, subject behavior suggests the need to study equilibria in asymmetric strategies.

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1 Introduction

Elections are typically expected to achieve better-informed decisions than what an individual could achieve alone.¹ The rationale is that if each voter can convey her privately-held information through her ballot, voting results will reveal the aggregate information dispersed in the electorate. However, this is a big “if”: in plurality elections, for instance, rational voters are typically expected to coordinate their ballots on only two alternatives, independently of the number of competing alternatives (Duverger’s Law). Therefore, unless the number of candidates is exactly two, information aggregation is dubious.

This limitation resonates with centuries of scholarly research on how to design an electoral system that can aggregate heterogeneous preferences and information in an efficacious way (see e.g. Condorcet 1785, Borda 1781, Myerson and Weber 1993, Myerson 1999, Piketty 2000, Bouton 2012). Frustration with plurality is also apparent in civil society: a large number of activists lobby in favor of reforming the electoral system² and many official proposals have been introduced.³ One of the most popular alternatives to plurality is approval voting (AV).⁴ Yet a major hurdle stands in the way of reform: the substantial lack of knowledge surrounding the capacity of AV (or other systems) to outperform plurality. We need a better understanding of the properties of new electoral systems to identify and implement meaningful reforms.

With this purpose in mind, we study the properties of plurality and AV when voters are strategic but imperfectly informed. We focus on the case in which a majority both needs to aggregate information and to coordinate ballots to defeat a minority alternative: the Condorcet loser. Our analysis features two main novelties: first, we study these systems both theoretically and experimentally. Second, instead of focusing on the limiting properties of these systems when the electorate is arbitrarily large, we study them for any electorate size. This means that our conclusions are equally valid for committees and general elections.

A first theoretical finding is that, in plurality, the need to aggregate information pro-

¹See a.o. Condorcet 1785, Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1996, 1997, Myerson 1998, Krishna and Morgan 2012, and the references therein. For limitations, see e.g. Bhattacharya 2012, Mandler 2012, and Morgan and Várdy 2012.

²See e.g. the Electoral Reform Society (www.electoral-reform.org.uk) and the Fair Vote Reforms initiative (www.fairvote.org).

³Two examples are North Dakota in 1987, where a bill to enact approval voting in some statewide elections passed the Senate but not the House and, more recently, the U.K., which held a national referendum in 2011 on whether to replace plurality voting with alternative voting.

⁴Under approval voting, voters can “approve of” as many candidates as they want. Each approval counts as one vote and the candidate that obtains the largest number of votes wins (Weber 1977, 1995, Brams and Fishburn 1978, 1983, Laslier 2010, Nuñez 2010, Bouton and Castanheira 2012).

duces an equilibrium in which voters vote *informatively* (that is, their ballot conveys their private information), despite the need to coordinate against the minority. This equilibrium is not “knife edge”, and may rationalize the oft-observed pattern that strictly more than two candidates receive positive but different vote shares, despite the predictions of Duverger’s Law. When the minority is small, this equilibrium supports information aggregation, in the sense that the alternative with the largest expected vote share is the full information Condorcet winner. In contrast, when the minority is large, the alternative with the largest vote share is the Condorcet loser, in which case this equilibrium is highly inefficient. This equilibrium exists even when majority voters would benefit from collectively deviating towards a Duverger’s Law equilibrium.

In the same setup, we show that AV can always produce strictly higher welfare than plurality. Having the opportunity to approve of multiple alternatives allows the electorate to achieve both better coordination and information aggregation. While we cannot establish a general proof fully characterizing the equilibrium in approval voting,⁵ we are able to formulate two substantiated conjectures: (i) the symmetric equilibrium is unique, and (ii) the equilibrium strategy is such that voters approve of the candidate they deem best and sometimes also approve of the other majority candidate. These conjectures find support in one formal result and many numerical simulations.

Our theoretical analysis poses an interesting trade-off between these two electoral systems. On the one hand, one could claim that AV is more complex than plurality because it extends the set of actions that each voter can take.⁶ Hence, there is a risk that actual voters make more mistakes under AV, which could wash out its favorable theoretical properties. On the other hand, our theoretical findings are that AV reduces the number of equilibria and therefore simplifies strategic interactions amongst voters. In other words, AV should facilitate the voters’ two-pronged goal of aggregating information and coordinating ballots to avoid a victory of the Condorcet loser.

We ran controlled laboratory experiments to assess the validity of these theoretical

⁵In contrast, Bouton and Castanheira (2012) fully characterize the equilibrium for arbitrarily large electorate sizes. In the presence of “doubt”, the equilibrium proves to be unique and implies *full information and coordination equivalence*. That is, the full information Condorcet winner always has the largest expected vote share. In contrast, Goertz and Maniquet (2011) provide an example in which aggregate information does not obtain if sufficiently many voters assign a probability zero to some states of nature.

⁶With three alternatives, plurality offers four possible actions: abstain, and vote for either one of the three alternatives. AV adds another four possible actions: three double approvals, and approving of all alternatives. Saari and Newenhizen (1988) argue that this may produce indeterminate outcomes, and Niemi (1984) argues that AV “begs voters to behave strategically”, in a highly elaborate manner. In contrast, Brams and Fishburn (1983, p28) show that the number of undominated strategies can be smaller under AV than under plurality.

findings. They reveal interesting patterns and support most predictions. We first study setups in which information is symmetric across states of nature. Under plurality, we observe the emergence of both types of equilibria: when the minority is sufficiently small, all groups stick to playing the informative equilibrium. By contrast, when the minority is “large”, in the sense that the informative equilibrium leads to the Condorcet loser winning with a high probability, all groups gave up aggregating information and coordinated their ballots on a same alternative, as predicted by Duverger’s Law. Under AV, some subjects double vote to increase the vote shares of both majority candidates. As predicted, the amount of double voting increases with the size of the minority. However, the absolute level of double voting is lower than predicted.

Comparing the two systems, we observe that subjects make fewer strategic mistakes under AV than under plurality. Moreover, when the minority is large, subjects need more time to reach equilibrium play in plurality than in AV. This suggests that voters handle more easily the larger set of voting possibilities offered by AV than the need to select an equilibrium under plurality.

Next, and in contrast with theory (which focuses on symmetric equilibria), individual behavior in AV displays substantial heterogeneity among subjects: many subjects always double vote, whereas many other subjects always single vote their signal. The observation that double-voting increases with the size of the minority is mainly driven by a switch in the relative number of subjects in each cluster. This pattern points to the need to extend the theory and consider equilibria in asymmetric strategies.⁷ Extending the model in this direction, we find that this type of behavior is indeed an equilibrium which performs particularly well in explaining the level of double-voting observed in the laboratory.

We then turn to those treatments in which the quality of information varies across states, and find that subjects adjust their behavior in line with theoretical predictions. In the case of plurality, the data provides further evidence that three-candidate equilibria are a natural focal point when majority voters have common values. In the case of AV, the results are even stronger, in the sense that voters converge faster to the theoretical prediction.

Last, we analyze the welfare properties of both electoral systems. A valuable feature of a common value setup is that it allows us to make clear welfare predictions: in equilibrium, the majority voters’ payoff should be strictly higher with AV than with plurality. This is exactly what we observe in all different treatments. Actually, information aggregation

⁷In two-candidate elections, Ladha et al. (1996) have identified situations in which there exists an asymmetric equilibrium in which voters who receive the same signal behave differently.

becomes so efficient with AV that realized payoffs become very close to what a social planner who observes all signals could achieve.

Beyond testing the very predictions of the model, these experiments also shed new light on voter rationality: determining whether voters behave strategically and respond to incentives is a central issue in the quest for better political institutions.⁸ The advantage of our setup is two pronged. First, the need to aggregate information produces different –often opposite– voting incentives from the need to coordinate ballots. Therefore, we can test whether and in which proportion subjects react to a change in incentives when we modify the relative value of coordination versus information aggregation. Second, studying multicandidate rather than two-candidate elections widens the set of electoral systems (and thus of voter incentives) that can be analyzed. In our case, the predicted behavior of voters is substantially different between plurality and AV. To the best of our knowledge, our paper is the first laboratory experiment which explores multi-alternative elections with common value voters.⁹ As should be clear from the above description of the results, it offers overwhelming support to voters behaving strategically in this context.

2 A common value model

We consider a voting game with an electorate of fixed and finite size who must elect one policy P out of three possible alternatives, A , B and C . The electorate is split in two groups: n active voters who constitute a majority, and n_C voters who constitute a minority. There are two states of nature: $\omega = \{a, b\}$, which materialize with probabilities $q(\omega) > 0$. While these probabilities are common knowledge, the actual state of nature is not observable before the election.

Active voters' utility depends both on the policy outcome and on the state of nature: utility is high ($U = V$) if A is elected and the state is a , or if B is elected and the state is b . It is intermediate ($U = v \in (0, V)$) if A wins and the state is b or if B wins and the state

⁸For evidence of strategic behavior in experimental settings with information aggregation, see Guarnaschelli, McKelvey and Palfrey (2000), Battaglini et al. (2008, 2012), Goeree and Yariv (2010), and Bhattacharya, Duffy and Kim (2012).

⁹Surprisingly, the experimental literature on multicandidate elections with private value voters is also quite slim. The seminal papers of Forsythe et al (1993, 1996) are closest to our paper. See also Rietz (2008) or Palfrey (2012) for detailed reviews of that literature. Van der Straeten et al. (2010) also study AV experimentally although in substantially different settings.

is a . Finally, utility is low (normalized to zero) if C is elected:

$$\begin{aligned} U(P|\omega) &= V \text{ if } (P, \omega) = (A, a) \text{ or } (B, b) \\ &= v \text{ if } (P, \omega) = (A, b) \text{ or } (B, a) \\ &= 0 \text{ if } P = C. \end{aligned} \tag{1}$$

For the sake of simplicity, minority voters are passive in the game: they always vote for C . Hence, C receives n_C ballots independently of the state of nature and the electoral system. Active voters must cast at least n_C ballots in favor of either A or B to avoid the victory of C . We focus on the interesting case in which C -voters represent a large minority: $n - 1 > n_C > n/2$. Thus, C is a Condorcet loser (it would lose both against A and B in a one-on-one contest), but it can win the election if active voters split their votes between A and B .

Timing. Before the election (at **time 0**), nature chooses whether the state is a or b . At **time 1**, each voter receives a signal $s \in S \equiv \{s_A, s_B\}$, with conditional probabilities $r(s|\omega) > 0$ and $r(s_A|\omega) + r(s_B|\omega) = 1$. Probabilities are common knowledge but signals are private. Signal s_A is more likely in state a than in state b :

$$r(s_A|a) > r(s_A|b), \text{ and therefore } r(s_B|a) < r(s_B|b).$$

The distribution of signals is *unbiased* if $r(s_A|a) = r(s_B|b)$. Note that $r(s_A|a) + r(s_A|b) = 1$ in this case. The distribution of signals is *biased* if $r(s_A|a) \neq r(s_B|b)$ and, by convention, we will focus on the case in which the “more abundant” signal is s_A : $r(s_A|a) + r(s_A|b) > 1$.

Having received her signal, the voter updates her beliefs about each state through Bayes’ rule: $q(\omega|s) = \frac{q(\omega)r(s|\omega)}{q(a)r(s|a) + q(b)r(s|b)}$. Like Bouton and Castanheira (2012), we assume that signals are sufficiently strong to create a *divided majority*:

$$q(a|s_A) > 1/2 > q(a|s_B). \tag{2}$$

That is, conditional on receiving signal s_A , alternative A yields strictly higher expected utility than alternative B , and conversely for a voter who receives signal s_B .

The election is held at **time 2**, when the actual state of nature is still unobserved, and payoffs realize at **time 3**: the winner of the election and the actual state of nature are revealed, and each voter receives utility $U(P, \omega)$.

Strategy space and equilibrium concept. The alternative winning the election is the one receiving the largest number of votes, with ties being broken by a fair dice. Still, the action space, *i.e.* which ballots are feasible, depends on the electoral rule. We consider two such rules: plurality and approval voting.

In *plurality*, each voter can vote for one alternative or abstain. The action set is then:

$$\Psi_{Plu} = \{A, B, C, \emptyset\},$$

where, by an abuse of notation, action A (respectively B, C) denotes a ballot in favor of A (resp. B, C) and \emptyset denotes abstention.¹⁰

In *approval voting*, each voter can approve of as many alternatives as she wishes:

$$\Psi_{AV} = \{A, B, C, AB, AC, BC, ABC, \emptyset\},$$

where, by an abuse of notation, action A denotes a ballot in favor of A only, action BC denotes a joint approval of B and C , etc. Each approval counts as one vote: when a voter only approves of A , then only alternative A is credited with a vote. If the voter approves of both A and B , then A and B are credited with one vote each, and so on. As in plurality, the alternative with the most votes wins the election.

The only difference between AV and plurality is that a voter can also cast a double or triple approval. Double approvals ($\psi = AB, BC$ and AC) can only be pivotal against one precise alternative. For instance, if the voter plays AC , her ballot can only be pivotal against B , either in favor of A or of C . A triple approval (ABC) can never be pivotal: it is strategically equivalent to abstention.

Let x_ψ denote the number of voters who played action $\psi \in \Psi_R$, $R \in \{PLU, AV\}$ at time 2. The total number of votes received by an alternative ψ is denoted by X_ψ . Under plurality the total number of votes received by alternative A , for instance, is simply: $X_A = x_A$. Under AV, it is: $X_A = x_A + x_{AB} + x_{AC} + x_{ABC}$.

A symmetric strategy is a mapping $\sigma : S \rightarrow \Delta(\Psi_R)$. We denote by $\sigma_s(\psi)$ the probability that some randomly sampled voter who received signal s plays ψ . Given a strategy σ , the *expected share of active voters* playing action ψ in state ω is thus:

$$\tau_\psi^\omega(\sigma) = \sum_s \sigma_s(\psi) \times r(s|\omega). \quad (3)$$

¹⁰ Abstention will turn out to be a dominated action in both rules. Hence, removing abstention from the choice set would not affect the analysis.

The expected number of ballots ψ is: $\mathbf{E}[x(\psi) | \omega, \sigma] = \tau_\psi^\omega(\sigma) \times n$.

Let an *action profile* x be the vector that lists, for each action ψ , the *realized* number of ballots ψ . Since we are focusing on symmetric strategies for the time being, and since the conditional probabilities of receiving a signal s are iid, the probability distribution over the possible action profiles is given by the multinomial probability distribution.

For this voting game, we analyze the properties of Bayesian Nash equilibria that (1) are in weakly undominated strategies and (2) satisfy what we call *sincere stability*. That is, the equilibrium must be robust to the case in which voters may tremble by voting sincerely (that is: $\sigma_{s_A}(A), \sigma_{s_B}(B) \geq \varepsilon > 0$. We look for sequences of equilibria with $\varepsilon \rightarrow 0$).

Some equilibrium refinement is necessary to get rid of equilibria that would only be sustainable when all pivot probabilities are exactly zero, and voters are then indifferent between all actions. Imagine for instance that all active voters play A . In that case, the number of votes for A is n and the number of votes for C is n_C , with probability 1. Voters are then indifferent between all possible actions, since a ballot can never be pivotal. Sincere stability, by imposing that a small fraction of the voters votes for their preferred alternative, implies that at least some pivot probabilities become strictly positive, and hence that indifference is broken.

The advantage of our sincere stability refinement is twofold: it captures the essence of properness in a very tractable way,¹¹ and it is behaviorally relevant. Indeed, experimental data (both in our experiments and others) suggest that some voters vote for their ex ante most preferred alternative no matter what.

3 Plurality

This section analyzes the equilibrium properties of plurality voting. We find that two types of equilibria coexist: in one, all active voters play the same (pure) strategy independently of their signal: they all vote either for A or for B . This type of equilibrium is known as a *Duverger's Law equilibrium*, in which only two alternatives receive a strictly positive vote share. In the second type of equilibrium, an active voter's strategy does depend on her signal. Depending on parameter values, this equilibrium either features *sincere voting*, that is voters with signal s_A (resp. s_B) vote A (resp. B) or a strictly mixed strategy in

¹¹We do not use more traditional refinement concepts such as *perfection* or *properness* because, in the voting context, the former does not have much bite since weakly dominated strategies are typically excluded from the equilibrium analysis. The latter is quite untractable since it requires a sophisticated comparison of pivot probabilities for totally mixed strategies.

which voters with the most abundant signal (s_A by convention) mix between A and B . Importantly, these three-party equilibria exist for any population size, are robust to signal biases, and do not feature any tie.

3.1 Pivot Probabilities and Payoffs

When deciding for which alternative to vote, a voter must first assess the expected value of each possible action, which depends on *pivot events*: unless the ballot affects the outcome of the election, it leaves the voter's utility unchanged. We denote by piv_{QP} the pivot event that one voter's ballot changes the outcome from a victory of P towards a victory of Q .

In our setup, the comparison between the three potentially relevant actions, A , B and C , is simplified by two elements: first, voting for C is a dominated action. Hence, we can set $\tau_C^\omega(\sigma)$ equal to zero. Second, a vote for A or for B can only be pivotal against C , since we impose that $n_C > n/2$. This implies that abstention is also a dominated action, and simplifies the other computations without affecting generality.

A ballot, say in favour of A can only be pivotal if the number of other A -ballots (x_A) is either the same as or one less than the number of C -ballots (n_C). To assess the probability of such an event, each active voter must identify the distribution of the other $n - 1$ votes, given the strategy σ . Dropping σ from the notation for the sake of readability, the pivot probabilities in favour of A and B are:

$$p_{AC}^\omega \equiv \Pr(piv_{AC}|\omega, \text{Plurality}) = \frac{(n-1)!}{2} \frac{(\tau_A^\omega)^{n_C-1} (\tau_B^\omega)^{n-n_C-1}}{(n_C-1)!(n-n_C-1)!} \left[\frac{\tau_A^\omega}{n_C} + \frac{\tau_B^\omega}{n-n_C} \right], \quad (4)$$

$$p_{BC}^\omega \equiv \Pr(piv_{BC}|\omega, \text{Plurality}) = \frac{(n-1)!}{2} \frac{(\tau_B^\omega)^{n_C-1} (\tau_A^\omega)^{n-n_C-1}}{(n_C-1)!(n-n_C-1)!} \left[\frac{\tau_B^\omega}{n_C} + \frac{\tau_A^\omega}{n-n_C} \right], \quad (5)$$

where the two terms between brackets represent the cases in which one vote respectively breaks and makes a tie. Note that pivot probabilities are continuous in τ_A^ω and τ_B^ω .

Let $G(\psi|s)$ denote the *expected gain* of an action $\psi \in \{A, B\}$ over abstention, \emptyset :

$$G(A|s) = q(a|s) p_{AC}^a V + q(b|s) p_{AC}^b v (> 0), \quad (6)$$

$$G(B|s) = q(a|s) p_{BC}^a v + q(b|s) p_{BC}^b V (> 0). \quad (7)$$

Since both actions yield higher payoffs than abstention, the latter is dominated. The pay-off

difference between actions A and B is:

$$G(A|s) - G(B|s) = q(a|s) [Vp_{AC}^a - vp_{BC}^a] + q(b|s) [vp_{AC}^b - Vp_{BC}^b]. \quad (8)$$

3.2 Duverger's Law Equilibria

The game theoretic version of Duverger's Law (Duverger 1963, Riker 1982, Palfrey 1989, Myerson and Weber 1993, Cox 1997) states that, when voters play strategically, only two alternatives should obtain a strictly positive fraction of the votes in plurality elections. In our setup, these equilibria are such that:

Definition 1 *A Duverger's Law equilibrium is a voting equilibrium in which only two alternatives obtain a strictly positive fraction of the votes. Majority voters thus concentrate all their ballots either on A or on B .*

These Duverger's Law equilibria feature pros and cons. On the one hand, they ensure that C cannot win the election. On the other hand, they prevent information aggregation. That is, the winner of the election is fully determined by voter coordination, and cannot vary with the state of nature. Our first proposition is that:

Proposition 1 *In plurality, Duverger's Law equilibria exist for any electorate size, prior probabilities of the two states, and distribution of signals.*

Proof. Consider e.g. $\sigma_{s_A}(A) = \varepsilon$ and $\sigma_{s_B}(B) = 1$. From (4) and (5), we have:

$$\frac{p_{AC}^\omega}{p_{BC}^\omega} = \left(\frac{\tau_A^\omega}{\tau_B^\omega} \right)^{2n_C - n} \frac{\tau_A^\omega (n - n_C) + \tau_B^\omega n_C}{\tau_A^\omega n_C + \tau_B^\omega (n - n_C)} \xrightarrow{\varepsilon \rightarrow 0} 0.$$

Hence, from (8), we have that $G(A|s) - G(B|s) < 0$ for any ε in the neighborhood of 0. ■

The reason why Duverger's Law equilibria exist in plurality elections is the classical one: voters do not want to waste their ballot on an alternative that is very unlikely to win. Consider for instance the strategy profile $\sigma(B|s_A) = 1 - \varepsilon$ and $\sigma(B|s_B) = 1$ with ε strictly positive but arbitrarily small. In that case, an A -ballot is much less likely to be pivotal against C than a B -ballot.¹² Therefore, the value of a B -ballot is larger than that of an A -ballot, both for s_A - and s_B -voters.

¹²For $\sigma(B|s_A) = 1 = \sigma(B|s_B)$, all pivot probabilities are equal to zero. In this case, voters are indifferent between all actions. Sincere stability means that we identify incentives for $\sigma(B|s_A) \rightarrow 1$. They imply that $G(B|s_A) > G(A|s_A)$ in the neighborhood of this Duverger's Law equilibrium.

3.3 Informative Equilibria

In Duverger’s Law equilibria, the information dispersed among active voters is therefore lost. Still, this type of equilibrium is typically considered as the only reasonable one if voters are *short-term instrumentally rational*, in Cox’s (1997) terminology. Indeed, in a private value setup, equilibria with more than two alternatives obtaining votes are typically “knife edge” and “expectationally unstable” (Palfrey 1989 and Fey 1997).¹³ Therefore, empirical research associates strategic voting with the voters’ propensity to abandon their preferred but non-viable candidates, and vote for more serious contenders (see Blais and Nadeau 1996, Cox 1997, Alvarez and Nagler 2000, Blais et al 2005). Observing that only relatively low fractions of the electorate switch to their second-best alternative in this way is then interpreted as evidence that few voters are instrumental or rational.

Yet, as shown by Propositions 2 and 3 below, common values among majority voters gives rise to other equilibria in which “short-term instrumentally rational voters” should actually deviate from either Duverger’s Law equilibria or “knife-edge” three-candidate equilibria. The key difference is that, in our setup, voters value the information generated by their own and by other voters’ ballots. Like in Austen-Smith and Banks (1996) and Myerson (1998), they compare pivot probabilities across states of nature. In what we call an *informative equilibrium*, these pivot probabilities are sufficiently close to one another and (i) all alternatives receive a strictly positive vote share, (ii) these vote shares are different across alternatives (no knife-edge equilibrium), and (iii) A is the strongest majority contender in state a , and B in state b .¹⁴

When information is close to being symmetric across states, voters vote *sincerely* in an informative equilibrium: a voter who receives signal s_A votes for A , whereas a voter who receives signal s_B votes for B . That is, abandoning one’s preferred candidate would *not* be a best response when one expects other voters to vote sincerely:

Proposition 2 *In the unbiased case $r(s_A|a) = r(s_B|b)$, the sincere voting equilibrium exists $\forall n, n_c$. Moreover, there exists a value $\delta(n, n_c) > 0$ such that sincere voting is an equilibrium for any asymmetric distribution satisfying $r(s_A|a) - r(s_B|b) < \delta(n, n_c)$.*

Proof. We start with the unbiased case, i.e. $r(s_A|a) = r(s_B|b)$. Under sincere voting, $\sigma_{s_A}(A) =$

¹³An exception is Dewan and Myatt (2007) and Myatt (2007) who emphasize the existence of three-candidate equilibria when there is aggregate uncertainty.

¹⁴If, in addition, the expected vote shares of A and of B in their respective state is sufficiently larger than C ’s, then the informative equilibrium is also *expectationally stable* in the sense of Fey (1997). See also Bouton and Castanheira (2009, Propositions 7.3 and 7.4).

$1 = \sigma_{s_B}(B)$, (4) and (5) imply $p_{AC}^a = p_{BC}^b > p_{AC}^b = p_{BC}^a$. Then, from (8):

$$G(A|s) - G(B|s) = [Vp_{AC}^a - vp_{BC}^a][q(a|s) - q(b|s)].$$

Since $q(a|s_A) - q(b|s_A) > 0 > q(a|s_B) - q(b|s_B)$, this implies $G(A|s_A) - G(B|s_A) > 0 > G(A|s_B) - G(B|s_B)$. Sincere voting is thus an equilibrium strategy. By the continuity of pivot probabilities with respect to τ_A^ω and τ_B^ω , it immediately follows that there must exist a value $\delta(n, n_C) > 0$ such that sincere voting is an equilibrium for any $|r(s_A|a) - r(s_B|b)| < \delta(n, n_C)$. ■

The intuition for the proof is simply that, in the unbiased case, sincere voting implies that the likelihood of being pivotal against C is the same with an A -ballot in state a as with a B -ballot in state b . Therefore, s_A -voters strictly prefer to vote for A and s_B -voters strictly prefer to vote for B . The pros and cons of sincere voting are the exact flipside of the ones identified for Duverger's Law equilibria: as illustrated by the following example, it allows for learning, but does not guarantee a defeat of the Condorcet loser.

Example 1 Consider a case in which $n = 12$, $n_C = 7$, and $r(s_A|a) = r(s_B|b) = 2/3$.¹⁵ Then, sincere voting implies that the best alternative (A in state a ; B in state b) has the highest expected vote share and wins with a probability of 73%. C has the second largest expected vote share and wins with a probability of 23% in either state. The alternative with the lowest –but strictly positive– vote share is B in state a and A in state b .

When n_C is 9, the alternative with the largest expected vote share is C , who then wins with a probability above 71%, whereas the best alternative wins with a probability below 29%.

Based on Proposition 2 and Example 1, one may be misled into thinking that informative equilibria require that signals are close to being unbiased.¹⁶ Yet, the fact that the signal structure becomes too biased to sustain sincere voting does not imply that voters switch to a Duverger's Law equilibrium: Proposition 3 instead shows that an informative equilibrium still exists. In that equilibrium, s_A -voters adopt a mixed strategy and vote for B with strictly positive probability. This allows them to lean against the bias in the signal structure:

¹⁵Each numerical example reproduces the parameters used in one of the treatments of our laboratory experiments (see Section 5). In all examples, the two states of nature are equally likely, and the payoffs are: $V = 200$; $v = 110$ and the value of C is 20. Normalizing the latter to 0 would also reduce the other payoffs by 20.

¹⁶Note that, for a given bias $r(s_A|a) - r(s_B|b) > 0$, sincere voting is only an equilibrium if electorate size is sufficiently small: as electorate size increases to infinity, given the biased signal structure, the ratio of pivot probabilities would either converge to zero or infinity if voters kept voting sincerely.

Proposition 3 *Let $r(s_A|a) - r(s_B|b) > \delta(n, n_c)$. Then, there exists a mixed strategy equilibrium with $\sigma_{s_A}(A) \in (0, 1)$ and $\sigma_{s_B}(B) = 1$, such that alternative A receives strictly more votes in state a than in state b , and conversely for alternative B .*

Proof. See Appendix A2. ■

The intuition for this result is that strong biases in the signal structure imply that the difference in pivot probabilities between states a and b becomes too large if voters keep voting sincerely. To compensate for this bias, s_A -voters must lend some support to B . The proof shows that one such strictly mixed strategy must be an equilibrium. It is such that s_A -voters are indifferent between voting A and B , whereas s_B -voters strictly prefer the latter. The intuition for the proof of this result is best conveyed with a second example:

Example 2 *Electorate size is $n = 12$ and $n_C = 7$, and the signal structure is $r(s_A|a) = 8/9 > 2/3 = r(s_B|b)$. For these parameter values, an s_A -voter would strictly prefer to vote for B if all the other voters were to vote sincerely. Indeed, sincere voting implies:¹⁷*

$$\varrho \equiv \frac{Vp_{BC}^b - vp_{AC}^b}{Vp_{AC}^a - vp_{BC}^a} = 13.6 > \frac{8}{3} = \frac{q(a|s_A)}{q(b|s_A)}. \quad (9)$$

That is, the probability of being pivotal in favour of B in state b is much larger than any other pivot probability, which implies $G(A|s_A) - G(B|s_A) < 0$. The mixed-strategy equilibrium is reached when $\sigma_{s_A}(A) = 0.915$ and $\sigma_{s_B}(B) = 1$: by reducing the expected vote share of A and increasing that of B , the relative probability of being pivotal in favour of A in state a increases to the point in which $\varrho = 8/3$.

As a consequence, s_A -voters are now indifferent between voting A and B , whereas s_B -voters still strictly prefer to vote B . Importantly, all vote shares are strictly positive and the full information Condorcet winner is the most likely winner in both states of nature (their winning probabilities are respectively 96% and 79% in states a and b):

$$\tau_A^a = 0.81 > \tau_B^b = 0.69 > \frac{n_C}{n} = 0.58 > \tau_A^b = 0.31 > \tau_B^a = 0.19.$$

This informative equilibrium gives C a strictly positive probability of victory (18% in state b and 3% in state a) but expected utility is higher in this equilibrium than in a Duverger's Law equilibrium.

The example illustrates that neither the existence nor the stability of this equilibrium

¹⁷Note that, by (8), $G(A|s) - G(B|s) > 0$ iff the LHS of (9) is smaller than the RHS.

relies on some form of symmetry between vote shares. Also, as proved by Bouton and Castanheira (2009), this mixed-strategy equilibrium also exists in large electorates, with the difference that the gap between τ_A^a and τ_B^b decreases to zero (*i.e.* $\lim_{n \rightarrow \infty} \tau_A^a = \lim_{n \rightarrow \infty} \tau_B^b$), and that stability relies on $r(s_A|a)$ being sufficiently larger than n_C/n .

4 Approval Voting

4.1 Payoffs and Dominated Strategies

Under AV, voters have access to a larger choice set, which makes their choice potentially more complex. Single approvals (A , B , C) have exactly the same effect as in plurality. Double or triple approvals instead ensure that one selectively abstains between the approved alternatives. For instance, an AB -ballot can only be pivotal against C . The following lemma shows that the set of undominated strategies is more restricted:

Lemma 1 *Independently of a voter's signal, the actions $\psi \in \{C, AC, BC, ABC, \emptyset\}$ are weakly dominated by some action in $\psi \in \{A, B, AB\}$. Hence, in equilibrium:*

$$\sigma_s(A) + \sigma_s(B) + \sigma_s(AB) = 1, \quad \forall s \in \{s_A, s_B\}. \quad (10)$$

Proof. *Straightforward.* ■

The intuition for the lemma is that abstaining or approving of C can only increase C 's probability of winning. In contrast, the actions in the undominated set (A , B , and AB) can only reduce it. The remaining question is how a voter may want to allocate her ballot across these undominated actions. This depends on the probability of each pivot event. Let π_{QP}^ω denote the probability that a single- Q ballot is pivotal in favor of Q at the expense of P in state $\omega \in \{a, b\}$ and the voting rule is AV. The derivation of these pivot probabilities are detailed in Appendix A1.

The expected value G^{AV} of a single- A ballot under AV is then:

$$G^{AV}(A|s) = q(a|s) [\pi_{AC}^a V + \pi_{AB}^a (V - v)] + q(b|s) [\pi_{AC}^b v + \pi_{AB}^b (v - V)]. \quad (11)$$

Note that the probability of being pivotal between A and B is no longer zero, since double voting can increase the score of both A and B above that of C . Similarly, the value of a

single- B ballot is:

$$G^{AV}(B|s) = q(a|s)[\pi_{BC}^a v + \pi_{BA}^a (v - V)] + q(b|s)[\pi_{BC}^b V + \pi_{BA}^b (V - v)]. \quad (12)$$

The value of a double ballot follows almost immediately from (11) and (12). Double voting cannot be pivotal between A and B , while adding up the chances of being pivotal against C , either in favor of A or in favor of B :

$$G^{AV}(AB|s) = q(a|s)[\pi_{AC}^a V + \pi_{BC}^a v - \phi^a] + q(b|s)[\pi_{AC}^b v + \pi_{BC}^b V - \phi^b], \quad (13)$$

where ϕ^a and ϕ^b are correcting terms for three-way ties (see Appendix A1 for a precise definition). These correcting terms become vanishingly small and can be omitted when the population size increases towards infinity. Yet, our purpose in this paper is to assess the properties of plurality and AV both for small-committee and for large-population elections, which implies that we need to take them into account.¹⁸

From (11) and (13), the payoff differential between actions A and AB is:

$$\begin{aligned} G^{AV}(A|s) - G^{AV}(AB|s) &= q(a|s)[\pi_{AB}^a (V - v) - \pi_{BC}^a v + \phi^a] \\ &\quad + q(b|s)[\pi_{AB}^b (v - V) - \pi_{BC}^b V + \phi^b]. \end{aligned} \quad (14)$$

With straightforward, although tedious, manipulations, one finds that the first term in (14) may either be positive or negative, whereas the second is strictly negative. Similarly, the first term in (15) is strictly negative:

$$\begin{aligned} G^{AV}(B|s) - G^{AV}(AB|s) &= q(a|s)[\pi_{BA}^a (v - V) - \pi_{AC}^a V + \phi^a] \\ &\quad + q(b|s)[\pi_{BA}^b (V - v) - \pi_{AC}^b v + \phi^b]. \end{aligned} \quad (15)$$

4.2 Equilibrium Analysis

The action set under AV is an extension of the action set under plurality. Therefore, in a common value setting as ours, there is always an equilibrium in AV for which welfare is (weakly) higher than for any equilibrium in plurality (Ahn and Oliveros 2011, Proposition 1).¹⁹ Furthermore, our setup imposes that the size of the minority is large. As we observed

¹⁸These correcting terms actually prove extremely relevant for the characterization of the asymmetric equilibria that we analyze in Section 6.3.1.

¹⁹Ahn and Oliveros (2011) exploit McLennan (1998) to show that, in a common value setup as ours, one can rank equilibrium outcomes under approval voting as opposed to plurality and negative voting. By

in Section 3, this implies that the probability of being pivotal between A and B is zero under plurality. Theorem 1 directly follows from that fact and from (14 – 15):

Theorem 1 *There always exists a sincerely stable equilibrium in AV for which expected welfare is strictly higher than for any equilibrium in plurality. In that equilibrium, some voters must double vote, and $\sigma_{s_A}(A), \sigma_{s_B}(B) > 0$.*

Proof. See Appendix A3. ■

The intuition for this result is as follows: when one compares the set of undominated actions in plurality and in AV, one sees that the only relevant difference is the possibility to double vote AB . When no “other” voter double votes (which is the case under any equilibrium strategy in plurality) any voter must realize that she can never be pivotal between A and B . In this case, she strictly prefers to double vote, to maximize her probability of being pivotal against C ($G^{AV}(P|s) - G^{AV}(AB|s) < 0, \forall P \in \{A, B\}$).²⁰ Moreover, since voters have common value preferences, if such a deviation is beneficial for one voter, it must also increase the other voters’ expected utility. Two corollaries follow from Theorem 1:

Corollary 1 *The strategies that are an equilibrium in plurality cannot be an equilibrium in AV. In particular, Duverger’s Law equilibria do not exist under AV.*

Double voting has pros and cons in terms of the election outcome. On the one hand, it reduces the risk that C wins the election. On the other hand, a voter who double votes does not reveal her signal. Yet, there can never be so much double voting that information aggregation is impossible:

Corollary 2 *Pure double voting is never an equilibrium in AV.*

The reason is straightforward: if all the other voters double vote, then voter i knows (a) that her vote cannot be pivotal against C and (b) that she is as likely to be pivotal in state a as in state b . Hence, her preferred reaction is to single vote her signal.

Pure double voting has been termed the *Burr dilemma* by Nagel (2007), who argues that approval voting is inherently biased towards such ties. He documents this with the

revealed preferences, since the action set in the two other rules is a strict subset of the action set under AV, “the maximal equilibrium utility under approval voting is greater than or equal to the maximal equilibrium utility under plurality voting or under negative voting.” (p. 3).

²⁰This is due to the fact that we focus on large minorities. If the size of the minority, n_C , falls towards zero, then the propensity to double vote may well drop to zero as well (see Bouton and Castanheira, 2012).

“[approval] *experiment* [that] *ended disastrously in 1800 with the infamous Electoral College tie between Jefferson and Burr*”. Lemma 2 shows why such a “disaster” cannot be an equilibrium when voting behavior is not dictated by party discipline.

Together, Corollaries 1 and 2 show that a voter’s best response is to double vote if the other voters single vote “excessively” and to single vote sincerely if the other voters double vote “excessively”. In a large Poisson game setup, Bouton and Castanheira (2012) shows that this pattern is monotonic, and that the relative value of the double and single votes cross only once. In other words, AV displays a unique equilibrium. In contrast, we do not focus on arbitrarily large electorates. This implies that one can no longer establish a general proof of equilibrium uniqueness. Yet, our next theorem pinpoints unique voting patterns for any *interior* equilibrium:

Theorem 2 *Whenever both s_A - and s_B -voters adopt a nondegenerate mixed strategy, then it must be that voters with signal s_A only mix between A and AB , and voters with signal s_B only mix between B and AB .*

Proof. See Appendix A3. ■

This theorem builds on the comparison between the preferences of s_A and s_B voters: conjecture for instance a case in which the former play B with strictly positive probabilities. Since a voter with signal s_B values B even more, it must only play B , which contradicts the very nature of an interior equilibrium. To extend this result to equilibria in which (one of the two groups of) voters play pure strategies, we would have to focus on larger electorates, which is not the purpose of our analysis. Yet, we can rely on numerical simulations. For all the parametric values we checked, the equilibrium was unique and such that voters with signal s_A never play B (i.e. they mix between A and AB), and voters with signal s_B never play A . This held both for interior equilibria and for equilibria in which (one of the two groups of) voters play a degenerate strategy.

Two additional examples are useful to better understand the features and comparative statics of voting equilibria in AV:

Example 3 *Consider the same set of parameters as in Example 1: $n = 12$, $n_C = 7$ or 9 , and $r(s_A|a) = r(s_B|b) = 2/3$. As just emphasized, the equilibrium is unique under AV.²¹*

²¹In the strategy space $(\sigma_{s_A}(A), \sigma_{s_B}(B))$, there is a unique cutoff for which $G(A|s_A) = G(AB|s_A)$, and the same holds for $G(B|s_B) = G(AB|s_B)$. The equilibrium lies at the intersection between these two reaction functions.

It is such that:

$$\begin{aligned} \sigma_{s_A}(A) = \sigma_{s_B}(B) = 0.64 \text{ and } \sigma_{s_A}(AB) = \sigma_{s_B}(AB) = 0.36 \text{ when } n_C = 7, \\ \sigma_{s_A}(A) = \sigma_{s_B}(B) = 0.30 \text{ and } \sigma_{s_A}(AB) = \sigma_{s_B}(AB) = 0.70 \text{ when } n_C = 9. \end{aligned}$$

When $n_C = 7$, these equilibrium profiles imply that A wins with a probability of 82% in state a (as does B in state b), whereas C 's probability of winning is below 1%. When $n_C = 9$, A wins with a probability of 73% in state a (as does B in state b), whereas C 's probability of winning remains as low as 1.5%. These values should be contrasted with the sincere voting equilibrium in plurality (see example 1), in which the probability of selecting the best outcome was substantially lower, and the risk that C wins was substantially larger.

Comparing equilibrium behavior with $n_C = 7$ and $n_C = 9$ in Example 3 shows that the larger n_C , the more double voting in equilibrium. This pattern was found to be monotonic and consistent across numerical examples for any value of n and signal structures.

Example 4 Consider the same set of parameters as in Example 3, except for $r(s_A|a) = 8/9$. This reproduces the biased signal setup of Example 2. Like in the previous example, the equilibrium is unique. It yields: $\sigma_{s_A}(A) = 0.26 < \sigma_{s_B}(B) = 0.52$ and $\sigma_{s_A}(AB) = 0.74 > \sigma_{s_B}(AB) = 0.48$. This equilibrium profile implies that A wins with a probability of 87% in state a , whereas B wins with a probability of 90% in state b . C 's winning probabilities are 0.5% in state a and 2.8% in state b .

The equilibrium with biased information has the property that the voters with the most abundant signal single vote less than the voters with the least abundant signal. The rationale for this result might be obvious to the readers knowledgeable about the Condorcet Jury Theorem: if s_A - and s_B -voters were to single vote with the same probability, A 's winning probabilities would be disproportionately higher than B 's. Moreover, the pivot probabilities between A and B would be lower in state a than in state b , which should induce all voters to put more value on being pivotal in favour of B .

5 Experimental Design and Procedures

To test our theoretical predictions we ran controlled laboratory experiments. Subjects were introduced to a game that had the very same structure as the one presented in the model of Section 2. All participants were given the role of an active voter, whereas passive voters

were simulated by the computer.²² Following the experimental literature on the Condorcet Jury Theorem initiated by Guarnaschelli et al (2000), the two states of the world were called *blue jar* and *red jar*, whereas the signals were called *blue ball* and *red ball*. The red jar contained six red balls and three blue balls. Depending on the treatment, the blue jar contained either six blue and three red balls (unbiased signals) or eight blue and one red ball (biased signals). One of the jars was selected randomly by the computer, with equal probability. The subjects were not told which jar had been selected, but were told how the probability of receiving a ball of each color depended on the selected jar. After seeing their ball, each subject could vote from a set of three candidates: *blue*, *red* or *gray*.²³ *Blue* and *red* were the two majority candidates and *gray* was the Condorcet loser. Subjects were told that the computer casts n_C votes for *gray* in each election (n_C varied across treatments).

The subjects' payoff depended on the color of the selected jar and that of the election winner. If the color of the winner matched that of the jar, the payoff to all members of the group was 200 euro cents. If the winner was blue and the jar red or the other way around, their payoff was 110 cents. Finally, if gray won, their payoff was 20 cents.

We consider three treatment variables, which leads to six different treatments. The first variable is the *voting mechanism*: in *PL* treatments, the voting mechanism was plurality. In this case, subjects could vote for only one of the three candidates. In *AV* treatments, the voting mechanism was approval voting. In this case, subjects could vote for any number of candidates.²⁴ With either mechanism, the candidate with the most votes wins, and ties were broken with equal probability. The second variable is the *size of the minority*, n_C , which was set to either 7 or 9. We will refer to them as *small* and *large minority*. The third variable is whether the signal structure is *unbiased* or *biased*. In unbiased treatments, signal precision was identical across states and set to $r(\text{blue ball} \mid \text{blue jar}) = r(\text{red ball} \mid \text{red jar}) = 2/3$. In biased treatments (which we indicate by *B*), $r(\text{blue ball} \mid \text{blue jar})$ was increased to $8/9$. Table 1 provides an overview of the different treatments.

Experiments were conducted at the BonnEconLab of the University of Bonn between July 2011 and January 2012. We ran a total of 18 sessions with 24 subjects each. No subject

²²Morton and Tyran (2012) show that preferences in one group are not affected by the preferences of an opposite group. Therefore, having computerized rather than human subjects should not alter the behavior of majority voters in a significant way. Having partisans (the equivalent to our passive voters) simulated by the computer has been used in previous studies – see Battaglini et al. (2008, 2010).

²³The colors that we used in the experiments were *blau*, *rot* and *schwarz*. Throughout the paper, however, we refer to blue, red and gray respectively.

²⁴As in Guarnaschelli et al. (2000), abstention was not allowed (remember that abstention is always a strictly dominated action). In a similar setting to ours, Forsythe et al (1993) allowed for abstention and found that the abstention rate was as low as 0.65%.

Treatment	Voting rule	Minority size (n_c)	Precision Blue State	Precision Red State	Sessions / Ind. Obs.	Group numbers
PL7	Plurality	7	2/3	2/3	3 / 6	1-6
PL9	Plurality	9	2/3	2/3	3 / 6	7-12
AV7	Approval	7	2/3	2/3	3 / 6	13-18
AV9	Approval	9	2/3	2/3	3 / 6	19-24
PL7B	Plurality	7	8/9	2/3	3 / 6	25-30
AV7B	Approval	7	8/9	2/3	3 / 6	31-36

Table 1: Treatment overview. Note: ind. obs. stands for “individual observations”.

participated in more than one session. Students were recruited through the online recruitment system ORSEE (Greiner 2004) and the experiment was programmed and conducted with the software z-Tree (Fischbacher 2007).

All experimental sessions were organized along the same procedure: subjects received detailed written instructions, which an instructor read aloud (see supplementary appendix). Each session proceeded in two parts: in the first part, subjects played one of the treatments in fixed groups for 100 periods.²⁵ Before starting, subjects were asked to answer a questionnaire to check their full understanding of the experimental design. In the second part, subjects received new instructions, and made 10 choices in simple lotteries, as in Holt and Laury (2002). We ran this second part to elicit subjects’ risk preferences.

To determine payment, the computer randomly selected four periods from the first part and one lottery from the second part.²⁶ In total, subjects earned an average of €13.47, including a showup-fee of €3. Each experimental session lasted approximately one hour.

6 Experimental Results

Section 6.1 presents our experimental results when information is unbiased, and Section 6.2 when it is biased. Section 6.3 turns to individual behavior and extends the model to asymmetric equilibria. Finally, Section 6.4 turns to aggregate outcomes and welfare.

²⁵In the setup of the Condorcet Jury Theorem, Ali et al (2008) find no significant difference between random matching (or *ad hoc committees*) and fixed matching (or *standing committees*).

²⁶In the first round of experiments (the seven sessions with the groups 1, 2, 7, 8, 9, 10, 13, 14, 15, 16, 19, 20, 21 and 22), we selected seven periods to determine payment. We reduced this to four periods after realizing that the experiment had taken much less time than expected. We find no difference in behavior between these two sets of sessions.

Treatment	Minority Size		Periods	Periods	Equilibrium	
			1-50	51-100	Sincere Voting	Duverger’s Law
PL7	Small	Signal	91.80	90.94	100.00	50.00
		Opposite	7.78	8.89	-	50.00
		Gray	0.42	0.17	-	-
PL9	Large	Signal	68.47	59.25	100.00	48.92*
		Opposite	31.11	40.67	-	51.08*
		Gray	0.41	0.08	-	-

Table 2: Aggregate voting behavior in plurality treatments with unbiased information, separated by first and second half, and equilibrium predictions. * In the case of Duverger’s Law in PL9, the prediction is adjusted to the color that each group converged to.

6.1 Unbiased Treatments

6.1.1 Plurality

As shown in Section 3, two types of equilibria coexist under plurality when information is unbiased: Duverger’s Law and sincere voting equilibria. In the former type of equilibria, participants should disregard their signal and coordinate on voting always blue or always red. In the latter instead, participants should vote their signal. Table 2 shows the average frequencies with which subjects voted sincerely (we call this *voting the signal*), for the majority color opposite to their signal (we will call this *voting opposite*) or for gray.²⁷

In the presence of a small minority, the participants’ voting behavior is consistent with sincere voting: taking an average across all groups and periods, 91.38% of the ballots were sincere in PL7, with a lowest value of 86.42% in one independent group. This behavior is quite stable over time: we regressed the frequency of “voting the signal” on the period number, and found that the coefficient was not significantly different from zero. Most deviations from sincere voting behavior consisted of “voting opposite”, which might be related to the “gambler fallacy”.²⁸ Finally, less than 0.5% of the votes went to gray.

Voting behavior is substantially different in the presence of a large minority (PL9). First, only 63.86% of the observations are consistent with sincere voting. Second, performing the

²⁷The figures with a ‘*’ report the predicted voting pattern for the last 50 periods, conditional on the color on which the group coordinated. For instance, if the group coordinated on blue, and if 40% of the voters obtain a blue ball in a given draw, then 40% should play “signal” and 60% “opposite”.

²⁸The gambler’s fallacy is the mistaken notion that the likelihood of an event that occurs with a fixed probability increases or decreases depending upon recent occurrences. The gambler’s fallacy has been documented extensively (see e.g. Tversky and Kahneman 1971). In our experiment, the gambler’s fallacy might lead subjects to disregard signals on the ground that the perceived likelihood of the signal being wrong is higher than the likelihood of the signal being right after some particular histories.

same regression on the period number, we found a clear and significant (p -value: 1%) trend: the frequency of voting the signal decreases over time. Participants begin the experiment by voting sincerely (94.44% of them voted their signal in the first period) but rapidly abandon that strategy (see below) and eventually adjust their behavior by voting against their signal (only 53.70% voted sincerely in the last period). This pattern is fully consistent with the progressive shift from a sincere voting equilibrium to a Duverger's Law equilibrium. Figure 1 illustrates this shift by plotting the observed frequency of voting blue, red and gray (irrespective of the signals subjects receive) for each group in the PL9 treatment. The horizontal dashed line displays the minimal vote share required to defeat gray (in case nobody plays the dominated strategy of voting gray). As one can see, all six groups converged to a Duverger's Law equilibrium.

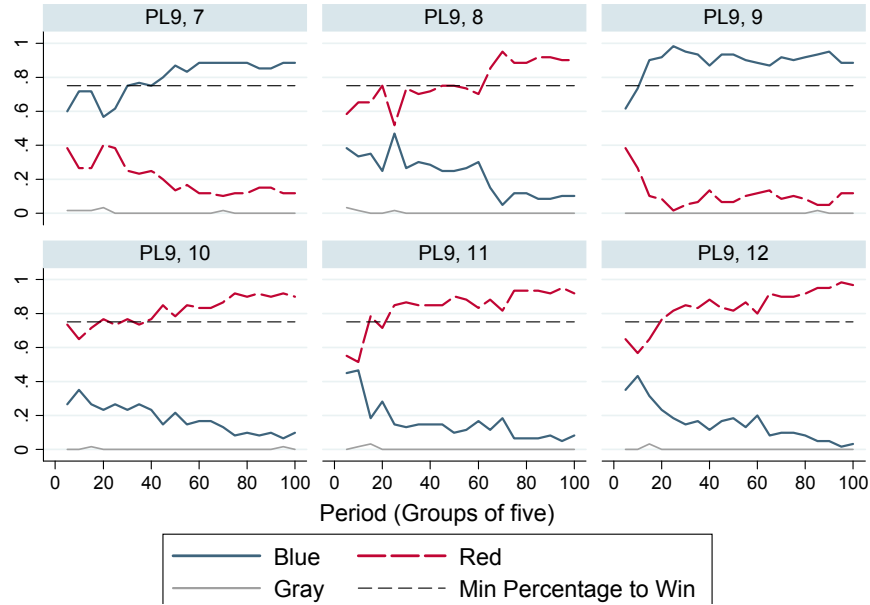


Figure 1: Frequency of voting blue, red and gray irrespective of the signal in groups of treatment PL9U. The dashed line indicates the minimum frequency of vote share required to defeat gray (in case nobody from the majority votes for the Condorcet loser).

This raises two empirical questions relating to equilibrium selection. The first one is *why* all groups selected a Duverger's Law equilibrium in the PL9 treatment, and the informative equilibrium in the PL7 treatment. The second question is *how* each PL9 group selected its Duverger's Law equilibrium.

We can identify at least two reasons why Duverger's Law equilibria are the most natural

focal point in PL9: first, the expected utility in the informative equilibrium is 69.76 in PL9, instead of 152.76 in PL7. This compares with an expected utility of 155 in a Duverger's Law equilibrium. The incentive to get away from sincere voting is thus substantial in PL9. Second, the range of strategy profiles for which sincere voting is a best response is quite narrow in the case of PL9. The phase diagrams in Figure 2 illustrate this graphically. The horizontal axis displays the other blue voters' propensity to vote blue, and the vertical axis displays the other red voters' propensity to vote red. The solid curve represents the locus of strategies for which a given voter is indifferent between playing blue and red if she receives a blue ball. To the left of that curve, her payoff of playing red is higher than that of playing blue, and conversely to the right of the curve. The dashed curve represents the equivalent locus for a voter who receives a red ball. Above the curve, she prefers red to blue, and conversely below the curve. The arrows display the attraction zones of each of the three equilibria mentioned: sincere voting in the top right corner, and the two Duverger's Law equilibria in the bottom right and top left corners. The attraction zone of the sincere voting equilibrium is much larger in PL7 than in PL9.²⁹ Therefore, even relatively small deviations from sincere voting make it optimal to vote for the leading majority candidate in PL9.

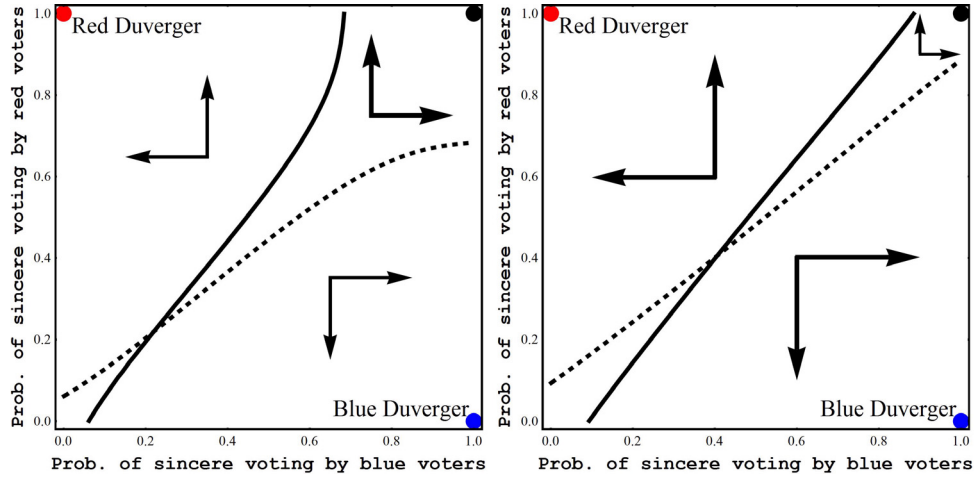


Figure 2: Phase diagram of treatments PL7 and PL9. The horizontal axis displays the probability of sincere voting by blue voters while the vertical axis displays the probability of sincere voting by red voters. The solid line indicates the indifference curve for the blue voters, while the dashed line indicates the indifference curve for the red voters.

²⁹Confronting the strategies actually played by the subjects to these theoretical predictions, we found that, even in early periods, the typical strategy falls outside the sincere voting attraction zone in PL9, and inside that zone in PL7.

Treatment	Minority Size		Periods	Periods	Equilibrium
			1-50	51-100	
AV7	Small	Signal	70.92	71.94	64.00
		Double Vote	22.22	24.36	36.00
		Opposite	6.50	3.69	-
		Gray	0.36	0.00	-
AV9	Large	Signal	47.08	43.33	30.00
		Double Vote	45.67	51.64	70.00
		Opposite	6.86	4.97	-
		Gray	0.39	0.06	-

Table 3: Aggregate voting behavior in approval voting treatments with unbiased information. Gray refers to voting for gray or a combination of gray and others.

Turning to the second question, most groups coordinate on the first color obtaining strictly more than six of the majority votes.³⁰ This is in line with the findings of Forsythe et al (1993, p235): *“a majority candidate who was ahead of the other in early elections tended to win the later elections, while the other majority candidate was driven out of subsequent races”*. Yet, the transition from sincere voting to the selected Duverger’s Law equilibrium can take a substantial amount of time: the first period from which either blue or red repeatedly obtained enough votes to win was 50, 59, 83, 63, 21 and 26 for groups 7–12 respectively. This shows that experiments using shorter horizons may fail to capture equilibrium convergence. A reason can be the time needed to learn which strategy is actually played by the other voters (see Fey 1997 for an analysis of such dynamics).

6.1.2 Approval Voting

Table 3 summarizes behavior in AV treatments. It displays the frequencies with which subjects single vote their signal, double vote red and blue, single vote opposite to their signal, and vote gray (possibly in combination with another candidate).

These two treatments reproduce the parametric cases covered in Example 3, which we found to display a unique symmetric equilibrium. In that equilibrium a voter should only single vote her signal or double vote blue and red. A huge majority of actions were in line with this theoretical prediction: 94.73% in the case of AV7 and 93.86% in the case of AV9. One could think that AV involves higher complexity and therefore higher frequency of mistakes, but we actually observed the opposite. We define mistakes as playing an action

³⁰It happened in period 1 for four groups and in period 2 for one group. The only exception is group 11, where blue got 7 votes in the first period and then red received more votes from period 2 onwards.

that is not a best response to the equilibrium. That is, in PL7, AV7 or AV9, a subject made a mistake when she voted opposite to her signal or for gray; in PL9, she made a mistake when voting for another color than the one the rest of the group converged to. We find that subjects made more mistakes under plurality than under AV. In treatments with a small minority, the percentages of mistakes in the second half (where equilibrium selection is clearer) were 3.69% in AV7 as opposed to 9.06% in PL7. In treatments with a large minority, they represented 5.03% of the ballots in AV9 as opposed to 11.06% in PL9. These differences are significant in both cases (Mann-Whitney, $z = 2.082$, $p = 0.0374$ with $c = 7$ and $z = 1.925$, $p = 0.0542$ with $c = 9$).³¹

The second theoretical prediction drawn from Example 3 refers to the effect of minority size: it should increase the frequency of double voting. The rationale is that voters need to double vote more to contain the risk that gray wins. Table 3 shows that this is indeed the way in which the subjects adapted their voting behavior: the percentage of double voting was multiplied by more than two, from 23.29% in treatment AV7 to 48.66% in treatment AV9. This difference is significant at 1% (Mann-Whitney, $z = 2.722$, $p < 0.01$).

Although the comparative statics go in the direction predicted by theory, one should notice that the amount of double-voting was well below theoretical predictions: 24.36% instead of 36.00% in treatment AV7, and 51.64% instead of 70.00% in treatment AV9. These differences are significant at 5% in both cases (Mann-Whitney, $z = 2.201$, $p < 0.05$). Section 6.3 returns to this discrepancy to show that asymmetric equilibria help explain this gap.

6.2 The Effects of Biased Information

In PL7, we observed that all independent groups coordinated on the sincere voting equilibrium. One reason might be the symmetry between the blue and red signals, which made coordination challenging for the subjects. In treatment PL7B, we instead made the signal structure strongly biased in favor of the blue signal by setting $r(\text{blue ball} \mid \text{blue jar}) = 8/9$. So, if the voters were to keep playing sincere, blue would win disproportionately more often than red. Together with Example 2, Propositions 1 and 3 show that voters may still coordinate on either the Duverger’s Law equilibrium or the informative equilibrium. Example 2 showed that, to aggregate information, blue voters should then adopt a strictly mixed strategy and vote red with probability 8.43%.

In the experiment, we observe that one independent group (group 28) out of six coordi-

³¹In all nonparametric tests we use a matching group as an independent observation.

	Periods	Periods	
	1-50	51-100	Equilibrium
Signal if blue	92.99	90.75	91.53
Opposite if blue	6.89	8.38	8.47
Signal if red	96.39	97.48	100
Opposite if red	3.13	1.74	0
Gray	0.27	0.83	0

Table 4: Aggregate voting behavior in treatment PL7B. Group 28 was excluded given that it converged to a Duverger’s Law equilibrium.

nated on the “blue” Duverger’s Law equilibrium, whereas the other five adopted a strategy coherent with the informative equilibrium of Example 2. Let us analyze each in turn: in group 28, almost all voters cast a blue ballot as of period 31: from that period onwards, blue obtained at least seven votes in all cases, and strictly more than seven in 94.29% of the cases.³² Table 4 summarizes the behavior of the other 5 independent groups. Focusing on the last fifty periods, we observe that blue voters indeed voted red with a probability close to the theoretical prediction, whereas red voters mainly voted sincerely. The difference between these two behaviors is statistically significant (Mann-Whitney, $z = 2.023$, $p < 0.043$). Moreover, we do not find any statistically significant difference between the theoretical prediction and the observed frequency of voting blue when getting a blue ball (Mann-Whitney, $z = 0.405$, $p = 0.6858$).

The model helps identify two reasons why the informative equilibrium is more likely to be selected. First, it yields a higher expected payoff than Duverger’s Law equilibria (178.37 instead of 155). Second, as identified by the phase diagram displayed in Figure 3, when starting from sincere voting (the top-right corner), the local dynamics of individual best responses point towards the informative mixed strategy equilibrium (the point MSE on the graph) rather than towards either Duverger’s Law equilibria.

These data provide ample evidence that three-candidate equilibria are a natural focal point when voters have common values. Note also that subjects do not simply play “sincere”. They actually adjusted their behavior to better aggregate information. We believe that our experimental results are the first to identify this pattern in a three-candidate setting.

Approval voting offers the opportunity to double vote to achieve the dual objectives of

³²The fact that the only group that converged to a Duverger’s Law equilibrium did coordinate on blue is coherent with the idea that biased signals foster coordination. However, this intuition cannot be tested since there is only one such group to study. Moreover, the results in PL9 offer an alternative rationale, which is that voters coordinate on the color that won in the first period(s).

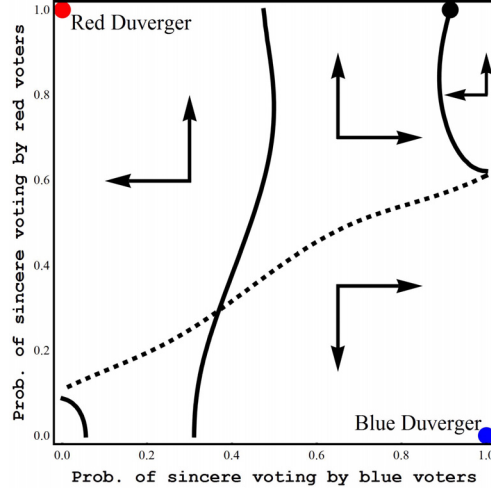


Figure 3: Phase diagram of treatment PL7B. The horizontal axis displays the probability of sincere voting by blue voters, while the vertical axis displays the probability of sincere voting by red voters. The solid line indicates the indifference curve for the blue voters; the dashed line indicates the indifference curve for the red voters.

information aggregation and coordination. Treatment AV7B is the same as PL7B with the only difference that subjects can exploit this opportunity. In this treatment, voters face the more complex challenge of having to deal with a broader choice set but, as identified in Example 4, their task is simplified by the fact that the equilibrium is now unique. Like in the informative equilibrium of PL7B, blue voters should play blue less often than red voters play red. The difference with PL7B is that blue voters should double vote instead of voting red. Table 5 shows that the subjects' behavior was in line with this prediction. Actually, the difference between the blue and red voters is significant not only for the second half of the sample but for the whole experiment (Mann-Whitney, $z = 2.201$, $p = 0.028$).

6.3 Individual Behavior

We begin by describing individual behavior in plurality treatments with unbiased information. These cases do not allow for much variation among players: in treatment PL7 most subjects voted sincerely throughout the entire experiment: 43.06% of the participants always did and 88.89% of the subjects voted sincerely more than 75% of the occasions. As we saw, behavior is somewhat different in PL9, since voters always converged to a Duverger's law equilibrium, although slowly. In the last half of the experiment, 88.94% voted for the

	Periods 1-50	Periods 51-100	Equilibrium
Signal if blue	66.95	61.16	50.1
Double Vote if blue	29.87	37.16	49.9
Signal if red	74.56	80.52	92.6
Double Vote if red	20.52	17.98	7.4
Opposite	2.94	1.56	0
Gray	0.89	0.06	0

Table 5: Aggregate Voting Behavior in treatment AV7B.

color their group converged to.

The case of AV is more interesting. Figure 4 disaggregates behavior at the individual level in the last fifty periods of treatments AV7 (left panel) and AV9 (right panel). The horizontal axis plots the frequency of voting the signal and the vertical axis plots the frequency of double voting. Each circle in the graph corresponds to the observed frequency of play. Its size represents the number of subjects who actually adopted that frequency: the larger the number of subjects, the bigger the circle.

According to Theorem 2, subjects should only mix between voting the signal and double-voting. If all subjects voted in this way, all the circles should be on the diagonal between (0,1) and (1,0). Most circles are indeed on this diagonal but, instead of observing a large number of voters playing the predicted mixed strategy, we observe significant heterogeneity with two opposite clusters: one that plays the pure strategy of always double voting and another one with subjects who always single vote their signal. The treatment effect observed in Section 6.1.2 is mainly driven by a switch in the relative number of subjects in each cluster.

This pattern points at the need to consider asymmetric strategies. Pushing the line of reasoning of McLennan (1998) and Ahn and Oliveros (2012) further, allowing for asymmetric strategies can be interpreted as an extension of the group’s choice set, which may only increase expected welfare. Allowing some voters to specialize in double or single voting may produce significant advantages. The challenge is to identify potential equilibria when allowing for asymmetric strategies.

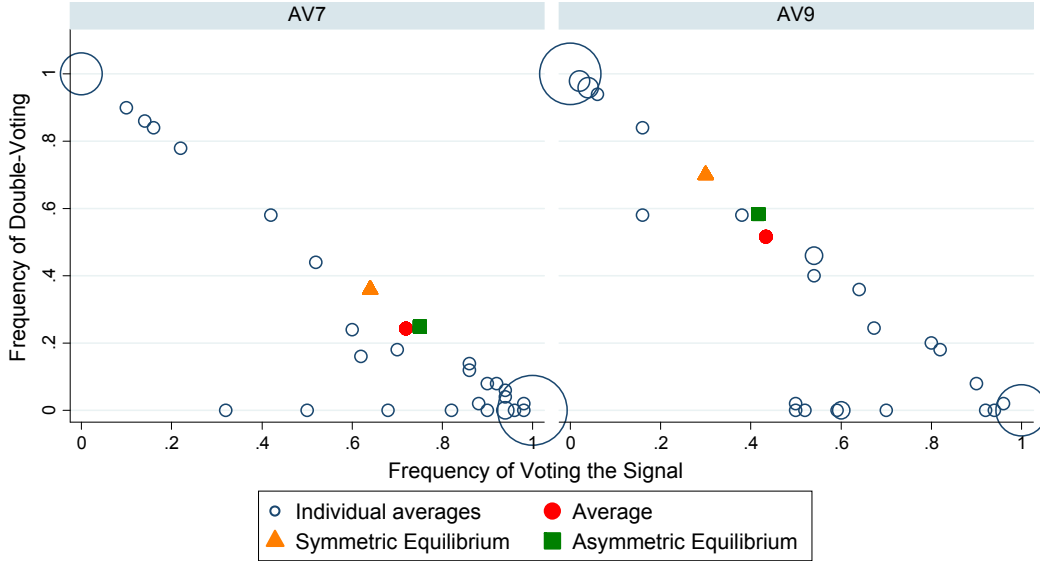


Figure 4: Individual behavior in AV treatments with unbiased information. Each hollow circle in the graph corresponds to the observed frequency of play: its size represents the number of subjects who actually adopted that frequency of play. The red circle represents the average frequency of play observed, the orange triangle represents the symmetric equilibrium prediction and the green square represents the asymmetric equilibrium prediction.

6.3.1 Asymmetric Equilibria with Approval Voting

In this subsection, we extend our theoretical analysis by relaxing the assumption of symmetric strategies, ubiquitous as it is in the voting literature.³³ That is, we no longer impose that voters who receive the same signal play the same strategy. The following proposition proves, for a broad set of parameter values (including the ones used in the experiment), the existence of at least one *asymmetric* equilibrium, i.e. in which voters play asymmetric strategies. We also characterize this asymmetric equilibrium: voters specialize independently of their signal in either single voting or double voting. That is, some voters always single vote and others always double vote. If the signal structure is sufficiently unbiased, all “single-voters” vote their signal, i.e. A if signal s_A and B if signal s_B . If the bias in the signal structure is stronger, then the voters receiving the less abundant signal vote sincerely whereas those who receive the more abundant signal mix between A and B . As discussed below, it appears that this equilibrium helps rationalize the behavior observed in

³³There are noticeable exceptions such as McLennan (1998), Ladha, Miller and Oppenheimer (2000) and Dekel and Piccione (2000).

the laboratory.

Proposition 4 *Suppose that $q(a) = q(b)$, $r(s_A|a) \geq r(s_B|b)$ and $V \leq 2v$. Any strategy profile satisfying the following conditions is an asymmetric equilibrium:*

1. $2n_C - n + 1$ voters always double vote;
2. The rest of the voters single vote informatively with $\sigma_{s_B}^{1v}(B) = 1$ and

$$\sigma_{s_A}^{1v}(A) = \begin{cases} \frac{\rho^{\frac{n-n_C}{n-n_C-1}} - 1}{\rho^{\frac{n-n_C}{n-n_C-1}} r(s_A|a) - r(s_A|b)} & \text{if } \rho^{\frac{n-n_C}{n-n_C-1}} > \frac{r(s_B|b)}{r(s_B|a)}, \text{ where } \rho = \frac{r(s_A|a)}{r(s_A|b)} \\ 1 & \text{if } \rho^{\frac{n-n_C}{n-n_C-1}} \leq \frac{r(s_B|b)}{r(s_B|a)} \end{cases},$$

where $\sigma_s^{1v}(\psi)$ is the probability that a single voter of type s plays action ψ .

Proof. See supplementary appendix. ■

Such an asymmetric behavior can be sustained in equilibrium because voters who specialize in single voting perceive the expected effect of any given ballot on the final outcome differently from voters specializing in double voting. In particular, “single voters” are pivotal only when A , B , and C receive exactly the same number of votes, whereas “double voters” are pivotal if either A is trailing behind by one vote or if it is leading by one vote. The best responses of these two groups of voters are thus different. The following example illustrates this result in more detail.

Example 5 *Assume (as in Example 3) $n = 12$, $n_C = 7$, and $q(s_A|a) = q(s_B|b) = 2/3$. In the asymmetric equilibrium, $2n_C - n + 1 = 3$ voters double vote, and the other 9 single vote their signal.*

Compared with the symmetric equilibrium, the aggregate level of double voting decreases from 36% to 25%, but this is enough to ensure that the Condorcet loser never wins the election. Indeed, with three double votes and nine single votes, one of the two majority alternatives must receive at least eight votes, i.e. strictly more than the Condorcet loser. Finally, the likelihood of choosing the best candidate increases from 82% in the symmetric equilibrium to 85.5%. The better aggregation of information holds because the (expected) number of voters who reveal their information, i.e. the single voters, is larger in this asymmetric equilibrium than in the symmetric one (9 vs. 7.68).

Such asymmetric equilibria appear to organize laboratory data better than the symmetric equilibrium. In treatment AV7, the predicted level of double voting in the asymmetric

Treatment	Periods 1-50	Periods 51-100	Equilibrium	
			Symmetric*	Asymmetric
PL7	136.70	138.50	154.87	-
PL9	101.15	147.80	156.20	-
PL7B	169.85	171.95	178.76	-
AV7	167.00	183.95	179.65	189.80
AV9	146.75	168.95	164.70	181.10
AV7B	188.90	192.50	193.58	194.68

Table 6: Average payoff and theoretical predictions. * In the case of Plurality, equilibrium predictions refer to the equilibrium where experimental groups converged to.

equilibrium is 25% compared to the observed 24.46%. The difference is not significant (Wilcoxon, $z = -0.524$ $p = 0.6002$). In the case of AV9, the predicted level of double voting is 58.33% compared to the observed 51.64%. The difference is still significant (Wilcoxon, $z = 2.201$, $p = 0.0277$) although the gap is much smaller.

The equilibrium described in Proposition 4 also makes an interesting prediction for the biased treatment AV7B. In the asymmetric equilibrium, the level of double voting is independent of the signal structure. This is not what we observe in the data (see Table 5). Although it is beyond the scope of this paper, it might be useful to explore other types of asymmetric equilibria in this type of setting.

6.4 Welfare and Outcomes

Previous multicandidate election setups used in laboratory experiments were based on theories that are inconclusive when it comes to comparing welfare across voting systems. The theoretical predictions of Myerson and Weber (1993) used in Forsythe et al. (1996), for instance, do not make a clear-cut comparisons between plurality and AV. A valuable feature of our common value setup is that it allows for clear welfare predictions: in equilibrium, the active voters' payoff should be strictly higher with AV than with plurality.

Table 6, columns 2 and 3, displays the average payment obtained by the subjects in each treatment, respectively for the first and second fifty periods. Comparing PL and AV treatments two by two, one can see that realized payoffs are systematically higher in AV treatments. All these differences are significant at a 1% confidence level.³⁴

It is also interesting to see the effect of the size of the minority. In plurality, the

³⁴Mann-Whitney tests are: $z = 2.882$ and p -value 0.0039 for AV7-PL7, $z = 2.722$ and p -value = 0.0065 for AV9-PL9, and $z = 2.913$ and p -value = 0.0036 for AV7B-PL7B.

expected payoff should be strictly decreasing in n_C in an informative equilibrium, whereas it is independent of n_C in a Duverger’s Law equilibrium. In the case of our experiment, the expected payoff of Duverger’s Law equilibria is 155. Table 6 shows an interesting reversal: in the first half, the average payoff is higher in treatment PL7 than in treatment PL9, while the opposite is true for the second half. This is explained by the progressive switch towards a Duverger’s Law equilibrium under PL9, and the selection of the sincere voting equilibrium in PL7. The latter happens in spite of the fact that Duverger’s Law equilibria payoff dominate sincere voting in both treatments. On the other hand, the slow convergence process in PL9 treatments explains why payoffs are so low for the first fifty periods. Across the entire experiment session, payoff is lower under PL9 than under PL7 (Mann-Whitney test, $z = 1.922$, $p\text{-value} = 0.0547$).

In the case of AV the theoretical prediction is unambiguous since the voters’ payoffs is predicted to be strictly decreasing with the size of the minority, both in the symmetric and asymmetric equilibria. As one can see from Table 6, this is what we observe in the data (Mann-Whitney test, $z = 2.242$, $p\text{-value} = 0.0250$). This is due to a remarkable increase in the frequency of victory of the best candidate, combined with a drop in the frequency of victory of the Condorcet loser. This observation can be made in all two-by-two comparisons, including PL9 against AV9, because gray won 10% of the times even in the second half of the PL9 experiment, due to slow convergence towards the equilibrium.

7 Conclusions

In this paper we studied the properties of plurality and approval voting both theoretically and experimentally. We considered a case in which the majority is divided between two alternatives as a result of information imperfections, while the minority backs a third alternative, which the majority views as strictly inferior. The majority thus faced two problems: aggregating information and coordinating to defeat the minority candidate.

In plurality, two types of equilibria coexist: Duverger’s Law equilibria, which fulfill the coordination purpose at the expense of information aggregation, and informative equilibria, in which majority voters aggregate information but open the door to a victory of the Condorcet loser. Interestingly, this equilibrium is not “knife edge”. This theoretical finding helps rationalize some empirical regularities in the literature that are oft-considered as supporting evidence for the lack of a “rational-instrumental” voting behavior. In approval voting (AV), the structure of incentives is quite different. In equilibrium, some majority

voters should double vote. This allows for information aggregation and a significant reduction in the threat posed by the Condorcet loser. As a consequence, AV produces strictly higher expected welfare.

We then tested our predictions with laboratory experiments: under plurality, we observed the emergence of both informative (when minority size was small) and Duverger’s Law equilibria (when minority size was large). Under AV, double voting increased welfare significantly: the subjects’ behavior allowed them to elect the full information Condorcet winner with a probability very close to what a social planner would have achieved after observing all available signals. Such behavior is statistically different from “sincere voting” and consistent with most theoretical predictions. However, in contrast with our theoretical prior, we also found that subjects used asymmetric strategies. This led us to extend the theoretical analysis in that direction.

We believe that this paper opens up many novel theoretical and experimental questions about multicandidate elections: how would other voting rules perform in such a common value setup? How would plurality and AV perform when majority voters have a mix of private and common values? What are the equilibria in asymmetric strategies under different voting rules? How do voter characteristics influence the role that each of them assumes within such equilibria? Last but not least, how do voters select between equilibria?

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Appendices

Appendix A1: Pivot Probabilities and Correcting Terms in AV

The pivotal event piv_{AC}^{AV} is defined as follows:

$$\begin{aligned} x_A &> x_B - 1 \text{ and } x_A + x_{AB} \in \{n_C - 1, n_C\} \\ x_A &= x_B \text{ and } x_A + x_{AB} = n_C, \text{ or} \\ x_A &= x_B - 1 \text{ and } x_B + x_{AB} = n_C. \end{aligned}$$

With the multinomial distribution: $\Pr(x|\omega) = n! \prod_{\psi \in \Psi_{AV}} \frac{\tau_{\psi}^{\omega}(\sigma)^{x(\psi)}}{x(\psi)!}$. Therefore, the probability of event piv_{AC} in state ω under AV is:

$$\begin{aligned}\pi_{AC}^\omega &\equiv \Pr(piv_{AC}^{AV}|\omega) = (n-1)! \sum_{i=0}^1 \sum_{x_{AB}=0}^{2(n_C-i)-n} \frac{(\tau_A^\omega)^{n_C-i-x_{AB}} (\tau_{AB}^\omega)^{x_{AB}} (\tau_B^\omega)^{(n-1)-(n_C-i)}}{2(n_C-i-x_{AB})! x_{AB}! (n-1-n_C+i)!} \\ &\quad + \frac{(n-1)!}{3} \frac{[\tau_A^\omega \tau_B^\omega]^{n-1-n_C}}{[(n-1-n_C)!]^2} \frac{(\tau_{AB}^\omega)^{2n_C+1-n}}{(2n_C-n+1)!} + \frac{(n-1)!}{6} \frac{(\tau_A^\omega)^{n-1-n_C} (\tau_{AB}^\omega)^{2n_C-n} (\tau_B^\omega)^{n-n_C}}{(n-1-n_C)! (n-n_C)! (2n_C-n)!}.\end{aligned}$$

π_{BC}^ω can be computed similarly. The pivot probability of piv_{AB}^{AV} is given by:

$$\begin{aligned}\pi_{AB}^\omega &\equiv \Pr(piv_{AB}^{AV}|\omega) = (n-1)! \sum_{i=0}^1 \sum_{k=i}^{n+2(i-1)-n_C} \frac{(\tau_A^\omega)^{k-i} (\tau_{AB}^\omega)^{n+(i-1)-2k} (\tau_B^\omega)^k}{2k!(k-i)!(n+i-1-2k)!} \\ &\quad + \frac{(n-1)!}{3} \frac{[\tau_A^\omega \tau_B^\omega]^{n-1-n_C}}{[(n-1-n_C)!]^2} \frac{(\tau_{AB}^\omega)^{2n_C+1-n}}{(2n_C-n+1)!} + \frac{(n-1)!}{6} \frac{(\tau_A^\omega)^{n-1-n_C} (\tau_{AB}^\omega)^{2n_C-n} (\tau_B^\omega)^{n-n_C}}{(n-1-n_C)! (n-n_C)! (2n_C-n)!},\end{aligned}$$

and π_{BA}^ω is defined in the same way.

Using the pivot probabilities and the expected gains of the different actions (i.e. (11), (12), and (13)), we can compute the correcting terms ϕ^a and ϕ^b :³⁵

$$\begin{aligned}\phi^a &= [\Pr(X_A = X_B = n_C - 1|a)(V + v) + \Pr(X_A = X_B + 1 = n_C|a)v + \dots \\ &\quad \dots + \Pr(X_A + 1 = X_B = n_C|a)V + \Pr(X_A = X_B = n_C|a)(V + v)]/6, \text{ and} \\ \phi^b &= [\Pr(X_A = X_B = n_C - 1|b)(V + v) + \Pr(X_A = X_B + 1 = n_C|b)V + \dots \\ &\quad \dots + \Pr(X_A + 1 = X_B = n_C|b)v + \Pr(X_A = X_B = n_C|b)(V + v)]/6.\end{aligned}$$

To understand what these correcting terms represent, consider the case in which, without voter i 's ballot, both alternatives A and B lose to C by one vote (that is, both obtain $n_C - 1$ votes). A single- A ballot creates a tie between A and C . Thus, the ballot is pivotal in favor of A with probability $1/2$. Likewise, a single- B ballot is pivotal in favor of B with probability $1/2$. Yet, a double vote AB creates a three-way tie, which still allows C to win with probability $1/3$: the winning probabilities of A and B are $1/3$ instead of $1/2$. Summing up the probabilities π_{AC}^a and π_{BC}^a thus overestimates the value of the double ballot by $(V + v)/6$. ϕ^a and ϕ^b correct for these overestimations in that and three other cases: when A trails behind both B and C by one vote, when B trails behind both A and C by one vote, and when A , B and C have the same number of votes. We directly see that $\phi^\omega = 0$ when $\tau_{AB}^\omega \in \{0, 1\}$, or $\tau_A^\omega = 0$, or $\tau_B^\omega = 0$.

7.1 Appendix A2: Plurality, Equilibrium Analysis

Proof of Proposition 3. Consider a distribution of signals such that $r(s_A|a) - r(s_B|b) > \delta(n, n_c)$, in which case sincere voting is not an equilibrium. That is, there exists a signal $\bar{s} \in \{s_A, s_B\}$ such that all the voters who received signal \bar{s} strictly prefer to deviate from a strategy profile $\sigma^{\text{sincere}} \equiv \{\sigma_{s_A}(A), \sigma_{s_B}(B)\} = \{1, 1\}$.

Case 1: $\bar{s} = s_A$. In this case, $\sigma^{\text{sincere}} \Rightarrow G(A|s_A) - G(B|s_A) < 0$. Now, consider a second strategy

³⁵Proof available upon request.

profile $\sigma' \equiv \left\{ [r(s_A|a) + r(s_A|b)]^{-1}, 1 \right\}$. With this profile, we have: $\tau_A^a = \tau_B^b$ and $\tau_A^b = \tau_B^a$, and thus $p_{BC}^b = p_{AC}^a > 0$ and $p_{AC}^b = p_{BC}^a > 0$ and, from (8):

$$G(A|s) - G(B|s) = [Vp_{AC}^a - vp_{BC}^a] [q(a|s) - q(b|s)], \quad (16)$$

where (i) $[Vp_{AC}^a - vp_{BC}^a]$ is positive, and (ii) $[q(a|s) - q(b|s)]$ is positive for s_A and negative for s_B . In other words, all voters would strictly prefer to deviate from σ' by voting sincerely. This means that the value of $G(A|s_A) - G(B|s_A)$ changes sign when $\sigma_{s_A}(A)$ is increased from $[r(s_A|a) + r(s_A|b)]^{-1}$ to 1.

Since all pivot probabilities are continuous in σ_{s_A} , the differential $G(A|s_A) - G(B|s_A)$ is also continuous in σ_{s_A} . This implies that there must exist a value $\sigma_{s_A}^*(A) \in \left([r(s_A|a) + r(s_A|b)]^{-1}, 1 \right)$ such that voters with signal s_A are indifferent between playing A and B .

Now, we prove that the strategy profile $\{\sigma_{s_A}(A), \sigma_{s_B}(B)\} = \{\sigma_{s_A}^*(A), 1\}$ is an equilibrium. This profile implies: $\tau_A^a \in \left(\frac{r(s_A|a)}{r(s_A|a) + r(s_A|b)}, r(s_A|a) \right)$ and $\tau_B^b \in \left(r(s_B|b), \frac{r(s_A|a)}{r(s_A|a) + r(s_A|b)} \right)$ and hence:

$$\tau_A^a > \tau_B^b > \frac{n_C}{n} > \tau_A^b > \tau_B^a; p_{AC}^a > p_{BC}^a \text{ and } p_{BC}^b > p_{AC}^b.$$

Since $G(A|s_A) - G(B|s_A) = 0$ for that strategy profile, we know from (8) that:

$$q(a|s_A) [Vp_{AC}^a - vp_{BC}^a] = q(b|s_A) [Vp_{BC}^b - vp_{AC}^b], \quad (17)$$

where both sides of the equality are strictly positive. Since $q(a|s_B) < q(a|s_A)$, (17) implies:

$$q(a|s_B) [Vp_{AC}^a - vp_{BC}^a] < q(b|s_B) [Vp_{BC}^b - vp_{AC}^b], \quad (18)$$

which means that a voter who received signal s_B strictly prefers to play B .

Case 2: $\bar{s} = s_B$. In this case, $\sigma^{\text{sincere}} \Rightarrow G(A|s) - G(B|s) > 0$ for both signals. Now, consider another strategy profile $\sigma'' \equiv \{\varepsilon, 1\}$, with $\varepsilon \rightarrow 0$ (and hence $\sigma_{s_A}(B) \rightarrow 1$). From Proposition 1, this strategy profile implies $G(A|s) - G(B|s) < 0$ for both signals. By the continuity of the payoffs with respect to $\sigma_{s_A}(A)$, there must therefore exist a value $\sigma_{s_A}^{**}(A) \in (0, 1)$ such that $G(A|s_A) - G(B|s_A) = 0$ and, by the same argument as in (17–18), $G(A|s_B) - G(B|s_B) < 0$. Hence, the strategy profile $\{\sigma_{s_A}(A), \sigma_{s_B}(B)\} = \{\sigma_{s_A}^{**}(A), 1\}$ is an equilibrium.

Note that sincere stability is not a binding restriction, since all voters vote for their preferred alternative with a probability strictly larger than 0. ■

Appendix A3: Approval Voting, Equilibrium Analysis

Lemma 2 *If there exists a signal s such that*

$$\begin{aligned} G^{AV}(A|s) - G^{AV}(AB|s) = 0 \text{ then } G^{AV}(A|s_A) - G^{AV}(AB|s_A) &> G^{AV}(A|s_B) - G^{AV}(AB|s_B) \quad (19) \\ G^{AV}(B|s) - G^{AV}(AB|s) = 0 \text{ then } G^{AV}(B|s_B) - G^{AV}(AB|s_B) &> G^{AV}(B|s_A) - G^{AV}(AB|s_A), \text{ and} \\ G^{AV}(A|s) - G^{AV}(B|s) = 0 \text{ then } G^{AV}(A|s_A) - G^{AV}(B|s_A) &> G^{AV}(A|s_B) - G^{AV}(B|s_B). \end{aligned}$$

Proof. We detail the proof for (19). It is similar for the other two implications. Remember that the second term in (14) is necessarily negative. Thus $G^{AV}(A|s) - G^{AV}(AB|s) = 0$ implies that the first term must be strictly positive. It follows immediately that:

$$G^{AV}(A|s) - G^{AV}(AB|s) \geq 0 \text{ iff } \frac{q(a|s)}{q(b|s)} \geq \frac{\pi_{AB}^b(V-v) + \pi_{BC}^bV - \phi^b}{\pi_{AB}^a(V-v) - \pi_{BC}^av + \phi^a}.$$

Thus, (19) follows from $\frac{q(a|s_A)}{q(b|s_A)} > \frac{q(a|s_B)}{q(b|s_B)}$. ■

Lemma 3 *In any voting equilibrium under AV, neither A nor B can be approved by all voters.*

Proof. We prove the proposition by contradiction and for the limit case in which $\varepsilon = 0$. By definition the results hold when $\varepsilon > 0$.

Policy A is approved by all voters if and only if $\sigma_{s_A}(A) + \sigma_{s_A}(AB) = 1 = \sigma_{s_B}(A) + \sigma_{s_B}(AB)$. In this case, we have: $x_A + x_{AB} = n$ and hence $\pi_{AC}^\omega = 0 = \pi_{BC}^\omega$ and $\phi^\omega = 0$. The only possible pivot events are when $x_{AB} = n - 1$ or $n - 2$. Hence:

$$\begin{aligned} G(A|s) - G(AB|s) &= [q(a|s)\pi_{AB}^a - q(b|s)\pi_{AB}^b](V-v) \geq 0 \\ G(B|s) - G(AB|s) &= [q(b|s)\pi_{BA}^b - q(a|s)\pi_{BA}^a](V-v) \geq 0. \end{aligned}$$

with: $\pi_{AB}^\omega = \frac{(\tau_{AB}^\omega)^{n-1}}{2}$, and $\pi_{BA}^\omega = \frac{(\tau_{AB}^\omega)^{n-2}}{2} [(n-1) + (2-n)\tau_{AB}^\omega]$. Therefore,

$$\frac{\pi_{BA}^b}{\pi_{BA}^a} = \left(\frac{\tau_{AB}^b}{\tau_{AB}^a} \right)^{n-2} \frac{(n-1) + (2-n)\tau_{AB}^b}{(n-1) + (2-n)\tau_{AB}^a}, \quad (20)$$

$$\frac{\pi_{AB}^a}{\pi_{AB}^b} = \left(\frac{\tau_{AB}^a}{\tau_{AB}^b} \right)^{n-1} \quad (21)$$

Now, we show that $\frac{\pi_{BA}^b}{\pi_{BA}^a}$ is increasing in $\frac{\tau_{AB}^b}{\tau_{AB}^a}$ (from (21), it is straightforward that $\frac{\pi_{AB}^a}{\pi_{AB}^b}$ is also increasing in $\frac{\tau_{AB}^a}{\tau_{AB}^b}$). Taking logs, we have that the right-hand side of (20) is

$$(n-2) [\log \tau_{AB}^b - \log \tau_{AB}^a] + \log [(n-1) + (2-n)\tau_{AB}^b] - \log [(n-1) + (2-n)\tau_{AB}^a]$$

Differentiating with respect to τ_{AB}^b yields:

$$\frac{n-2}{\tau_{AB}^b} - \frac{n-2}{(n-1) + (2-n)\tau_{AB}^b}.$$

This is non-negative if and only if $\tau_{AB}^b \leq 1$. Therefore, we have that $\pi_{AB}^b > \pi_{AB}^a$ and $\pi_{BA}^b > \pi_{BA}^a$ when $\tau_{AB}^b > \tau_{AB}^a$, and conversely.

We now use this result to prove that A cannot be approved by all voters. From Theorem 1, Lemma 2, and Lemma 2 (in this Appendix), there are 2 cases to check: (i) $\sigma_{s_A}(A) = 1$ and $\sigma_{s_B}(A) \in [0, 1)$, and (ii) $\sigma_{s_B}(A) = 0$ and $\sigma_{s_A}(A) \in (0, 1]$. If $\sigma_{s_A}(A) = 1$ and $\sigma_{s_B}(A) \in [0, 1)$, then $\tau_{AB}^b > \tau_{AB}^a$. Hence, we have that $\pi_{BA}^b > \pi_{BA}^a$, which implies $G(B|s_B) - G(AB|s_B) > 0$. Thus,

there cannot be any equilibrium in which $\sigma_{s_A}(A) = 1$ and $\sigma_{s_B}(A) \in [0, 1)$. If $\sigma_{s_A}(A) \in (0, 1]$ and $\sigma_{s_B}(A) = 0$, then either $\tau_{AB}^a > \tau_{AB}^b$ or $\tau_{AB}^a < \tau_{AB}^b$. If $\tau_{AB}^a > \tau_{AB}^b$, then $\pi_{AB}^a > \pi_{AB}^b$, and thus $G(A|s_A) - G(AB|s_A) > 0$. If $\tau_{AB}^a < \tau_{AB}^b$, then $\pi_{BA}^a > \pi_{BA}^b$, and thus $G(B|s_B) - G(AB|s_B) > 0$. Therefore, there cannot be any equilibrium in which $\sigma_{s_B}(A) = 0$ and $\sigma_{s_A}(A) \in (0, 1]$. ■

Proof of Theorem 1. From McLennan (1998), a strategy that maximizes expected utility must be an equilibrium of such a common value game (and any finite Bayesian game like ours must have an equilibrium). Now, conjecture some strategy profile σ that can be played under plurality. That is, $\sigma_s(AB) = 0$ for $s = s_A, s_B$. In this case, $\pi_{AB}^\omega = \pi_{BA}^\omega = \phi^\omega = 0 < \pi_{AC}^\omega, \pi_{BC}^\omega$, $\omega = a, b$. Therefore, $G^{AV}(A|s) - G^{AV}(AB|s) < 0$ and $G^{AV}(B|s) - G^{AV}(AB|s) < 0$, $\forall s$. This means that $\tau_{AB}^\omega = 0$ cannot be part of an equilibrium under AV, and that the welfare-maximizing equilibrium under AV must produce strictly higher expected utility than plurality.

It remains to show that this equilibrium is sincerely stable. We actually show the stronger statement that, to maximize expected welfare, a strategy must satisfy $\sigma_{s_A}(A), \sigma_{s_B}(B) > 0$. We show this by contradiction: suppose that $\hat{\sigma}$ maximizes expected welfare and is such that $\hat{\sigma}_{s_A}(A) = 0$. By Lemma 3, we have $\tau_A^\omega, \tau_B^\omega, \tau_{AB}^\omega > 0$ and hence $\hat{\sigma}_{s_B}(A) > 0$. Then, compare $\hat{\sigma}$ with some other strategy σ' in which s_A -voters transfer some of their votes from B towards AB , whereas s_B -voters adapt their voting strategy so as to maintain all vote shares unchanged in state b .³⁶

As a result, the total vote share of A in state a must increase (i.e. $\tau_A^a(\sigma') + \tau_{AB}^a(\sigma') > \tau_A^a(\hat{\sigma}) + \tau_{AB}^a(\hat{\sigma})$), whereas the expected fraction of double votes increases (the total vote share of B remains unchanged). As a result, in state a , the probability that A wins must increase, whereas the probability that C wins decreases weakly. In state b , winning probabilities are unchanged. Hence, $\hat{\sigma}$ cannot maximize expected welfare: a contradiction. ■

Proof of Theorem 2. We prove the Theorem in two steps. First, we show that there is no interior equilibrium in which a voter strictly mixes across the three actions A , B , and AB . Second, we show that s_A -voters never play B , nor s_B -voters play A in an interior equilibrium. It follows that the only possible interior equilibrium is such that voters with signal s_A mix between A and AB , and voters with signal s_B mix between B and AB .

First, conjecture an equilibrium in which $\sigma_{s_A}(A), \sigma_{s_A}(B), \sigma_{s_A}(AB) > 0$. This requires $G(A|s_A) = G(B|s_A) = G(AB|s_A)$. In this case, by Lemma 2 (in this Appendix), s_B -voters must be playing B with probability 1, i.e. $\sigma_{s_B}(B) = 1$. The equilibrium is therefore not interior, a contradiction. Similarly, s_A -voters must play A with probability 1 if s_B -voters strictly mix between A , B , and AB .

Second, imagine that s_B -voters play A with strictly positive probability in equilibrium: $\sigma_{s_B}(A) \in (0, 1)$. This requires either (i) $G(A|s_B) = G(AB|s_B) \geq G(B|s_B)$ or (ii) $G(A|s_B) = G(B|s_B) \geq G(AB|s_B)$. By Lemma 2, both (i) and (ii) imply that $G(A|s_A) > G(AB|s_A), G(B|s_A)$, and hence that A 's strategy cannot be interior. By symmetry, $\sigma_{s_A}(B) \in (0, 1)$ cannot be part of an interior equilibrium either. ■

³⁶If $\hat{\sigma}_{s_A}(AB) = 1$, then one must consider a transfer of s_A -votes from AB towards A , and s_B -voters adapt their strategy to maintain all τ_ψ^a unchanged.