

The disadvantages of winning an election.*

Enriqueta Aragonés[†]
Institut d'Anàlisi Econòmica, CSIC

Santiago Sánchez-Pagés[‡]
University of Edinburgh

January 2010

Abstract

After an election, the winner has to decide what to do in front of popular initiatives or policy proposals coming from different sources. We analyze the problem that an incumbent faces during the legislature when deciding how to react to them. This paper shows the disadvantages that the incumbent has won together with the election. We also analyze the decision of the incumbent when facing his reelection, and we show under which conditions the advantages that the incumbent obtains can overcome the initial disadvantages.

1 Introduction

The incumbency advantage states that in general an incumbent is more likely to be reelected than a challenger. This fact is well documented in the literature. This paper focuses on some disadvantages that incumbents may face.

*Aragonés acknowledges financial support from the Spanish Ministry of Education and Science, grant number ECO2009-08820, the Barcelona GSE and the Catalan Government. We would like to thank Miguel Angel Ballester, Micael Castanheira, Luca Merlino, Norman Schofield, Jim Snyder, Joel Sobel, participants of the first APE-X meeting in Brussels and participants of the IAE's informal seminar for their useful comments.

[†]Institut d'Anàlisi Econòmica, CSIC. Campus UAB. 08193 Bellaterra (Spain). enriqueta.aragones@iae.csic.es

[‡]Edinburgh School of Economics, University of Edinburgh, 50 George Square, EH8 9JY Edinburgh (United Kingdom). santiago.sanchez-pages@ed.ac.uk.

After winning an election the incumbent is supposed to decide and implement some policies, but he will be facing some restrictions on the policies that he can implement. His policy choices may turn out to be costly in terms of reelection chances. We show how these disadvantages may be overcome by the advantages, and we also find some conditions under which this is not the case.

There is large amount of literature devoted to the incumbency advantage. On the one hand, empirical studies such as Gelman and King (1990) and Lee (2008) show the success of an incumbent when facing reelection on US House. These results provide large evidence of the incumbency advantage as a fact. In addition there are some studies that also analyze the reasons of the incumbency advantage. Most of these studies assume that incumbents have better ways to influence the voters decision than challengers, and they can do so through different mechanisms such as: redistricting (Levitt and Wolfram 1997, Cox and Katz 2002), seniority (McKelvey and Reizman 1992), informational advantages (Krehbiel and Wright 1983), access to campaign resources (Goodliffe 2001, Jacobson 2001), legislative irresponsibility (Fiorina 1989), pork barrel politics (Cain, Ferejohn and Fiorina 1987, Ansolabehere, Snyder and Stewart 2000).

Ansolabehere, Snyder (2002) measured the incumbency advantage in all state executive and found similar empirical evidence of the incumbent advantage. However they argue that the incumbency advantage is not originated by the incumbent's strategic choice but it is obtained by incumbents independently of the politicians choices. Following this argument there are some studies that investigate the incumbent characteristics that may cause him to enjoy an advantage. On this line we find that Bevia and Llavador (2009) show that only good quality incumbents may enjoy an advantage, and Asworth and de Mesquita (2008) show that on average incumbents' quality and ability are higher than challengers'. Gowrisankaran, Mitchell and Moro (2006) find that incumbents face weaker challengers than candidates that face open seats and Stone, Maisel and Maestas (2004) find that incumbents' personal qualities deter strong challengers.

The contribution of this paper is to explain how the incumbency advantage coexists with an incumbency disadvantage. We characterize under which conditions the disadvantage is compensated by the advantage and the incumbent can still run as favorite in the electoral campaign. We show that a net incumbency advantage holds in most cases. However, we characterize the cases in which the net incumbency effect is negative. In those cases, even if the incumbent can choose a strategy that would allow him to win the next election, he prefers not to do so because he finds it too costly, and he chooses a different strategy that allows the challenger to win. We show that this

result holds when both incumbent and citizens have strong preferences for a certain issue, and their preferences on this issue are not aligned.

Most models of elections assume that after the election the incumbent can choose to implement any policy that he likes. His policy choice will depend only on his objective function. If the incumbent is mostly policy oriented he will choose a policy close to his ideal point. But in real life this is not always the case. When an incumbent is deciding which policies to implement, he has to take into account that some policy choices might have a large negative effect on his chances of success at being reelected in the next election. Notice that jeopardizing his chances of reelection is not optimal for the incumbent even when he cares about policy, because the policy implemented in case he loses will be worse from his point of view than the policy that he could have chosen if he had won the election.

There are several reasons why some policy choices might have a significant effect on the incumbent's chances of being re-elected. We discuss two different types: referenda, and participatory democracy.

Referenda may be mandatory or facultative. It is mandatory if the law (usually the constitution) directs authorities to hold referendums on specific matters. This is normally the case for amending constitutions, impeaching heads of state, ratifying international treaties, etc.... Otherwise, when it is facultative, it may be initiated at the will of a public authority or at the will of some organized group of citizens (in this case it is also known as a popular initiative).

By the nature of their effects, referenda may be either binding or non-binding. A non-binding referendum is merely consultative or advisory. It is left to the government or legislature to interpret its results and they may even choose to ignore them. A binding referendum forces the incumbent to implement its policy outcome. A binding referendum may also require the support of a supermajority of votes cast or a minimum turn-out of the electorate (Herrera and Mattozzi, 2010).

If a non-binding referendum of any of these types is called during the time of the legislature, the incumbent will have to react to it with a given policy implementation or by choosing to ignore it. If he chooses not to implement the policy corresponding to the referendum outcome he might be punished by the voters. The relevance of this policy choice on the voters' when deciding whether to re-elect the incumbent will depend on the proportion of voters for whom this issue is relevant.

A referendum initiated by the incumbent himself might have a weaker effect on the voters' reaction than a referendum that originates with a popular initiative. The first type of referendum requires a more complicated strategy from the incumbent because he has to decide whether and when it is optimal

to call it. The analysis of these strategies is beyond the scope of this paper (see for instance, Xefteris (2008) for an analysis of the incumbent's decision about when to call a referendum).

The policy proposal received by the incumbent may have its origin in an organized group of citizens without calling for a referendum. Participatory democracy maybe thought of as an extended version of a system of representative democracy that allows for the existence of policy proposals made by organized groups of citizens and presented to the incumbent. Real cases of participatory democracy can be found in the town meetings of New England. It is a form of local government practiced since colonial times. It can also be found in the participatory budgeting system of many latin-american cities, which has also been applied for school, university, and public housing budgets. The implications of a participatory democracy system over the behavior of citizens and politicians and over the policy outcomes are analyzed in Aragonés and Sánchez-Pagés (2009).

There are some empirical studies that have shown that the systems of direct democracy such as referenda and participatory democracy are very effective in satisfying voters preferences, and overall increasing the voters wellbeing (see Frey (1994), Frey and Bohnet (1993) and Frey and Slutzer (2000)).

In all these cases, if citizens care enough about a certain policy being implemented on a certain issue, their expected benefits might overcome the costs of coordination and a policy proposal might be produced and sent to the incumbent. If the support to the policy proposal is significant in the population, then the incumbent's chances of being re-elected might depend on his policy decision on that issue. That is, if voters are very much interested on that issue, they will be paying attention to the choices of the incumbent in that respect, and react to them accordingly at the next voting opportunity. That will induce the incumbent to implement policies that are close to what the voters demand.

The characteristics that both referenda and participatory democracy have in common are: (1) there is an issue that is considered very important for a significant part of the population; (2) there is a policy proposal received by the incumbent on this issue; (3) the incumbent has to make a decision regarding that issue: either he implements a particular policy or does not implement any policy on that issue; (4) there is a significant proportion of voters that may base their voting decision on that issue.

Our claim is that an incumbent facing this kind of situation has a disadvantage compared not only to an incumbent who has not received any proposal, but also compared to the challenger. The challenger is not required to react to a policy proposal on any issue. In fact, the position of the

challenger does not allow him to do anything with respect to it. The incumbent's disadvantage will be larger the more policy motivated the incumbent is.

Of course, the policy proposal that comes out of any popular initiative, such as the ones described above, will not represent a decision problem for the incumbent in systems where the proposals received are considered as binding. In those instances the incumbent does not have a choice. He only has to implement the proposed policy. This policy implementation might harm the incumbent's payoffs, but he will not be facing the kind of problem that we aim to analyze here.

In order to perform the analysis of the incumbent decision over which policy to implement we build a formal model of electoral competition with two candidates, two issues and three stages. In the first stage of the game, the incumbent faces an exogenously given policy proposal on an issue, the popular issue, and he has to make a policy choice on that issue. The implementation of this policy choice takes place during the legislature and before the beginning of the electoral campaign. Both the policy proposal and the incumbent's policy choice are common knowledge by all candidates and voters. In the second stage of the game, both candidates announce simultaneously their policy platforms on a different issue, the electoral issue. In the third stage of the game, the voters vote for their most preferred candidate.

The model presented includes two different types of asymmetries. First, there is an asymmetry in how the two issues are treated. On the one hand, the electoral issue is defined in the same way as most models of electoral competition, and the candidates choices on the electoral issue represent their campaign promises. On the other hand, by definition, the popular issue is regarded as very important by a significant part of the population, and its corresponding policy choice is known, since it is implemented during the legislature, and it is only responsibility of the incumbent. In addition, when voters evaluate the candidates' performance they assign different weights to the specific performance on each one the issues.

The second type of asymmetry refers to the way in which voters evaluate candidates. We assume that voters use all the information they have available in order to decide to whom to give their vote. Thus when evaluating the incumbent, in addition to considering his campaign promises, they have to take into account the policy choices he made during the legislature, and how they relate to the corresponding policy proposals made before. However, when evaluating the challenger all the information voters have comes from his campaign promises.

Since the decisions on the two dimensions of the model are made sequentially, one at each stage, we can solve it as a one dimensional model in each

stage. However, given the asymmetry implied by the decisions on the popular issue made in the first stage, in the second stage the standard median voter analysis does not apply. Indeed, in some cases we may obtain that voters with ideal points at the two extremes of the distribution decide to vote for the incumbent or for the challenger. This kind of situations does not arise in equilibrium, but it needs to be considered for the equilibrium analysis.

The optimal policy choices of the incumbent in both issues reflect the incumbent's trade-off between her own policy preferences and her benefit from being re-elected. For all parameter values the incumbent has a strategy that allows his reelection. The question is then whether this winning strategy is always optimal from the incumbent's point of view. And the answer is no.

There are some instances where the incumbent prefers to forgo his reelection and guarantee a good payoff in terms of policies. For this to happen we need three conditions to be satisfied: (1) the incumbent cares enough about policy (that is, the value he obtains from holding office is low enough); (2) there is enough conflict of interest between the voters and the incumbent regarding the popular issue; and (3) the voters care about the incumbent's performance on both issues. In those cases, the policy implemented by the incumbent on the popular issue coincides with the incumbent's ideal point, that is, the incumbent does not bear any utility cost from the policy implemented on the popular issue; and the policy proposed by the incumbent on the electoral issue is close to the median voter's ideal point in order to force the policy choice of the challenger to be as moderated as possible.

Otherwise, in equilibrium the incumbent chooses a winning strategy that consists of a combination of policies that depend on the weight that voters assign to his performance on each issue. The larger the weight that the voters assign to the popular issue, the more the incumbent will satisfy the popular demand; and the larger the weight that the voters assign to the electoral issue, the more the incumbent will satisfy the voters on the electoral issue.

The intuition is as follows: the incumbent has a disadvantage from the popular issue whenever he does not fully satisfy the voters demand on that issue, but he has always an advantage from the electoral issue. In particular, the incumbent enjoys the largest advantage at the electoral competition stage when he fully satisfies the demands of the voters in the popular issue. However, this is a costly strategy for a policy motivated candidate. If the incumbent is policy motivated he will choose the winning strategy that is cheapest in terms of policy, from his view point. And when this strategy is too costly he will decide to give up on the reelection.

The paper is organized as follows: in the next section we describe the formal model. Section 3 presents and discusses the results obtained. Finally section 4, offers some concluding remarks.

2 The model

We assume that electoral competition takes place across two dimensions, denoted by x and y . Each dimension is represented by a unit interval of the real line $[0, 1]$. Dimension x represents the electoral issue and dimension y represents the popular issue. There are two candidates: the incumbent and the challenger. The game proceeds in three stages. The first stage takes place during the legislature: the incumbent receives a policy proposal on the popular issue and given that, he has to implement a policy on that issue. Both the policy proposed and the policy implemented on the popular issue are common knowledge for all candidates and all voters. The second stage is the electoral campaign: both candidates make policy announcements simultaneously on the electoral issue. Again all policy announcements are common knowledge by all candidates and all voters. It is assumed that the winner will implement the announced policy on that issue. In the third stage of the game the election takes place: voters decide whether to reelect the incumbent or vote for the challenger. The winner is selected by majority rule and implements the policy announced on the electoral issue.

2.1 Candidates

The two candidates are denoted by L and R . Candidate L is assumed to be the incumbent. Candidates have single peaked preferences over the electoral issue. Without any loss of generality we assume that on the electoral issue the ideal point of candidate L is represented by $x_L = 0$ and the ideal point of candidate R is represented by $x_R = 1$. We assume that the incumbent has single-peaked preferences over the popular issue, that are independent of his preferences on the electoral issue. We assume that the incumbent's ideal point on the popular issue is represented by $y_L = 0$. Given the features of the game we analyze, it is not necessary to specify the properties of the preferences on the popular issue for voters and challenger.

The choice of the incumbent over the popular issue is being made and implemented during the legislature. Thus at the time of the electoral campaign this choice has already been made and it taken as given. Let us denote by $y(L)$ the policy chosen by the incumbent on this issue during the legislature. At the end of the legislature, elections take place and we assume that they are represented by a standard model of electoral competition on the electoral issue: the incumbent and the challenger simultaneously announce policy platforms on the electoral issue, represented by $x(L)$ and $x(R)$ respectively. We assume full commitment, that is, the winner of the election will implement the policy he announced during the campaign on the electoral

issue.

We assume that candidates have preferences over policies but they are also office-motivated. Candidates' payoffs are represented by the following utility functions that depend on the policy choice by the incumbent on the popular issue and the policy announcements of both candidates on the electoral issue:

$$\begin{aligned} V_L(y(L), x(L), x(R)) &= -|y_L - y(L)| + \pi_L(K - |x_L - x(L)|) - (1 - \pi_L)|x_L - x(R)| \\ V_R(y(L), x(L), x(R)) &= (1 - \pi_L)(K - |x_R - x(R)|) - \pi_L(|x_R - x(L)|) \end{aligned}$$

where $\pi_L = \pi_L(y(L), x(L), x(R))$ represents the probability that candidate L wins the election, and $1 - \pi_L$ denotes the probability that candidate R wins the election. The probability with which the incumbent is reelected depends on the policy choices made during the legislature and the policy announcements made during the campaign.

K is assumed to be a non-negative number that represents the utility of holding office. $K = 0$ implies that candidates do not obtain any extra utility for holding office, they only derive utility from the policy implemented. In this case we have two candidates that are only policy motivated. On the other hand, the larger the value of K the more candidates value being in office. Thus for larger values of K candidates care more about winning than about the policies they need to implement or commit to in order to win. By increasing the value of K we obtain that candidates become purely office motivated.

There is an asymmetry embedded in the definition of the candidates' payoff functions. The incumbent obtains a negative payoff whenever he implements a policy on the popular issue that does not coincide with what the society wants to be implemented. While the challenger's payoffs are not affected by the incumbent's policy choice on the popular issue. This assumption is justified because the challenger cannot do anything with respect to the policy implementation on the popular issue, since this takes place during the legislature when he does not have any implementation power. Therefore, he does not have to suffer any cost from that policy choice. Relaxing this assumption would not imply qualitative changes in the results obtained.

For simplicity, we have assumed that the incumbent cares equally about the two issues. Introducing a parameter in the incumbent's payoff function that represents the relative weight that each issue has on the incumbent overall payoffs would not change the main qualitative results obtained.

2.2 Voters

There is an infinite number of voters. Voters have single-peaked preferences over the electoral issue x . We assume that their ideal points are uniformly

distributed over this issue x , thus the ideal point of the median voter on the electoral issue is $x_m = \frac{1}{2}$. We assume that voters have homogeneous preferences on the popular issue y . Let the ideal point of society in issue y be denoted by $y_m > 0$. The parameter y_m is considered exogenous in our model. It is to be interpreted as the outcome of a referendum or a process of participatory democracy. As a simplifying assumption we consider that it represents the will of all citizens, and therefore it has the support of all the constituency. This assumption does not affect our qualitative results and its effects are discussed in the last section of the paper.

Notice that since the ideal point of the incumbent on the popular issue is assumed to be $y_L = 0$, the value of y_m measures the magnitude of the conflict of interest between the incumbent and the citizens with respect to the popular issue.

When facing the election, voters observe the policies announced by both candidates on the electoral issue, $x(L)$ and $x(R)$, take into account the policy implemented by the incumbent on the popular issue, $y(L)$, and decide whether to reelect the incumbent. Voters use all the information available in order to evaluate the two candidates. Since they have different kinds of information about each candidate, their decision rule must have to exhibit some sort of asymmetry.

We assume that voter i evaluates the incumbent according to the following function:

$$U_i(L) = -(1 - \mu) |y_m - y(L)| - \mu |x_i - x(L)|$$

where μ is a parameter that measures the relative weight that voters assign to the electoral issue with respect to the popular issue. We assume that $0 \leq \mu \leq 1$. The parameter μ thus measures the importance of the electoral issue over the popular issue. Values of μ close to one are to be interpreted as if the society considers that the popular issue is not very important, thus their payoffs would not be much affected by the incumbent's policy choice on that issue. Values of μ close to zero mean that the popular issue is regarded as very important from the voters' point of view and their payoffs will be largely affected by the incumbent's policy choice. Observe that voters evaluate the policy implemented on the popular issue comparing it to the policy proposed initially, and they evaluate the electoral issue comparing it to their own ideal point.

The challenger's performance on the popular issue cannot be evaluated, since he has not been able to do anything on that issue during the present legislature. Thus voters can only evaluate the challenger according to his policy promises on the electoral issue. We assume that voter i evaluates the

challenger according to the following function:

$$U_i(R) = -|x_i - x(R)|$$

Therefore, voter i will vote for candidate L if and only if

$$-(1 - \mu)|y_m - y(L)| - \mu|x_i - x(L)| \geq -|x_i - x(R)| \quad (1)$$

The lower the value of μ the more weight past choices have on the evaluation of the incumbent. That in turn will affect electoral competition, to the extent that a citizen with ideal point $x_i = x(L)$ will vote for candidate R whenever

$$\mu \leq 1 - \frac{|x_i - x(R)|}{|y_m - y(L)|}.$$

This shows that the existence of a policy proposed on the popular issue imposes a severe constraint on the incumbent's choices. If the distance between the incumbent's choice and the citizens' ideal point on the popular issue is large enough, it may be the case that the set of voters that decide to vote for the incumbent becomes non-connected.

Given this rule, the incumbent is reelected if and only if the set of voters that prefers the incumbent to the challenger contains a majority of the population. We assume that if there is a tie the incumbent is reelected.

This specification encompasses as particular cases some standard models of two-party competition. If $\mu = 1$, that is, voters care only about the electoral issue, we have a standard model of electoral competition. In this case, for very large values of K candidates are purely opportunistic and the model describes a Downsian framework. Instead, for relatively small values of K , candidates behave as mostly policy motivated, and our model reproduces Wittman (1983) model of electoral competition. On the other hand, the case of $\mu = 0$ boils down to a more general version of our previous work on participatory democracy (Aragonès and Sánchez-Pagés, 2009).

Thus, we have then set up a game in three stages: First, the incumbent implements a policy on the popular dimension. In the second stage, both candidates simultaneously announce policy platforms and in the third stage citizens vote to reelect or not the incumbent. In the next section we analyze the equilibrium of this game for all values of the parameters K and μ .

3 Equilibrium results

In order to solve the game described above we look for its subgame perfect equilibrium, solving the game by backward induction. Thus we start analyzing the electoral stage, taking as given the choice of the incumbent on the

popular issue. Citizens evaluate the incumbent according to his performance on the popular issue, so he does not enter the new election on the same grounds as the challenger. However, it would be wrong to conclude that the incumbent is indeed at a disadvantage. When the incumbent expects to face a tough competition on the electoral issue, he can soften it by conceding more on the popular issue. Similarly, when the incumbent expects to face a soft competition on the electoral issue, he can compensate his payoffs by not satisfying the voters on the popular issue. This is a strategic move that only the incumbent may afford. The following lemma illustrates this point.

Lemma 1 *If $x(L) = x(R)$, then L obtains at least $1 - 2|y(L) - y_m|$ of the votes and R obtains at most $2|y(L) - y_m|$.*

Proof. If $x(L) = x(R)$ then $-(1 - \mu)|y_m - y(L)| - \mu|x_i - x(L)| \geq -|x_i - x(R)|$ becomes $|y_m - y(L)| \leq |x_i - x(R)|$. Thus L obtains votes from all i such that are at a distance from $x(R) = x(L)$ of at least $|y(L) - y_m|$. This means that R obtains at most $2|y(L) - y_m|$ votes, therefore L obtains at least $1 - 2|y(L) - y_m|$ votes. Notice that in this case R obtains exactly $2|y(L) - y_m|$ if $|y(L) - y_m| \leq x(L) = x(R) \leq 1 - |y(L) - y_m|$. ■

This lemma shows how the presence of the popular issue affects the electoral competition. When both candidates choose the same position on the electoral issue, that is when $x(L) = x(R)$, only citizens at a distance of at least $|y(L) - y_m|$ from the policy proposed by both candidates vote for the incumbent. Thus, it is possible that the vote of the extremists is captured by the incumbent given a specific performance on the popular issue. It also implies that the incumbent's chances of winning are better if he does not depart too much from the society's most preferred policy on the popular issue. As a matter of fact, the threshold on this distance is critical in ascertaining whether the incumbent has an advantage. The next proposition describes the policy choices on the popular issue that guarantee the incumbent reelection in equilibrium.

Proposition 2 *If $|y(L) - y_m| < 1/4$, then L wins in equilibrium.*

Proof. First suppose that $\{y(L), x(L), x(R)\}$ is an equilibrium outcome such that $x(R) = x(L)$. Then R cannot win because by the previous lemma R at most can obtain $2|y(L) - y_m| < 1/2$ votes.

Next suppose that $\{y(L), x(L), x(R)\}$ is an equilibrium outcome such that $x(R) \neq x(L)$ and R wins. Then we must have

$$U_L(y(L), x(L), x(R)) = -y(L) - x(R).$$

Consider that L chooses instead $x'(L) = x(R)$. Then by the previous lemma L obtains at least $1 - 2|y(L) - y_m| > 1/2$ votes. Thus L wins and his utility is

$$U_L(y(L), x(R), x(R)) = -y(L) + K - x(R) > -y(L) - x(R),$$

In this case L prefers to win and has a winning strategy. Thus R cannot win in equilibrium. ■

When the incumbent decides to satisfy the citizens with his policy choice on the popular issue, the advantage that he obtains guarantees the existence of a winning strategy at the electoral stage. If the policy choice of the popular issue is close enough to the policy proposal, then it is also optimal for the incumbent to use the strategy that guarantees a sure reelection. That is what he will do in equilibrium.

On the other hand, if the incumbent does not satisfy the electorate with his performance on the participatory issue he will suffer a disadvantage at the electoral stage. In this case the incumbent cannot always guarantee a winning strategy at the electoral stage, and even when he can use a winning strategy, he prefers to lose in equilibrium. This is what the next proposition shows.

Proposition 3 *If $|y(L) - y_m| > 1/4$, R wins in equilibrium.*

Proof. First suppose that $|y(L) - y_m| > 1/2$. If L is winning in equilibrium with $x(L)$ and $x(R)$, then consider $x'(R)$ such that $x'(R) = x(L)$ and notice that in this case R obtains more than $|y(L) - y_m|$ votes, that is, more than half of the total. The reason is that if $x(L) \leq |y(L) - y_m|$ then R obtains $x(L) + |y(L) - y_m|$. Similarly if $x(L) \geq |y(L) - y_m|$ then R obtains $1 - x(L) + |y(L) - y_m|$. Thus L cannot win in equilibrium with $|y(L) - y_m| > 1/2$.

Next suppose that $|y(L) - y_m| \in (\frac{1}{4}, \frac{1}{2})$

If $x(L) \in [\frac{1}{2} - |y(L) - y_m|, \frac{1}{2} + |y(L) - y_m|]$ then R can defeat it with $x(R) = x(L)$ and he prefer to do so since he obtains K by mimicking L.

If $x(L) \in [0, \frac{1}{2} - |y(L) - y_m|)$ then R can defeat L with $x(R) \in (\frac{3-2\mu}{4}, \frac{3}{4})$. To show this, note that the set of supporters of R is the interval $[\frac{x(R) + \mu x(L)}{1 + \mu} - \frac{1-\mu}{1+\mu}|y(L) - y_m|, 1]$ whenever $x(R) > (1 - \mu)(1 - |y(L) - y_m|) + \mu x(L)$. In addition, this number of voters constitutes a majority if and only if $x(R) < \frac{1+\mu}{2} + (1 - \mu)|y(L) - y_m| - \mu x(L)$. This defines an interval of platforms that R can use to defeat L. Given the restrictions on $|y(L) - y_m|$ and the assumption on $x(L)$, this interval is at least as large as the interval $(\frac{3-2\mu}{4}, \frac{3}{4})$. Hence, any

platform in this interval guarantees R a victory against $x(L)$. Note again that R prefers to win rather than to let L win because $\frac{1}{2} - |y(L) - y_m| < \frac{3-2\mu}{4}$.

If $x(L) \in (\frac{1}{2} + |y(L) - y_m|, 1]$ then the best winning policy for R is $x(R) = \mu x(L) + (1 - \mu)(\frac{1}{2} + |y(L) - y_m|)$. We show this by following the same procedure as above to define the set of R's supporters and then check when it constitutes a majority. Next we need to see whether R actually uses this winning strategy. For this to be the case it need to hold that

$$\begin{aligned} K - 1 + \mu x(L) + (1 - \mu)(\frac{1}{2} + |y(L) - y_m|) &> -1 + x(L) \\ \Leftrightarrow x(L) &< \frac{K}{1 - \mu} + \frac{1}{2} + |y(L) - y_m|. \end{aligned}$$

Hence, L will not be able to win with a $x(L)$ in $(\frac{1}{2} + |y(L) - y_m|, 1]$ if $K > (1 - \mu)(\frac{1}{2} - |y(L) - y_m|)$. If $K < (1 - \mu)(\frac{1}{2} - |y(L) - y_m|)$ we need to check whether L prefers to win the election with such rightist policy. The best case scenario for L if he wants to win is when $x(L) = \frac{K}{1 - \mu} + \frac{1}{2} + |y(L) - y_m|$. In that case, his payoff is just $-y(L) + K - \frac{K}{1 - \mu} - \frac{1}{2} - |y(L) - y_m|$. The best case scenario for L if in the contrary he decides to lose is to set $x(L) = \frac{1}{2} + |y(L) - y_m|$ given that that forces R to choose the same policy. His payoff is just $-y(L) - \frac{1}{2} - |y(L) - y_m|$, so he actually prefers to lose. ■

The two previous propositions show that the incumbent obtains a decisive advantage only when she concedes enough to citizens on the popular issue. If, on the contrary, she departs considerably from y_m then she is doomed to lose reelection. In order to solve the problem of the incumbent when choosing what policy to implement on the popular issue, we need to fully characterize the equilibrium of the political competition stage. The following two propositions describe the equilibrium strategies used by the winner of the election in equilibrium. These strategies define the equilibrium policy outcome of the electoral stage as well. First we find the equilibrium outcomes of the electoral stage for the case in which the incumbent is reelected in equilibrium. In this case, the equilibrium policy outcome coincides with the strategies used by the incumbent in equilibrium at the electoral stage.

Proposition 4 *If $|y(L) - y_m| \leq \frac{1}{4}$, then L's equilibrium strategies at the electoral stage are:*

- i) $x^*(L) = 0$ if $|y(L) - y_m| \leq \frac{1-3\mu}{4(1-\mu)}$
- ii) $x^*(L) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu} |y(L) - y_m|$ if $|y(L) - y_m| \geq \frac{1-3\mu}{4(1-\mu)}$

Proof. From the previous proposition we know that in this case L wins in equilibrium. Suppose that $x(L)$ and $x(R)$ is an equilibrium outcome such that L wins and $x(R) < x(L)$. Then we must have $U_L(y(L), x(L), x(R)) = -y(L) + K - x(L)$. Consider that L chooses instead $x'(L) = x(R)$. Then by lemma 1 L obtains at least $1 - 2|y(L) - y_m| > 1/2$ votes and his utility is $U_L(y(L), x(R), x(R)) = -y(L) + K - x(R)$.

Notice that $U_L(y(L), x(R), x(R)) = -y(L) + K - x(R) > -y(L) + K - x(L) = U_L(y(L), x(L), x(R))$ since we assumed that $x(R) < x(L)$. Thus, $x(L)$ and $x(R)$ such that $x(R) < x(L)$ cannot be part of an equilibrium strategy and we must have $x(L) \leq x(R)$.

Let us first characterize the sets of voters that vote for candidate L given $y(L)$, $x(L)$ and $x(R)$.

The set of voters with $x_i < x(L)$ that vote for L is given by all x_i such that

$$x_i < \frac{x(R) - \mu x(L)}{1 - \mu} - |y(L) - y_m| \equiv \underline{x}_i.$$

Similarly, the set of voters with $x_i > x(R)$ that vote for L is given by all x_i such that

$$x_i > \frac{x(R) - \mu x(L)}{1 - \mu} + |y(L) - y_m| \equiv \overline{x}_i$$

Since by proposition 2 $\frac{x(R) - \mu x(L)}{1 - \mu} > x(R)$ then we have that $\overline{x}_i > x(R)$. Notice that if $\underline{x}_i < 0$ then $\overline{x}_i < 1$ for all $|y(L) - y_m| < \frac{1}{2}$.

Finally, the set of voters with $x(L) < x_i < x(R)$ that vote for L is given by all x_i such that

$$x_i < \frac{x(R) + \mu x(L)}{1 + \mu} - \frac{1 - \mu}{1 + \mu} |y(L) - y_m| \equiv \tilde{x}_i$$

Since by proposition 2 $\frac{x(R) + \mu x(L)}{1 + \mu} < x(R)$ then we have that $\tilde{x}_i < x(R) < \overline{x}_i$. However, the comparison between $\underline{x}_i$ and $\tilde{x}_i$ is not clear-cut. We have that $\underline{x}_i < \tilde{x}_i < x(L)$ if and only if

$$x(R) - x(L) < (1 - \mu) |y(L) - y_m|.$$

Thus, two cases can emerge:

Case 1: If $x(R) - x(L) \geq (1 - \mu) |y(L) - y_m|$ then we have that the votes that L obtains are given by $\tilde{x}_i + \max\{0, 1 - \overline{x}_i\}$.

Case 2: If $x(R) - x(L) < (1 - \mu) |y(L) - y_m|$ then we have that the votes that L obtains are given by $\max\{0, \underline{x}_i\} + \max\{0, 1 - \overline{x}_i\}$.

Case 1: If $x(R) - x(L) \geq (1 - \mu) |y(L) - y_m|$

In this case we have that the votes that L obtains are given by $\tilde{x}_i + \max\{0, 1 - \bar{x}_i\}$.

Suppose in the first place that $x(L) = 0$. Then the number of votes that L receives are

$$\#L = \begin{cases} 1 - |y(L) - y_m| - \frac{x(R)}{1-\mu} & \text{if } x(R) < (1-\mu)|y(L) - y_m| \\ 1 - \frac{2\mu}{1-\mu^2}x(R) - \frac{2}{1+\mu}|y(L) - y_m| & \text{if } x(R) \in [(1-\mu)|y(L) - y_m|, (1-\mu)(1 - |y(L) - y_m|)] \\ \frac{x(R)}{1+\mu} - \frac{1-\mu}{1+\mu}|y(L) - y_m| & \text{if } x(R) > (1-\mu)(1 - |y(L) - y_m|) \end{cases}$$

that attains a minimum when $x(R) = (1-\mu)(1 - |y(L) - y_m|)$. The number of votes in that case is greater than $\frac{1}{2}$ if and only if

$$|y(L) - y_m| \leq \frac{1 - 3\mu}{4(1 - \mu)}.$$

Note that if this holds, $x(L) = 0$ is a winning strategy for L . Otherwise, there exists a platform $x(R)$ that can defeat $x(L) = 0$.

Second, suppose that $\frac{1-3\mu}{4(1-\mu)} < |y(L) - y_m| > \frac{1}{4}$. Let us first show that any platform $x(L) \in (0, \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y_m|)$ can be defeated by $x(R) = \frac{3-\mu}{4}$. First, note that we are in Case 1 since

$$x(R) - x(L) > (1-\mu)|y(L) - y_m| \Leftrightarrow x(L) < \frac{3-\mu}{4} - (1-\mu)|y(L) - y_m|$$

and in addition we have by assumption that

$$x(L) < \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y(A)| < \frac{3-\mu}{4} - (1-\mu)|y(L) - y(A)|$$

where the last inequality follows from simple algebra. One can also show that our assumption on $x(L)$ also implies that $\bar{x}_i > 1$ which means that the number of votes obtained by L is just $\tilde{x}_i$ which in turn is smaller than $\frac{1}{2}$ if and only if

$$x(L) < \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y_m|,$$

which holds by assumption. Hence, L is defeated if he chooses a platform in that interval. From the remainder, let us now show that $x(L) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y_m|$ is a dominant strategy.

Again case we have to consider two cases:

1. Suppose that $x(R) > \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu}|y(L) - y_m|$. In that case, the number of citizens who vote for the incumbent are given by $\min\{1, \bar{x}_i\} - \tilde{x}_i$. We need to consider two subcases depending on the value of the extremes of this interval.

i. If $\bar{x}_i > 1$ then R gets $1 - \tilde{x}_i$ votes and wins the election if and only if

$$\tilde{x}_i < \frac{1}{2} \rightarrow x(R) < \frac{3-\mu}{4}$$

Since $\bar{x}_i > 1$ if and only if $x(R) > \frac{3-\mu}{4}$ then this case cannot arise.

ii. If $\bar{x}_i < 1$ then R gets $\bar{x}_i - \tilde{x}_i$ votes. This number of votes is greater than $\frac{1}{2}$ if and only if $x(R) > \frac{3-\mu}{4}$. Since $\bar{x}_i < 1$ if and only if $x(R) < \frac{3-\mu}{4}$ again this case is not possible.

2. Suppose instead that $x(R) < \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu} |y(L) - y_m|$. This means necessarily that $\bar{x}_i < 1$ and that the challenger collects votes in $(\max\{0, \underline{x}_i\}, \bar{x}_i)$. We need to consider then two different subcases:

i. If $\underline{x}_i < 0$ the challenger gets $\bar{x}_i$ votes and wins if and only if $x(R) \geq \frac{1+\mu}{4}$. But this leads to a contradiction because

$$\frac{1+\mu}{4} > \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu} |y(L) - y_m| \Leftrightarrow \frac{1-\mu}{4(1+\mu)} > -|y(L) - y_m|.$$

ii. If $\underline{x}_i > 0$ then R gets $\bar{x}_i - \underline{x}_i = 2|y(L) - y_m|$ votes. So here R cannot win either.

Thus R cannot win the election for any $x(R)$ he may choose. Still, observe that $x(R) = \frac{3-\mu}{4}$ is a dominant strategy for her.

Since we have shown that L wins in equilibrium when $|y(L) - y_m| \leq \frac{1}{4}$, we have that L's most preferred best response is an equilibrium strategy. ■

This proposition illustrates the type of trade-off the incumbent faces when he tries to win the election. The more he pleases the electorate on the popular issue, the more he will be able to implement his preferred policy in the electoral issue. In particular, the incumbent is able to guarantee his reelection implementing his ideal point on the electoral issue if he satisfies enough the voters on the popular issue. In order to be able to be reelected by proposing his ideal point on the electoral issue the incumbent will have to concede more on the popular issue the larger the value of μ since $\frac{1-3\mu}{4(1-\mu)}$ decreases with μ .

Otherwise, if he implements a policy on the popular issue that departs significantly from the policy proposal, then in equilibrium he still decides to use a winning strategy, but in this case this strategy implies that he has to concede on the electoral issue to some extent. In this latest case, in order to guarantee a sure reelection the policy in the electoral issue that he has to

announce will lie between the incumbent's ideal point and the median voter's ideal point, and it will be closer to the median voter's ideal point the larger the distance between the policy implemented by the incumbent and the policy proposed on the popular issue. This equilibrium policy choice will be closer to the median voter's ideal point the tougher the competition at the electoral stage, that is the larger the value of μ , since $\frac{\partial x^*(L)}{\partial \mu} = \frac{1}{\mu^2} \left(\frac{1}{4} - |y(L) - y_m| \right) \geq$

0. In the limit, as the competition at the electoral stage dominates the game (μ increases), the policy announced by the incumbent on the electoral issue approaches the median voter's ideal point. Similarly, as the popular issue dominates the game (μ decreases), the policy announced by the incumbent on the electoral issue approaches the incumbent's ideal point.

The next proposition describes the equilibrium outcome of the electoral stage for the case in which the incumbent decides to forgo reelection in equilibrium. In this case the equilibrium policy outcome coincides with the strategies used by the challenger at the electoral stage in equilibrium.

Proposition 5 *If $|y(L) - y_m| > \frac{1}{4}$, then R's equilibrium strategy at the electoral stage is $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu} |y(L) - y_m|$*

Proof. First, suppose that $|y(L) - y_m| > 1/2$. If $x(L) > x(R)$ in equilibrium, consider $x'(R)$ such that $x'(R) = x(L)$ and notice that: 1) in this case R obtains more than $|y(L) - y_m|$ votes, that is, more than half of the total; and 2) the equilibrium policy outcome is larger, therefore better off for R'. Thus this is a profitable deviation for R and it implies that $x(L) > x(R)$ cannot hold in equilibrium.

Since we know that in equilibrium $x(L) \leq x(R)$ R's best winning strategy is defined by $\bar{x}_i > 1$ and $\tilde{x}_i < \frac{1}{2}$. This implies that

$$\bar{x}_i = \frac{x(R) - \mu x(L)}{1 - \mu} + |y(L) - y_m| > 1$$

and

$$\tilde{x}_i = \frac{x(R) + \mu x(L)}{1 + \mu} - \frac{1 - \mu}{1 + \mu} |y(L) - y_m| < \frac{1}{2}$$

Thus the set of winnings strategies for R is defined by

$$(1 - \mu)(1 - |y(L) - y_m|) + \mu x(L) < x(R) < \frac{1 + \mu}{2} + (1 - \mu)|y(L) - y_m| - \mu x(L)$$

and among them R prefers the largest one $x(R) = \frac{1 + \mu}{2} + (1 - \mu)|y(L) - y_m| - \mu x(L)$.

The best response for L in this case is the largest possible value of $x(L)$. So that R's best response to it corresponds to its smallest possible value. Since in equilibrium we need to have $x(L) \leq x(R)$ then $x(L) \leq \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$ implies $x(L) \leq \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$. Thus in equilibrium $x(L) = x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$.

Now suppose that $1/4 < |y(L) - y_m| < 1/2$. If $x(L) \in [0, \frac{1}{2} + |y(L) - y_m|]$ then R's best response, as in the previous proposition, is defined by $\bar{x}_i > 1$ and $\tilde{x}_i < \frac{1}{2}$.

This implies that

$$\bar{x}_i = \frac{x(R) - \mu x(L)}{1 - \mu} + |y(L) - y_m| > 1$$

and

$$\tilde{x}_i = \frac{x(R) + \mu x(L)}{1 + \mu} - \frac{1 - \mu}{1 + \mu}|y(L) - y_m| < \frac{1}{2}$$

Thus the set of winnings strategies for R is defined by $(1 - \mu)(1 - |y(L) - y_m|) + \mu x(L) < x(R) < \frac{1+\mu}{2} + (1 - \mu)|y(L) - y_m| - \mu x(L)$

and among them R prefers $x(R) = \frac{1+\mu}{2} + (1 - \mu)|y(L) - y_m| - \mu x(L)$

And the best response for L in this case is the largest possible value of $x(L)$. Since in equilibrium we need to have $x(L) \leq x(R)$ then $x(L) \leq \frac{1+\mu}{2} + (1 - \mu)|y(L) - y_m| - \mu x(L)$ implies $x(L) \leq \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$. Thus for $x(L) \in [0, \frac{1}{2} + |y(L) - y_m|]$ R's best response is $x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$.

Given that if $x(L) \in [\frac{1}{2} + |y(L) - y_m|, 1]$ we have that R's best response is $x(R) \in [\frac{1}{2} + |y(L) - y_m|, 1]$, and for $x(L) \in [0, \frac{1}{2} - |y(L) - y_m|]$ we have that R's best response is $x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m| < \frac{1}{2} + |y(L) - y_m|$, this implies that L's optimal strategy will not be in $[\frac{1}{2} + |y(L) - y_m|, 1]$.

Therefore the equilibrium if $\frac{1}{4} < |y(L) - y_m| < \frac{1}{2}$ is given by $x(L) = x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$. ■

When the incumbent has departed significantly from the citizens' ideal point in the popular issue, he prefers to lose the election and in this cases he chooses a moderate position on the electoral issue in order to force the challenger to choose a moderate policy as well in order to win. Observe that $\frac{\partial x^*(R)}{\partial \mu} = -\frac{2}{(1+\mu)^2}|y(L) - y_m| \leq 0$. Thus, as before, the tougher the competition at the electoral stage the closer the policy outcome will be to the median voter's ideal point. And the larger the distance between the policy proposal and the policy implemented on the popular issue the closer the policy outcome on the electoral issue will be from the challenger's ideal point.

After solving for the equilibrium strategies of the electoral stage of the game, we move forward in order to find the incumbent's best responses for the first stage of the game given the payoffs obtained from the continuation of the game.

Proposition 6 *L's best winning strategies are*

- (i) $y^*(L) = \max \left\{ y_m - \frac{1-3\mu}{4(1-\mu)}, 0 \right\}$ and $x^*(L) = 0$ if $\mu \leq \frac{1}{3}$
- (ii) $y^*(L) = y_m$ and $x^*(L) = \frac{3\mu-1}{4\mu}$ if $\frac{1}{3} \leq \mu \leq \frac{1}{2}$
- (iii) $y^*(L) = \max \left\{ y_m - \frac{1}{4}, 0 \right\}$ and $x^*(L) = \frac{1}{2}$ if $\mu \geq \frac{1}{2}$

Proof. Let us start with the case when $|y(L) - y_m| \leq \frac{1-3\mu}{4(1-\mu)}$. Notice that it can emerge if and only if $\mu \leq \frac{1}{3}$. In that case the incumbent's payoff is increasing with $|y(L) - y_m|$ so his most preferred value of $y(L)$ in this range corresponds to $y(L) = y_m - \frac{1-3\mu}{4(1-\mu)}$. We already know from previous results that in this case that he will then set $x^*(L) = 0$

When $\frac{1-3\mu}{4(1-\mu)} \leq |y(L) - y_m| \leq \frac{1}{4}$, after plugging the incumbent's equilibrium platforms in the electoral issue, it is possible to rewrite his payoff as

$$V_L = -y_m + K - \frac{3\mu - 1}{4\mu} - \frac{1 - 2\mu}{\mu} |y(L) - y_m|, \quad (2)$$

which is decreasing with $|y(L) - y_m|$ as long as $\mu \leq \frac{1}{2}$ and increasing otherwise. In the former case, L's most preferred value of $y(L)$ corresponds to the minimal value of $|y(L) - y_m|$ in this range, that is, $y(L) = \max \left\{ y_m - \frac{1-3\mu}{4(1-\mu)}, y_m \right\}$. Hence, if $\mu \leq \frac{1}{3}$ he will set again $y(L) = y_m - \frac{1-3\mu}{4(1-\mu)}$ (and then $x^*(L) = 0$) whereas if $\frac{1}{3} \leq \mu \leq \frac{1}{2}$ he must set $y(L) = y_m$ which in turn implies that $x^*(L) = \frac{3\mu-1}{4\mu}$.

The third case occurs when $\mu \geq \frac{1}{2}$. Then (2) is increasing with $|y(L) - y_m|$. Thus while staying in this range his most preferred value of $y(L)$ corresponds to the one that maximizes $|y(L) - y_m|$, that is, $y(L) = y_m - \frac{1}{4}$, that from previous results it implies $x^*(L) = \frac{1}{2}$. ■

When competition on the electoral issue is rather soft ($\mu \leq \frac{1}{3}$) the incumbent will concede enough on the popular issue so that he can guarantee his reelection implementing his ideal point on the electoral issue. The tougher the electoral competition (the larger the value of μ) the more he has to concede on the popular issue. For higher values of μ , the incumbent prefers to satisfy completely the policy proposed on the popular issue and win the election

by choosing an electoral policy as close as possible to his ideal point. This policy will be larger (less favorable for the incumbent, but always smaller than $\frac{1}{4}$) the tougher the electoral competition, that is, the larger the value of μ . Finally, when the electoral competition becomes very tough ($\mu \geq \frac{1}{2}$) the incumbent prefers to win the election by conceding enough on the popular issue, that is, he has to implement the median voter's ideal point on the electoral issue. See figure 1.

Notice that the policy implemented in equilibrium on the electoral issue is always increasing with the value of μ . That is, the tougher the electoral competition the less favorable the policy outcome on the electoral issue is for the incumbent. However, the policy implemented in equilibrium on the popular issue is not a monotone functions of μ .

Next we find the incumbent's best losing strategy and the corresponding best response of the challenger.

Lemma 7 Proposition 8 *The incumbent best losing strategy is to set $y^*(L) = 0$ which in turn implies that $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}y_m$.*

Proof. We know from previous results that if the incumbent decides to lose by setting $|y(L) - y_m| > \frac{1}{4}$, the challenger will win the election and set $x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$. In that case, the incumbent receives the payoff

$$V_L = -y_m - \frac{1}{2} - \frac{2\mu}{1+\mu}|y(L) - y_m|,$$

which is increasing in $|y(L) - y_m|$. Thus while staying in this range, his most preferred value of $y(L)$ corresponds to the one that maximizes $|y(L) - y_m|$, that is, $y^*(L) = 0$, which implies that the challenger's best response in this case is $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}y_m$. ■

Finally, the last step of the analysis amounts to characterize when the incumbent will prefer to win the election. The parameters that determine whether the incumbent prefers to win the election are: the policy proposed on the popular issue, y_m , the incumbent's value for holding office, K , and the relative weight that voters assign to the different issues, μ . In particular, we have that if the policy proposed on the popular issue is close enough to the ideal point of the incumbent on this issue, then the incumbent prefers to win for all values of K and all values of μ . Otherwise, if the preferences of the incumbent on the popular issue are not aligned with the policy proposal on this issue, then the incumbent may decide to forgo the reelection. In this

case, he will do so only when he is mostly policy motivated (for low values of K). The softer the electoral competition the lower the value of K that will induce the incumbent to forgo the election. Intuitively, the more intense electoral competition and the more costly is to please voters in the popular issue, the more likely is that the incumbent will prefer to lose.

Proposition 9 *If $y_m \leq \frac{1}{4}$, the incumbent wins in equilibrium for any $K \geq 0$ and any $0 \leq \mu \leq 1$.*

If $\frac{1}{4} \leq y_m \leq \frac{3}{8}$ the incumbent wins the election in equilibrium if and only if

$$K > \begin{cases} 0 & \text{if } \mu \leq \frac{1}{8y_m+1} \\ \frac{2\mu}{1+\mu}y_m - \frac{1}{4} & \text{if } \mu \geq \frac{1}{8y_m+1} \end{cases}$$

If $y_m \geq \frac{3}{8}$ the incumbent wins the election in equilibrium if and only if

$$K > \begin{cases} \max \left\{ \frac{2\mu}{1+\mu}y_m + \frac{5\mu-3}{4(1-\mu)}, 0 \right\} & \text{if } \mu \leq \frac{1}{2} \\ \frac{2\mu}{1+\mu}y_m - \frac{1}{4} & \text{if } \mu \geq \frac{1}{2} \end{cases}$$

Proof. Previous results show that since $y_m < \frac{1}{4}$ implies that $|y(L) - y_m| < \frac{1}{4}$ then L prefers to win in this case.

If $y_m \geq \frac{1}{4}$, if the incumbent decides to lose then he receives a payoff equal to

$$V_L = -\frac{1}{2} - \frac{1-\mu}{1+\mu}y_m.$$

If the incumbent decides to use his best winnings strategy then he receives a payoff equal to

when $\mu \leq \frac{1}{3}$ his payoff boils down to

$$V_L = -y_m + K - \frac{3\mu-1}{4(1-\mu)} \text{ if } \mu \leq \frac{1}{2} \quad (3)$$

and

$$V_L = -y_m + K - \frac{1}{4} \text{ if } \mu \geq \frac{1}{2} \quad (4)$$

Thus, when $\mu \geq \frac{1}{2}$ he prefers to use his winning strategy as long as $-y_m + K - \frac{1}{4} \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu}y_m$, that is, for

$$K > \frac{2\mu}{1+\mu}y_m - \frac{1}{4}$$

Notice that this value is strictly positive for all values of $\mu \in [0, 1]$ as long as $y_m > \frac{3}{8}$. For $\frac{1}{4} \leq y_m \leq \frac{3}{8}$ we will have that the incumbent will decide to

use a winning strategy for all values of K whenever $\frac{2\mu}{1+\mu}y_m - \frac{1}{4} > 0$, that is, $\mu > \frac{1}{8y_m-1}$. Notice that the incumbent decides to win for all K whenever $y_m = \frac{1}{4}$. Furthermore, the incumbent always decides to forgo reelection for some positive values of K whenever $y_m > \frac{3}{8}$.

Similarly, when $\mu \leq \frac{1}{2}$ he prefers to use his winning strategy as long as $-y_m + K - \frac{3\mu-1}{4(1-\mu)} \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu}y_m$, that is, for

$$K > \frac{5\mu-3}{4(1-\mu)} + \frac{2\mu}{1+\mu}y_m,$$

Notice that this value is strictly negative for small values of μ (in particular for all $\mu \leq \frac{1}{3}$). For those values the incumbent decides to win the election for all K . The set of values of K for which the incumbent decides to use a winning strategy is smaller for larger values of μ in this area. ■

As we have proven before, when the preferences of the incumbent on the popular issue are aligned with those of the society ($y_m \leq \frac{1}{4}$) the incumbent always prefers to use a winning strategy. When that is not the case we find that for some combinations of values for K and μ the incumbent may prefer to forgo reelection. That happens only for large enough values of μ , and for small enough values of K . See figure 2.

The formal analysis of the strategic behavior of the incumbent facing a policy proposal originated by a popular initiative shows that incumbents that are mostly policy motivated might suffer a disadvantage from being in office. They may find it too costly to make a policy investment that would guarantee their reelection when their preferences are not aligned with society's preferences. But in general, the incumbent's strategic advantage may overcome the disadvantage that incumbents receive from popular policy proposals.

4 Concluding remarks

The success of representative democracy relies on the willingness of incumbents to deliver policies that satisfy the preferences of voters. The incentives that such a system offers to politicians often do not go in this direction. Incumbents that are policy motivated, as opposed to office motivated, do not take into account the voters' preferences, and often they ignore them. The systems of direct democracy analyzed in this paper, referenda and participatory democracy, are supposed to build a bridge between candidates and voters over which (1) the information about the voters' preferences may be transmitted from voters to candidates, and (2) voters may offer incentives to incumbents to satisfy their policy proposals.

Voters are interested in investing time and effort in elaborating a policy proposal to be submitted to the incumbent on those issues for which the voters preferences are very intense and the incumbents' preferences are very weak. That is, if they think that the incumbent is planning to make a more or less satisfactory policy choice on a certain issue, voters will not go through the trouble of organizing and making a proposal. On the other hand, if voters think that the incumbent is not going to act on an issue that they regard as important in a satisfactory way, then they will have incentives to submit a proposal and use it a threat.

Incumbents are more likely to receive policy proposals on issues where there is a conflict of preferences between the incumbent and society. In these cases it is reasonable to assume that the voters have incentives to organize themselves, offer a policy proposal to the incumbent, and base their vote on the incumbent's performance on that issue.

In fact, there is empirical evidence that voters welfare increases when making use of these direct democracy systems. As an example of their relevance we offer the following quotes from Frey and Bohnet (1993):

"A recent referendum made it clear that the political elite's interests do not always correspond with voters' preferences. In September 1992, the citizens of Switzerland turned down two proposals seeking to increase substantially the salaries and the staff of Swiss members of Parliament. Both issues would have become law without Swiss voters taking the optional referendum, and both issues would clearly have been to the benefit of the elected officials."

"... two referenda on Switzerland joining international organizations or agreements: the United Nations in 1986 and the European Economic Area in 1992.

Both proposals were rejected by the citizens, even though the political elite strongly supported them. These referenda were universally supported by all major political parties; all pressure groups, including both employers and trade unions; a huge majority of the members of Parliament; and the executive branch. However, the popular referendum of Switzerland joining the United Nations resulted in a rejection by 76 percent of the voters; on 6 December 1992, 50.3 percent of the population and a majority of the cantons (sixteen out of twenty-three) voted against Switzerland becoming part of the European Economic Area."

"These two examples of the citizens voting differently than the public officials in power are not exceptions: in 39 percent of the 250 referenda held in Switzerland between 1948 and 1990, the will of the majority of the voters differed from the opinion of the Parliament. Thus, in a representative system, the decision by the Parliament would have deviated from the people's preferences in 39 percent of all cases where referenda were held.

Econometric cross-section studies for Switzerland, moreover, reveal that political decisions with respect to publicly supplied goods correspond better with the voters' preferences when the institutions of direct political participation are more extensively developed. Because it is the individual tax'payers and not the elected officials per se who have to bear the costs of government activities, it is not surprising that public expenditures are *ceteris paribus* lower in communities where the taxpayers themselves can decide on such matters.

Taxpayers however do reward politicians' performance by a high tax morale if they are satisfied with policies in their community."

These direct democracy systems allow voters to destroy the agenda control of politicians, and bring implemented policies closer to what satisfies voters' preferences. One could think that if these direct democracy systems are so effective in selecting policies, then lobbies would have strong incentives in manipulating them. However, Frey and Bohnet (1993) argue that lobbying is less successful when these systems of direct democracy are in place. They show that in Switzerland, even if pressure groups and the political class are united they cannot always have their way particularly on important issues.

A novel feature of our approach is that the model we built combines elements of both retrospective voting and prospective voting. Voters use retrospective voting to evaluate the performance of the incumbent with respect to the popular issues, those issues that matter enough for voters. And voters use prospective voting to evaluate the campaign promises that candidates announce during the electoral campaign. In order to use all the information they have available at the time to make the voting decision, voters will have to combine these two different kinds of evaluations in a unique payoff function.

We could extend the model analyzed in this paper by internalizing the stage where the popular policy proposal originates. This stage would have to take into account who decides to bring the proposal forward: whether it is the citizens, the government, a party in the opposition, a lobby, etc. . . Each different case will have different consequences over the strategic behavior of all agents in the following stages of the game and over the final outcomes. In particular, depending on who initiates the proposal, the intensity of the voters response will be different and the balance between the incumbent's advantage and disadvantage will change.

We have assumed that voters use an asymmetric rule in order to evaluate the candidates. The reason is that we have identified two different kinds of asymmetries that we had to take into account: (1) only the incumbent is responsible for the policy implemented on the popular issue, and (2) there is a policy proposal made only on the popular issue. Thus, we have assumed

that voters assign different weights to the two issues, and they evaluate the incumbent according to his performance on the two issues and the challenger only according to the electoral issue.

We can relax this assumption by assuming that both candidates are evaluated according to both issues. The evaluation of the challenger with respect to the popular issue can only be a parameter, because during the period analyzed by the model the challenger cannot make any policy implementation. This parameter would represent the performance of the challenger with respect to popular issues in the past. We can even assume that the weights that voters assign to the different issues are different depending on the candidate that they are evaluating. In this case, we would have that the incumbent is more likely to have an advantage the larger it is the weight that the voters assign to the challenger on the electoral issue.

In addition to the systems discussed in this paper one may think that the results of polls and surveys that are not initiated by incumbent may imply similar effects to the ones obtained in this paper.

On the other hand, the use of primaries to decide the parties' candidates, open lists of candidates instead of closed lists, or even a federal structure instead of a centralized one, are additional ways that help to improve this transmission of information. However the effects of these instruments cannot be analyzed with the present model.

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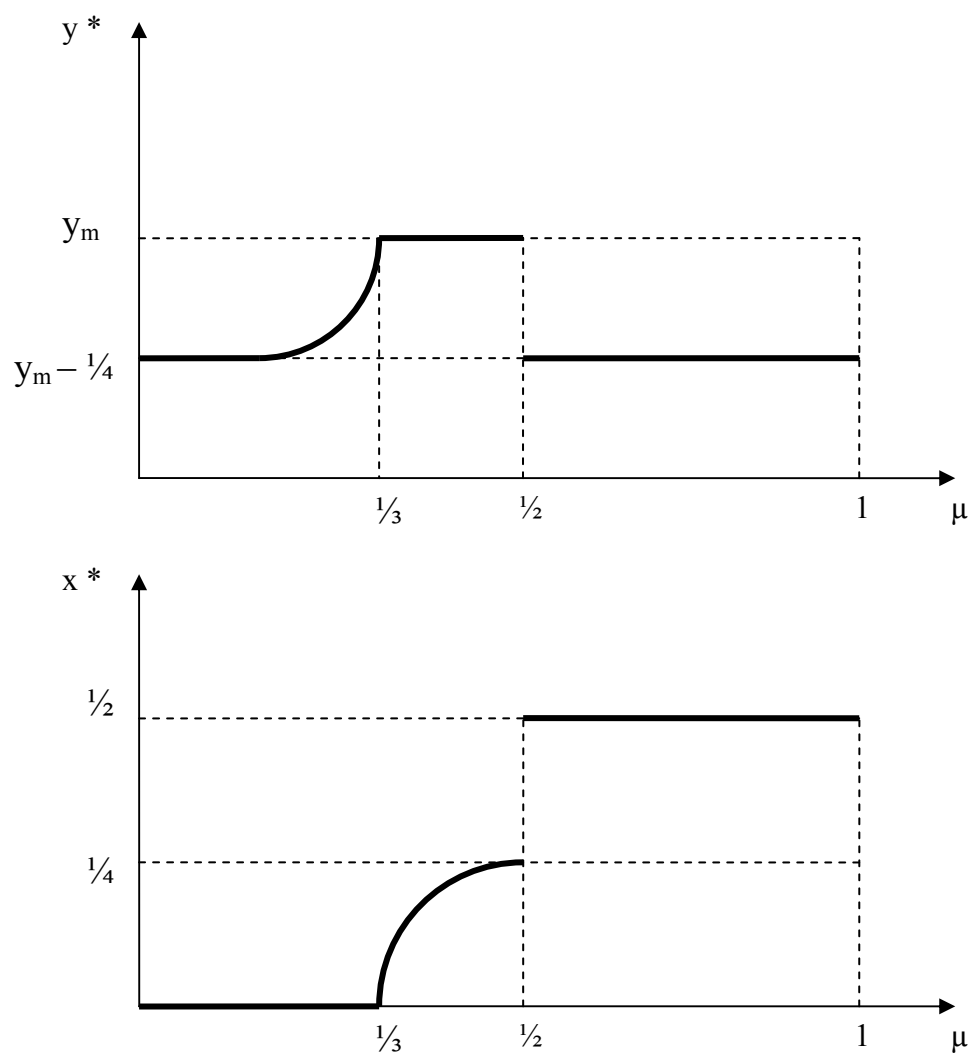


Figure 1: Incumbent's best winning strategies.

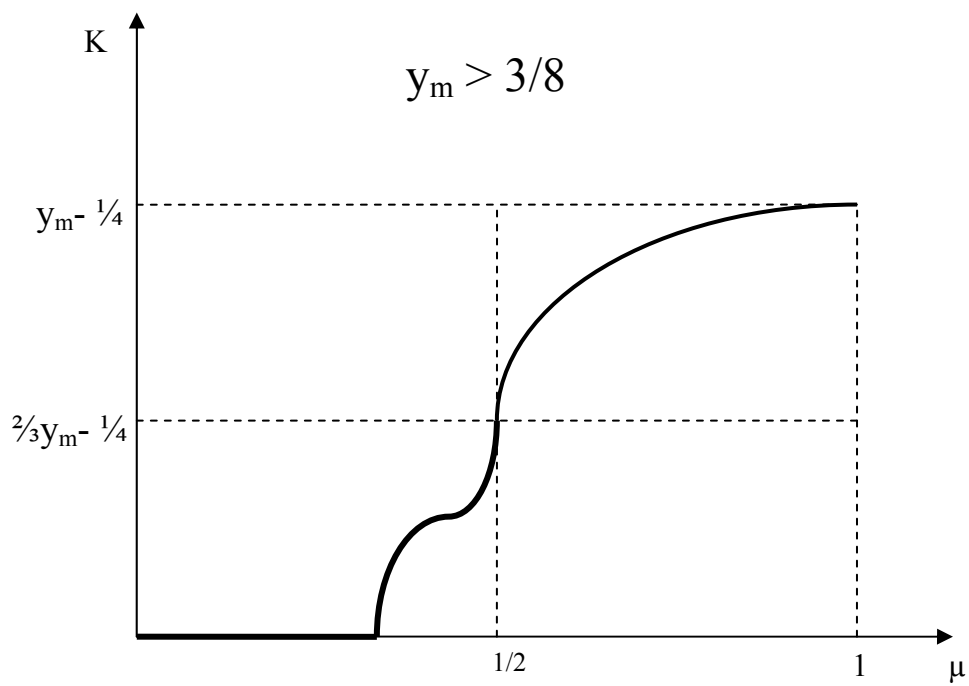
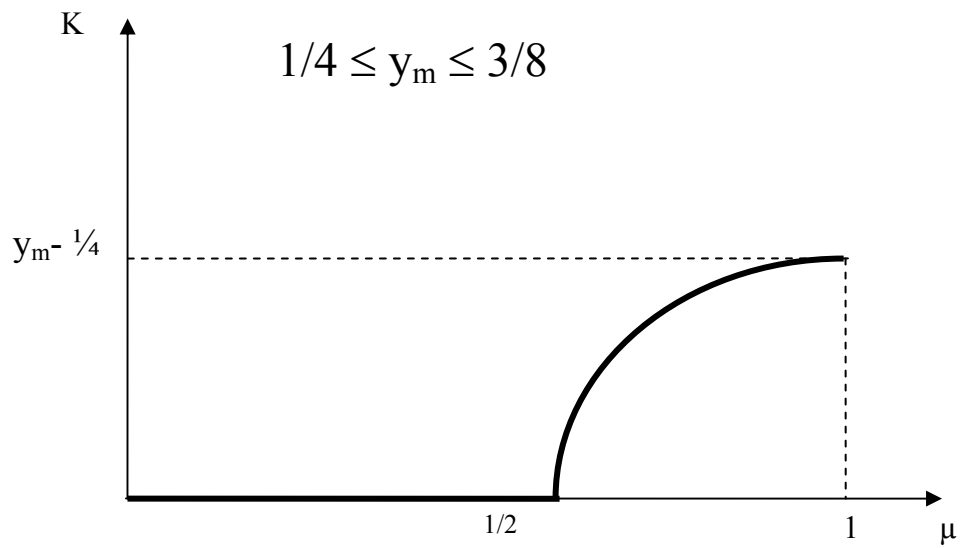


Figure 2 : Minimal values of K for which the incumbent prefers to use a winning strategy in equilibrium.