Testing for changes in the unconditional variance of financial time series.*

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Abstract

Inclan and Tiao (1994) proposed a test for the detection of changes of the unconditional variance which has been used in financial time series analysis. In this article we show some serious drawbacks for using this test with this type of data. Specifically, it suffers important size distortions for leptokurtic and platykurtic innovations. Moreover, the size distortions are more extreme for heteroskedastic conditional variance processes. These results invalidate in practice the use of the test for financial time series. To overcome these problems we propose new tests that explicitly consider the fourth moment properties of the disturbances and the conditional heteroskedasticity. Monte Carlo experiments show the good performance of these tests. The application of the new tests to the same series in Aggarwal, Inclan and Leal (1999) reveal that the changes in variance they detect are spurious.

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1 Introduction

Inclan and Tiao (1994) -IT hereafter- proposed a statistic to test for changes in the unconditional variance of a stochastic process. This test is based on the assumption that the disturbances are independent and Gaussian distributed, conditions that could be considered as extreme for financial time series provided that they usually show empirical distributions with fat tails (leptokurtic) and persistence in the conditional variance. Despite of this, the test has been extensively used for detecting changes in the volatility of financial time series such as returns, see, among others, Wilson *et al.* (1996), Aggarwal, Inclan and Leal (1999) and Huang and Yang (2001). For instance, Figure 1 shows the detected changes in the unconditional variance using the IT procedure by Aggarwal, Inclan and Leal (1999). As can be seen, several breaks are detected, some of them lasting few observations, which casts doubts on the real number of the changes that can be obtained by the application of the IT method.

[insert figure 1 about here]

In this paper we show that the asymptotic distribution of the IT test is free of nuisance parameters only when the stochastic process is mesokurtic and the conditional variance is constant. Otherwise, the distribution depends on some parameters and one would expect to find size distortions for the test when the process is non-mesokurtic and/or there is some persistence in the conditional variance. This will drive to find spurious changes in the unconditional variance. To overcome these problems, we propose new tests that take into account the fourth moment of the process and the persistence in the variance. These tests have an asymptotic distribution free of nuisance parameters and belong to the CUSUM-type tests family –see Andreou and Ghysels (2002) for a discussion on the recent literature. Moreover, we will also show that the IT test diverges when the disturbances are IGARCH.

The plan of the paper is as follows. Section 2 considers in some detail the IT test and its asymptotic distribution for both mesokurtic and non-mesokurtic processes. A new test that explicitly considers the fourth moment of the process is introduced. Section 3 focus on processes with persistence in the conditional variance. It is shown that the preceding tests, which do not consider such a persistence, have asymptotic distributions which depend on nuisance parameters. Subsequently, a modified version of the IT is proposed. Moreover, the asymptotic behavior of the three tests for IGARCH processes are also considered. Section 4, considers the Iterated Cumulative Sum of Squares (ICSS) algorithm suggested by Inclan and Tiao (1994) and adapts it to the suggested new tests. Given that this procedure needs to compute the tests for different sample sizes, we estimate response surfaces to generate critical values for any sample size. In Section 5, some Monte Carlo experiments confirm that the limit results derived in the preceding sections are also relevant in finite samples. The main conclusion of these simulations is that the κ_2 test we propose, which considers both the persistence in the variance as well as the kurtosis of the distribution, outperforms the other two tests and therefore should be used instead in applied research. In Section 6 we apply the ICSS procedure based on the new tests to the same series considered in Aggarwal, Inclan and Leal (1999) and we show that the changes in variance they detect are spurious. Finally, Section 7 concludes. The proofs of all propositions are collected in the Appendix.

2 The Inclan-Tiao test

In order to test the null hypothesis of constant unconditional variance, Inclan and Tiao (1994) proposed to use the statistic given by

$$IT = \sup_k \left| \sqrt{T/2} D_k \right|$$

where

$$D_k = \frac{C_k}{C_T} - \frac{k}{T}$$

and $C_k = \sum_{t=1}^{k} \varepsilon_t^2$, $k = 1, ..., T$, is the cumulative sum of squares of ε_t. Under the assumption that ε_t are a zero-mean, normally, identically and independently distributed random variables, $\varepsilon_t \sim iidN\left(0, \sigma^2\right)$, the asymptotic distribution of the test is given by:

$$IT \Rightarrow \sup_r \left| W^* (r) \right| \tag{1}$$

where $W^*(r) \equiv W(r) - rW(1)$ is a Brownian Bridge, $W(r)$ is a standard Brownian motion and $\Rightarrow$ stands for weak convergence of the associated probability measures.

The most serious drawback of the IT test is that its asymptotic distribution free of nuisance parameters critically depends on the assumption of normally, independently and identically distributed random variables ε_t. The following proposition establishes the asymptotic distribution of the test for the rather general case $\varepsilon_t \sim iid\left(0, \sigma^2\right)$.

Proposition 1 *If $\varepsilon_t \sim iid\left(0, \sigma^2\right)$, and $E\left(\varepsilon_t^4\right) \equiv \eta_4 < \infty$, then*

$$IT \Rightarrow \sqrt{\frac{\eta_4 - \sigma^4}{2\sigma^4}} \sup_r \left| W^* (r) \right| .$$

Hence, the distribution is not free of nuisance parameters and size distortions should be expected when using the critical values of the supremum of a Brownian Bridge. Note that for Gaussian processes $\eta_4 = 3\ \sigma^4$ and $IT \Rightarrow \sup_r |W^*(r)|$. When $\eta_4 > 3\ \sigma^4$, the distribution is leptokurtic (heavily tailed) and too many rejections of the null hypothesis of constant variance should be expected, with an effective size greater than the nominal one. Contrarily, when $\eta_4 < 3\ \sigma^4$ the test will be too conservative. In section 6 the finite sample performance of IT in such cases will be studied.

Proposition 1 suggests the following correction to the previous test that will be free of nuisance parameters for identical and independent zero-mean random variables:

$$\kappa_1 = \sup_k \left| T^{-1/2} B_k \right|$$

where

$$B_k = \frac{C_k - \frac{k}{T} C_T}{\sqrt{\widehat{\eta}_4 - \widehat{\sigma}^4}},$$

$\widehat{\eta}_4 = T^{-1} \sum_{t=1}^{T} \varepsilon_t^4$ and $\widehat{\sigma}^2 = T^{-1} C_T$. Its asymptotic distribution is established in the following proposition.

Proposition 2 *If* $\varepsilon_t \sim iid\left(0, \sigma^2\right)$, *and* $E\left(\varepsilon_t^4\right) \equiv \eta_4 < \infty$, *then* $\kappa_1 \Rightarrow \sup_r \left| W^*\left(r\right)\right|.$

Table 1 shows the finite sample critical values for κ_1. They have been computed from 50,000 replications of $\varepsilon_t \sim iidN\left(0, 1\right)$, $t = 1, ..., T$. A response surface to generate critical values for a wider range of samples sizes will be presented in Section 5.

[insert Table 1 about here]

Given that this statistic is free of nuisance parameters, we will expect a correct size when the disturbances are *iid*. Section 6 will examine the finite sample performance for both the IT and κ_1 tests. Before that, we consider the case of a conditionally heteroskedastic process.

3 Conditionally heteroskedastic processes

Both tests, IT and κ_1 in the previous Section, depend on the independence of the random variables. This is a very strong assumption for financial data, where there is evidence of conditional heteroskedasticity, see, for instance, Bera and Higgins (1993), Bollerslev *et al.* (1992, 1994) and Taylor (1986). In order to consider this situation explicitly, an estimation of the persistence may be used to correct the cumulative sum of squares. Nevertheless, some assumptions on ε_t are required.

Assumptions A1: *Assume that the sequence of random variables* $\{\varepsilon_t\}_{t=1}^{\infty}$ *satisfies:*

1. $E\left(\varepsilon_t\right) = 0$ *and* $E\left(\varepsilon_t^2\right) = \sigma^2 < \infty$ *for all* $t \geq 1$;

2. $\sup_t E\left(\left|\varepsilon_t\right|^{\psi+\epsilon}\right) < \infty$ *for some* $\psi \geq 4$ *and* $\epsilon > 0$;

3. $\omega_4 = \lim_{T\to\infty} E\left(T^{-1}\left(\sum_{t=1}^{T}\left(\varepsilon_t^2 - \sigma^2\right)\right)^2\right) < \infty$ *exists, and*

4. $\{\varepsilon_t\}$ *is* $\alpha-$*mixing with coefficients* α_j *which satisfy* $\sum_{j=1}^{\infty} \alpha_j^{(1-2/\psi)} < \infty$.

This set of assumptions is similar to that of Herrndorf (1984) and Phillips and Perron (1988) but here we need to impose the existence of moments greater than four and a common unconditional variance for all the variables of the sequence, which is the hypothesis we wish to test. Obviously, the existence of the fourth moments restricts the processes we can deal with. For instance, if ε_t is independent and identically distributed as a t-Student with three degrees of freedom, this sequence does not fulfil conditions 2 and 3. Note that the second condition does not impose a common fourth moment so that some sort of non-stationarity is allowed. ω_4 can be interpreted as the long-run fourth moment of ε_t or the long-run variance of the zero-mean variable $\xi_t \equiv \varepsilon_t^2 - \sigma^2$.[1] Condition 4 controls for the "degree of independence" of the sequence and shows a trade-off between the serial dependence and the existence of high order moments. In our case, by imposing the finiteness of the fourth moments we allow for a greater degree of serial dependence.

This brings us to propose the following statistic:

$$\kappa_2 = \sup_k \left| T^{-1/2} G_k \right|$$

where

$$G_k = \widehat{\omega}_4^{-1/2} \left(C_k - \frac{k}{T} C_T \right)$$

and $\widehat{\omega}_4$ is a consistent estimator of ω_4. One possibility is to use a non-parametric estimator of ω_4,

$$\widehat{\omega}_4 = \frac{1}{T} \sum_{t=1}^{T} \left(\varepsilon_t^2 - \widehat{\sigma}^2 \right)^2 + \frac{2}{T} \sum_{l=1}^{m} w\left(l, m\right) \sum_{t=l+1}^{T} \left(\varepsilon_t^2 - \widehat{\sigma}^2 \right) \left(\varepsilon_{t-l}^2 - \widehat{\sigma}^2 \right)$$

where $w\left(l, m\right)$ is a lag window, such as the Bartlett, defined as $w\left(l, m\right) = 1 - l/\left(m+1\right)$, or the quadratic spectral. This estimator depends on the selection of the bandwidth m, which can be chosen using an automatic procedure as proposed by Newey-West (1994).[2] Note that if $\xi_t = \varepsilon_t^2 - \widehat{\sigma}^2$ is not correlated, then $\widehat{\omega}_4 \to E\left(\xi_t^2\right) = \eta_4 - \sigma^4$. Kokoszka and Leipus (2000) proposed a test that is similar to κ_2 but departing from a different set of assumptions. Specifically, they assume an ARCH(∞) process. As can be seen, our framework is more general than the one of Kokoszka and Leipus (2000).

The limit distribution of the statistics for variance persistent processes is established in the next proposition.

Proposition 3 *Under assumption A1,*

[1] Note that when ε_t is a strictly stationary sequence $\omega_4 = 2\pi f_\xi\left(0\right)$, where $f_\xi\left(\lambda\right)$, $-\pi \leq \lambda \leq \pi$, is the spectrum of ξ_t.

[2] Another possibility is to use a parametric estimation of the long-run variance of ξ_t based on the Akaike estimator of the spectrum. That is $\widetilde{\omega}_4 = (1 - \widehat{\lambda}\left(1\right))^{-2} T^{-1} \sum_{t=1}^{T} e_t^2$, where $\widehat{\lambda}\left(1\right) = \sum_{j=1}^{p} \widehat{\lambda}_j$, $\widehat{\lambda}_j$ and e_t are obtained from the autoregression: $\xi_t = \widehat{\delta} + \sum_{j=1}^{p} \widehat{\lambda}_j \xi_{t-j} + e_t$. Andreou and Ghysels (2002), when computing the Kokoszka and Leipus (2000) test, use the VARHAC estimator of den Hann and Levin (1997) for ω_4.

a) $IT \Rightarrow \sqrt{\frac{\omega_4}{2\sigma^4}}\sup_r |W^*(r)|$
b) $\kappa_1 \Rightarrow \sqrt{\frac{\omega_4}{\eta_4-\sigma^4}}\sup_r |W^*(r)|$
c) $\kappa_2 \Rightarrow \sup_r |W^*(r)|$.

Table 1 shows some finite sample critical values for κ_2 computed from 50,000 replications of $\varepsilon_t \sim iidN(0,1)$, $t = 1, ..., T$. A response surface to summarize the finite sample critical values will be presented in Section 5.

For conditionally heteroskedastic processes one would expect the long-run fourth moment to be greater than its short-run counterpart $\eta_4 - \sigma^4$ and, consequently, an oversize for IT and κ_1. Let us consider some simple cases. For the ARCH(1) process (see Engle, 1982), $\varepsilon_t = u_t\sqrt{h_t}$, where $u_t \sim iidN(0,1)$ and $h_t = \delta + \gamma\varepsilon_{t-1}^2$, conditional on ε_0^2, with $\delta \geq 0$ and $0 < \gamma < 1$, it holds:

$$\eta_4 = \frac{\delta^2}{(1-\gamma)^2}\frac{3\left(1-\gamma^2\right)}{(1-3\gamma^2)}$$

and

$$\omega_4 = \frac{2\delta^2}{(1-\gamma)^4(1-3\gamma^2)}.$$

In this circumstances, $\frac{\omega_4}{2\sigma^4} = \frac{1}{(1-\gamma)^2(1-3\gamma^2)} \geq 1$ and the IT test will tend to overreject the null hypothesis of constant variance. For the κ_1 test $\frac{\omega_4}{\eta_4-\sigma^4} = \frac{1}{(1-\gamma)^2} \geq 1$ and we shall expect also an overrejection of the null of constant unconditional variance. In Section 4 these findings are confirmed for finite samples.

For the GARCH(1,1) processes (see Bollerslev, 1986) the conditional variance is given by:

$$h_t = \delta + \beta h_{t-1} + \gamma\varepsilon_{t-1}^2 \tag{2}$$

The fourth moment exists if $\beta^2 + 2\beta\gamma + 3\gamma^2 < 1$ and is given by:

$$\eta_4 = \frac{3\delta^2(1+\gamma+\beta)}{(1-\gamma-\beta)\left(1-\beta^2-2\beta\gamma-3\gamma^2\right)}$$

with coefficient of kurtosis:

$$\frac{\eta_4}{\sigma^4} - 3 = \frac{6\gamma^2}{1-\beta^2-2\beta\gamma-3\gamma^2} > 0$$

For the long-run fourth moment we have that

$$\omega_4 = \frac{2\delta^2\left(1-2\beta\gamma-\beta^2\right)(1-\beta)^2}{(1-\gamma-\beta)^4\left(1-\beta^2-2\beta\gamma-3\gamma^2\right)}$$

Then, if $\beta^2+2\beta\gamma+3\gamma^2 < 1$,which is the condition for the existence of the fourth moment $\frac{\omega_4}{2\sigma^4} = \frac{\left(1-2\beta\gamma-\beta^2\right)(1-\beta)^2}{(1-\gamma-\beta)^2(1-\beta^2-2\beta\gamma-3\gamma^2)} > 1$, and $\frac{\omega_4}{\eta_4-\sigma^4} = \frac{(1-\beta)^2}{(1-\gamma-\beta)^2} > 1$. Hence, as in the ARCH(1) case, we expect that the effective size of IT and κ_1 will be greater than the nominal one.

Similar results are expected when dealing with higher order GARCH processes.[3] To sum up, we would expect an overrejection of the null hypothesis for the IT and the κ_1 tests when they are applied to conditionally heteroskedastic processes.

4 Non-constant fourth moment

As it has been shown in the previous section, the existence of the fourth moment, rather than its constancy, as well as the finiteness of the long-run fourth moments, are required to establish the asymptotic distribution of the tests. This restricts the class of (G)ARCH processes we can deal with using this theory. In any case, although the results of Proposition 3 are no longer applicable to all situations, we can try to shed light on some special cases.

Let us consider a simple case, such as the covariance-stationary GARCH(1,1) process given by (2) but with non-constant fourth moment. That is, $\beta^2+2\beta\gamma+3\gamma^2 \geq 1$ and $\beta+\gamma<1$. In this case, as shown by Ding and Granger (1996), equation (A.16),

$$E\left(\varepsilon_t^4\right)=\eta_{4,t}=3\delta^2\frac{1+\gamma+\beta}{1-\gamma-\beta}\sum_{i=0}^{t}\left(\beta^2+2\beta\gamma+3\gamma^2\right)^i$$

tends to infinity. Then, the long-run fourth moment is also time varying and will tend to infinity. As a consequence, and according to Proposition 3, we would expect that the IT test will diverge and will tend to detect changes in variance. Note that this result holds irrespective of whether $T^{-1/2}\left(C_k-\frac{k}{T}C_T\right)$, the numerator of the statistic, diverges or not.

Moreover, assuming a distant starting point for the process, the autocorrelation function of ξ_t^2 is constant and is approximately given by $\rho_k \approx \left(\gamma+\frac{1}{3}\beta\right)(\gamma+\beta)^{k-1}$, which decreases exponentially, as it shown by Ding and Granger (1996). Then,

$$\begin{aligned}
\frac{\omega_{4,t}}{E\left(\varepsilon_t^4\right)-\sigma^4} &= \left(1+2\sum_{j=1}^{\infty}\rho_j\right)\\
&\approx 1+2\sum_{j=1}^{\infty}\left(\gamma+\frac{1}{3}\beta\right)(\gamma+\beta)^{j-1}\\
&= \frac{1+\gamma-\frac{1}{3}\beta}{1-\gamma-\beta}>1
\end{aligned}$$

so that, according to Proposition 3, we would expect an overrejection for the κ_1 test. If $T^{-1/2}\left(C_k-\frac{k}{T}C_T\right)$ also diverges, then the distortions in the size of the test will be greater.

[3] The conditions for the existence of the fourth moments in the wide family of GARCH processes where $h_t^\lambda=g\left(u_{t-1}\right)+c\left(u_{t-1}\right)h_{t-1}^\lambda$, $\lambda>0$, can be found in Ling and McLeer (2002).

For the κ_2 test, ideally computed from $G_k = \omega_4^{-1/2} T^{-1/2} \left(C_k - \frac{k}{T} C_T\right)$, we may expect that the numerator as well as $\omega_4^{1/2}$ will tend to diverge, so it is difficult to guess how will the test be affected in this case. Some Monte Carlo experiments in Section 6 show that the κ_2 is not seriously affected whereas IT or κ_1 have dramatic size distortions.

Let us now consider the case of non-covariance-stationary processes. We will restrict ourself to the case of IGARCH(1,1) disturbances, although the generalization to IGARCH(p,q) is straightforward. The following proposition establishes the distribution of the tests for IGARCH disturbances.

Proposition 4 *If ε_t is an IGARCH(1,1) process then:*
a) $IT \approx O_p\left(T^{1/2}\right)$;
b) $\kappa_1 \approx O_p\left(T^{1/2}\right)$;
c) $\kappa_2 \approx O_p\left((T/m)^{1/2}\right)$.

As a consequence, and provided that $m/T \to 0$, the tests diverge and will tend to reject the null hypothesis of constant unconditional variance. This means that for IGARCH processes one will find that the tests indicate that the variance is not constant. In this case, the correct procedure is to estimate an IGARCH process rather than trying to model the changes in the unconditional variance. The intuition behind this result is that the aforementioned test, as the usual unit root tests, cannot distinguish between I(1) processes and those with structural breaks (see, for instance, Perron,1990).

5 Iterative procedure

The iterative procedure proposed by Inclan and Tiao (1994) for detecting multiple changes in variance, known as Iterated Cumulative Sum of Squares (ICSS), can also be used with the κ_1 and κ_2 tests. A detailed description of the algorithm can be found in this reference. The method implies to compute the test several times for different sample sizes. However, using a single critical value for any sample size may distort the performance of the iterative procedure. To overcome this drawback, we fitted response surfaces to the finite sample critical values of the three tests. More formally, the idea is to fit a regression of the type:

$$q_{i,T}^{\alpha} = \sum_{j=1}^{m} \theta_{i,p_j}^{\alpha} T^{p_j} + v_{i,T} \tag{3}$$

where $q_{i,T}^{\alpha}$ is the quantile α of test $i = \{IT, \kappa_1, \kappa_2\}$ for a sample size T; θ_{i,p_j}^{α}, $j = \{1, ..., m\}$, are a set of parameters and the regressors are powers of the sample size. The values of $q_{i,T}^{\alpha}$ were obtained from Monte Carlo experiments, each of them consisting of 50,000 replications of the process $\varepsilon_t \sim iidN(0,1)$, $t = \{1, ..., T\}$ and the corresponding test and the empirical quantiles have been computed. The sample sizes considered were $T = \{15, 16, ..., 30, 32, ..., 50, 55, ..., 100, 110, ..., 200, 225, ..., 400, 450, ..., 700, 800, 900, 1000\}$. Therefore, 63 experiments for each test were carried out, obtaining 63 observations of $q^{\alpha}_{i,T}$ which vary with T. Finally, response surfaces as in (3) were fitted to the empirical quantiles. Table 2 shows the final estimates of the response surfaces for a 5% significance level, $\widehat{\theta}^{0.05}_{i,p_j}$, as well as some diagnostics.[4]

[insert Table 2 about here]

6 Monte Carlo experiments

In this section we will study the finite sample performance of the three considered tests as well as the ICSS algorithm. Although these have been extensively applied in empirical analysis of financial time series, few attention has been paid to the study of their finite sample properties. An exception is Andreou and Ghysels (2002). Our simulation experiments complement the afore mentioned article. Specifically, we will consider their size for *iid* non-mesokurtic sequences, for ARCH(1) and for IGARCH(1,1) processes, and their power when there are some breaks in the unconditional variance. Obviously, the applied researcher will be interested in the iterative procedure. Nevertheless, to shed light on the performance of this method when used with the three tests, we begin by analyzing the size and power of the individual tests.

6.1 Size and power of the tests

The first Monte Carlo experiment has consisted in generating sequences of *iid* zero-mean random variables with different coefficients of kurtosis. Specifically, we have taken into account the Uniform distribution on $U(-0.5, 0.5)$, the standard Normal, $N(0,1)$, the standard Logistic, the standard Laplace, the standard exponential (with parameter 1) and the standard Lognormal. The following table shows the rejection frequencies for the tests.

[insert Table 3 about here]

As can be seen, the IT test suffers from severe distortions for non-mesokurtic processes. As predicted from our asymptotic results, it tends to never reject for platikurtic distributions whereas tends to overreject for leptokurtic sequences. The two proposed test are not seriously affected.

The following table shows the rejection frequencies of the three tests when the data generation process is an ARCH(1) process. As expected from our theoretical analysis, all tests but κ_2 suffer from severe size distortions, as they ignore the persistence in the conditional variance. Contrarily, κ_2 seems to have a good size properties, even for ARCH processes without constant fourth moment.

[4]The complete set of results for the significance levels 1%, 2.5% and 10% are available from the authors upon request. A GAUSS routine to compute the ICSS algorithm with (any of) the three tests is also available on request. Also, OX routines implemented by Michail Karoglou and based on our GAUSS code are available.

[insert Table 4 about here]

Next table shows the rejection frequencies for IGARCH(1,1) processes. Here all three tests tend to reject the null hypothesis of constant variance when the DGP is an IGARCH processes. This overrejection is even worse for large samples (say $T = 500$). For large values of γ, say greater than 0.7, the size of κ_2 is not really seriously distorted. For these values, the autocorrelations of ε_t^2 –given by $\rho_k \approx \frac{1}{3}(1+2\gamma)\left(1+2\gamma^2\right)^{-k/2}$ (see Ding and Granger, 1996)– quickly tend to zero. Contrarily, for small values of γ, the persistence of ε_t^2 is large, and κ_2 also shows severe distortions.

[insert Table 5 about here]

Let us consider now the power of the different tests when there is a change in the unconditional variance of the processes. As can be seen from Table 6, κ_2 is the less powerful test, although in no case this lack of power is very extreme.

[insert Table 6 about here]

6.2 Size and power of the iterative procedure

We will study here the performance of the ICSS algorithm when based in one of the three tests. Given that the empirical applications of Section 7 have a sample size of about $T = 500$, this was the one considered. Similar qualitative results were obtained for $T = 100$ which are available upon request.

As in the preceding subsection, we will begin by considering non mesokurtic independent random sequences. Table 7 shows the frequency of detected changes in the variances when the ICSS procedure is used with the three tests. The more kurtosis the process has, the greater the number of time breaks erroneously detected by the iterative procedure with the IT test. In contrast, few of them are found with κ_1 or κ_2.

[insert Table 7 about here]

For conditional variance heteroskedastic sequences the picture is similar than for the individual tests: the iterative method based on IT or κ_1 tends to discover too many changes in variance, as can be seen in Table 8. The procedure based on κ_2 has a good performance and hardly ever detects any spurious time break. For IGARCH processes, as can be seen in Table 9, this last procedure also outperforms the other two, finding few spurious changes in variance except for small values of γ.

[insert Table 8 about here]
[insert Table 9 about here]

Finally, Table 10 shows the power of the ICSS procedure when there are two changes in the unconditional variance of an independent sequence. The procedure based on κ_2 is slightly less powerful than the other two, although the difference is not important.

[insert Table 10 about here]

Thus, we may conclude that the procedures based on IT or κ_1 show large size distortions that invalidate their use in practice for financial time series, which are leptokurtic and show persistence in the conditional variance. The procedure based on κ_2 is not affected by these distortions and attains a similar power profile.

7 Empirical application

In this section we check for the constancy of the unconditional variance of the four financial time series that have been already studied in Aggarwal *et al.* (1999), who detected several changes in variance for these series. Data consist of closing values for the stock indexes S&P500 (USA), Nikkei Average (Japan), FT100 (UK) and Hang-Seng (Hong-Kong). The period covers from May 1985 to April 1995. We have calculated the weekly returns for Wednesdays. When there was no trading on a given Wednesday, the trading day before Wednesday was used to compute the return.

Table 11 presents the descriptive statistics for each of the series aforementioned. All the series show excess kurtosis. The Ljung-Box statistic on the squared series and Engle's Lagrange multiplier test (Engle,1982) for the existence of ARCH effects provide strong evidence of non-constant conditional variance for the four series. Then, as concluded from the asymptotic theory as well as from the Monte Carlo experiments, we may expect too many rejections of the Inclan-Tiao test.

[insert Table 11 about here]

Table 12 presents the results obtained from using the ICSS algorithm. The second column gives the points of structural changes in variance obtained by Aggarwal et. al. (1999), whereas the rest of columns present those when the iterative procedure is implemented using the response surfaces shown in Section 4. Comparing the four sets of time breaks detected, several conclusions arise. First, comparing the second and the third column, less changes in variance are detected when the critical values are adapted to the effective sample size. Second, controlling for the kurtosis of the series dramatically reduces the number of time breaks. In this case, only four changes are detected for the Nikkei (instead of 6 with the IT test), one for the S&P index (instead of 8 or 2), none for the FT100 (instead of 2 or 1) and Hang-Seng (instead of 6 or 5). Finally, applying the ICSS(κ_2) procedure no changes are observed. According to our theoretical results, the Monte Carlo experiments as well as the descriptive analysis, we can conclude that the detected changes obtained by Aggarwal *et al.* (1999) and with the ICSS(IT) method are spurious.

[insert Table 12 about here]

8 Conclusions

In this article we have proven that the test used as a base for the implementation of the ICSS of Inclan and Tiao (1994) has two serious drawbacks that invalidate its use for financial time series. First, it neglects the fourth moment properties of the process and, second, it does not allow for conditional heteroskedasticity. The κ_2 test we have proposed in this paper explicitly considers this two features. Monte Carlo experiments detected extreme size distortions for the IT test whereas κ_2 is correctly sized in almost all the scenarios considered and it turns out to be only slightly less powerful.

These theoretical findings lead us to recommend the use of the ICSS procedure implemented with κ_2 and to be skeptical about the results obtained with the method based on the IT test. As an example of this, we have applied the ICSS method using the three tests considered in this paper to four of the financial time series analyzed in Aggarwal *et al.* (1999). These authors detected several time breaks in their financial data. The descriptive statistics show that these series are leptokurtic as well as conditionally heteroskedastic, the two situations where the IT test does not work properly. The ICSS procedure computed using the suggested κ_2 test does not detect any change in the unconditional variance. Hence, the time breaks detected by Aggarwal *et al.* (1999) are, given our findings, spurious.

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9 Appendix: proof of the propositions

We shall make use of the following asymptotic result:

Lemma 5 *Let* $\{\varepsilon_t\}_{t=0}^{\infty}$ *be a sequence of random variables that satisfies assumptions A1. Define* $\xi_t \equiv \varepsilon_t^2 - \sigma^2$. *Then, for* $r \in [0,1]$: $T^{-1/2}\omega^{-1/2}\sum_{t=1}^{[rT]}\xi_t \Rightarrow W(r)$, *a standard Brownian motion.*

Proof. First, note that if $\{\varepsilon_t\}$ is $\alpha-$mixing, then also it is ξ_t. Next, the set of assumptions A1 is a restricted case of the conditions of the Herrndorf's Theorem and, hence, the limit distribution stated in the previous lemma follows directly from that Theorem. ■

Note that the assumptions on ε_t of Propositions 1 and 2 fulfil the set of assumptions A1.

Proof. Propositions 1 and 2. This proof follows most of the steps of Inclan-Tiao so that we will only sketch it. First, note that $V(\xi_t) = E\left(\varepsilon_t^2 - \sigma^2\right)^2 = \eta_4 - \sigma^4 \equiv \omega$, where $\eta_4 \equiv E\left(\varepsilon_t^4\right)$. Only for mesokurtic random variables $V(\xi_t) = 2\sigma^4$. Moreover, $T^{-1}C_T = T^{-1}\sum_{t=1}^{T}\varepsilon_t^2 \to \sigma^2$, where $\to$ stands for convergence in probability, and

$$\begin{aligned} T^{-1/2}\omega^{-1/2}\left(C_k - \frac{k}{T}C_T\right) &= T^{-1/2}\omega^{-1/2}\left(\sum_{t=1}^{k}\varepsilon_t^2 - \frac{k}{T}\sum_{t=1}^{T}\varepsilon_t^2\right) \\ &= T^{-1/2}\omega^{-1/2}\left(\sum_{t=1}^{k}\left(\varepsilon_t^2 - \sigma^2\right) - \frac{k}{T}\sum_{t=1}^{T}\left(\varepsilon_t^2 - \sigma^2\right)\right) \\ &= T^{-1/2}\omega^{-1/2}\left(\sum_{t=1}^{k}\xi_t - \frac{k}{T}\sum_{t=1}^{T}\xi_t\right) \\ &\Rightarrow W(r) - rW(1) \equiv W^*(r) \end{aligned}$$

where $r \equiv \frac{k}{T} \in [0,1]$. Thus, $T^{-1/2}\left(C_k - \frac{k}{T}C_T\right) \Rightarrow \sqrt{\omega}W^*(r)$,

$$\sqrt{T/2}D_k = \sqrt{T/2}\left(\frac{C_k}{C_T} - \frac{k}{T}\right) \Rightarrow \sqrt{\frac{\omega}{2\sigma^4}}W^*(r),$$

and, applying the Continuous Mapping Theorem (CMT), Proposition 1 is proven. Proposition 2 follows immediately from the previous one. ■

Proof. Proposition 3. In this situation, the ξ_i are no longer independent. Then,

$$T^{-1/2}\omega_4^{-1/2}\left(C_k - \frac{k}{T}C_T\right) = T^{-1/2}\omega_4^{-1/2}\left(\sum_{t=1}^{k}\xi_t - \frac{k}{T}\sum_{t=1}^{T}\xi_t\right) \Rightarrow W^*(r).$$

Provided that $\widehat{\omega}_4$ is a consistent estimator, $T^{-1/2}\widehat{\omega}_4^{-1/2}\left(C_k - \frac{k}{T}C_T\right) = T^{-1/2}G_k \Rightarrow W^*(r)$ and, applying the CMT, result c) is proven. Given that $T^{-1/2}\left(C_k - \frac{k}{T}C_T\right) \Rightarrow$

$\omega_4^{1/2} W^*(r)$, it follows that $\sqrt{T/2} D_k = \sqrt{T/2}\left(\frac{C_k}{C_T} - \frac{k}{T}\right) \Rightarrow \sqrt{\frac{\omega_4}{2\sigma^4}} W^*(r)$ and $T^{-1/2} B_k = T^{-1/2} \frac{C_k - \frac{k}{T} C_T}{\sqrt{\widehat{\eta}_4 - \widehat{\sigma}^4}} \Rightarrow \sqrt{\frac{\omega_4}{\eta_4 - \sigma^4}} W^*(r)$. Hence, applying the CMT, a) and b) are proven. ■

We will consider the most simple case of IGARCH(1,1) processes, although the generalization to any IGARCH(p,q) is straightforward. The following lemma collects some intermediate results needed to proof Proposition 4.

Lemma 6 *Let $\varepsilon_t = u_t \sqrt{h_t}$, where $u_t \sim iidN(0,1)$ and $h_t = \delta + \beta h_{t-1} + \gamma \varepsilon_{t-1}^2$ with $\beta + \gamma = 1$, $\delta > 0$, $0 \leq \beta < 1$ and $0 < \gamma < 1$, conditional on h_0 and ε_0^2. Assume also that $E\left[\ln\left(\beta + \gamma \varepsilon_t^2\right)\right] < 0$ and $E\left[\left(\beta + \gamma \varepsilon_t^2\right)^{p+\lambda}\right] < 1$ for $0 < p < \psi/2$ and $\lambda > 0$, which ensures the existence of the fourth moment -see Nelson (1990) Theorem 4. Denote the long-run variance of $v_t \equiv w_t - \beta w_{t-1}$ as $\omega_v = \lim_{T\to\infty} E\left(T^{-1}\left(\sum_{t=1}^T v_t\right)^2\right) < \infty$, where $w_t \equiv \varepsilon_t^2 - h_t$. Define $r \equiv \frac{k}{T} \in [0,1]$. Then:*

L1) $T^{-2} C_k \to \frac{\delta}{2} r^2$;

L2) $T^{-3} \sum_{t=1}^T \varepsilon_t^4 = T^{-2} \widehat{\eta}_4 \to \frac{\delta^2}{3}$.

Proof. We can write: $\varepsilon_t^2 = \delta + (\beta + \gamma)\varepsilon_{t-1}^2 + w_t - \beta w_{t-1} = \delta + \varepsilon_{t-1}^2 + v_t$. Then, v_t is an invertible MA(1) process. Recursive substitution gives: $\varepsilon_t^2 = \varepsilon_0^2 + \delta t + S_t$, where $S_t = \sum_{j=1}^t v_j$. Then, it is well-known that $\omega_v^{-1/2} T^{-1/2} S_{[rT]} \Rightarrow W(r)$, $r \in [0,1]$.

Let us now consider the cumulative sum of squares:

$$\begin{aligned}
T^{-2} C_k &= T^{-2} \sum_{t=1}^k \varepsilon_t^2 = T^{-2} \sum_{t=1}^k \left(\varepsilon_0^2 + \delta t + S_t\right) \\
&= \frac{k}{T^2} \varepsilon_0^2 + \frac{1}{T^2} \frac{\delta}{2} k(k+1) + T^{-2} \sum_{t=1}^k S_t \\
&= \frac{\delta}{2}\left(\frac{k^2}{T^2} + \frac{k}{T^2}\right) + o_p(1) \\
&\to \frac{\delta}{2} r^2
\end{aligned}$$

$r \equiv \frac{k}{T} \in [0,1]$, when $T \to \infty$, provided that $\sum_{t=1}^k S_t \approx O_p\left(k^{3/2}\right)$. Then L1 is proven.

For result L2 note that, $\varepsilon_t^4 = \left(\varepsilon_0^2 + \delta t + S_t\right)^2 = \varepsilon_0^4 + \delta^2 t^2 + S_t^2 + 2\varepsilon_0^2 \delta t + 2\varepsilon_0^2 S_t + 2\delta t S_t$. Then,

$$\begin{aligned}
T^{-3}\sum_{t=1}^{T}\varepsilon_t^4 &= T^{-3}\left(T\varepsilon_0^4 + \delta^2\left(\frac{1}{3}T^3 + \frac{1}{2}T^2 + \frac{1}{6}T\right) + \sum_{t=1}^{T} S_t^2\right. \\
&\qquad \left. + \varepsilon_0^2\delta\left(T^2 + T\right) + 2\varepsilon_0^2\sum_{t=1}^{T} S_t + 2\delta\sum_{t=1}^{T} tS_t\right) \\
&= \frac{\delta^2}{3} + o_p(1) \\
&\to \frac{\delta^2}{3}
\end{aligned}$$

provided that $\sum_{t=1}^{T} S_t^2 \approx O_p\left(T^2\right), \sum_{t=1}^{T} tS_t \approx O_p\left(T^{5/2}\right)$ and $T^{-3}\varepsilon_0^2\sum_{t=1}^{T} S_t \approx o_p(1)$. Hence, $T^{-2}\widehat{\eta}_4 \to \frac{\delta^2}{3}$ and L2 is proven. ■

Proof. Proposition 4. From L1 it follows:

$$\begin{aligned}
D_k &= \frac{T^{-2}C_k}{T^{-2}C_T} - \frac{k}{T} \\
&\to r^2 - r.
\end{aligned}$$

Thus, $\sqrt{T/2}D_k \approx O_p\left(T^{1/2}\right)$ and it diverges. Hence, result a) is proven.

For result b) we have that $T^{-2}\widehat{\sigma}^4 \to \frac{\delta^2}{2^2}$, from L1, and using L2:

$$T^{-1}B_k = \frac{T^{-2}\left(C_k - \frac{k}{T}C_T\right)}{T^{-1}\sqrt{\widehat{\eta}_4 - \widehat{\sigma}^4}} \to \frac{\frac{\delta}{2}\left(r^2 - r\right)}{\sqrt{\frac{\delta^2}{3} - \frac{\delta^2}{2^2}}} = r(r-1)\sqrt{3}$$

and then $T^{-1/2}B_k \approx O_p\left(T^{1/2}\right)$, so that it diverges.

For result c) we have that

$$\begin{aligned}
\widehat{\omega}_4 &= \frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^2 - T^{-1}C_T\right)^2 + \frac{2}{T}\sum_{l=1}^{m} w(l,m)\sum_{t=l+1}^{T}\left(\varepsilon_t^2 - T^{-1}C_T\right)\left(\varepsilon_{t-l}^2 - T^{-1}C_T\right) \\
&= T^{-1}\sum_{t=1}^{T}\varepsilon_t^4 - T^{-2}C_T^2 + \\
&\quad 2\sum_{l=1}^{m} w(l,m)\left(T^{-1}\sum_{t=l+1}^{T}\varepsilon_t^2\varepsilon_{t-l}^2 - T^{-2}C_T\sum_{t=l+1}^{T}\varepsilon_t^2 - T^{-2}C_T\sum_{t=l+1}^{T}\varepsilon_{t-l}^2 + T^{-2}C_T^2\right)
\end{aligned}$$

Hence, $T^{-4}C_k^2 \to \left(\frac{\delta}{2}r^2\right)^2$, $T^{-3}\sum_{t=1}^{T}\varepsilon_t^4 \to \frac{\delta^2}{3}$,

$$\begin{aligned}
T^{-3}\sum_{t=l+1}^{T}\varepsilon_t^2\varepsilon_{t-l}^2 &= T^{-3}\sum_{t=l+1}^{T}\left(\varepsilon_0^2+\delta t+S_t\right)\left(\varepsilon_0^2+\delta\left(t-l\right)+S_{t-l}\right)\\
&= T^{-2}\varepsilon_0^4+T^{-3}\delta\varepsilon_0^2\left(\frac{1}{2}T^2+\frac{1}{2}T-\frac{1}{2}l^2-\frac{1}{2}l\right)+\\
&\quad T^{-3}\varepsilon_0^2\sum_{t=l+1}^{T}S_t+T^{-3}\delta\varepsilon_0^2\left(\frac{1}{2}T^2+\frac{1}{2}T-lT-\frac{1}{2}l+\frac{1}{2}l^2\right)+\\
&\quad \delta^2\left(-\frac{1}{6}lT^{-3}+\frac{1}{3}+\frac{1}{2}T^{-1}+\frac{1}{6}T^{-2}+\frac{1}{6}l^3T^{-3}-\frac{1}{2}lT^{-1}-\frac{1}{2}lT^{-2}\right)+\\
&\quad T^{-3}\delta\sum_{t=l+1}^{T}\left(t-l\right)S_t+T^{-3}\varepsilon_0^2\sum_{t=l+1}^{T}S_{t-l}+T^{-3}\delta\sum_{t=l+1}^{T}tS_{t-l}+\\
&\quad T^{-3}\sum_{t=l+1}^{T}S_tS_{t-l}\\
&= \frac{\delta^2}{3}+o_p\left(1\right)
\end{aligned}$$

and $T^{-2}\sum_{t=l+1}^{T}\varepsilon_t^2=\left(\frac{T-l}{T}\right)^2\left(T-l\right)^{-2}\sum_{t=l+1}^{T}\varepsilon_t^2 \to \frac{\delta}{2}$, provided that $l/T \to 0$, so that, $T^{-4}C_T\sum_{t=l+1}^{T}\varepsilon_t^2-T^{-4}C_T\sum_{t=l+1}^{T}\varepsilon_{t-l}^2+T^{-4}C_T^2 \to \left(\frac{\delta}{2}\right)^2$. Thus, for the Bartlett window, $w\left(l,m\right)=1-l/\left(m+1\right)$, and

$$\begin{aligned}
T^{-2}\widehat{\omega}_4 &= \left(\frac{\delta^2}{3}-\left(\frac{\delta}{2}\right)^2\right)+2\sum_{l=1}^{m}w\left(l,m\right)\left(\frac{\delta^2}{3}-\left(\frac{\delta}{2}\right)^2\right)+o_p\left(1\right)\\
&= \sum_{l=-m}^{m}w\left(l,m\right)\left(\frac{\delta^2}{3}-\left(\frac{\delta}{2}\right)^2\right)+o_p\left(1\right)\\
&\to \frac{1}{12}\left(2m+1\right)\delta^2
\end{aligned}$$

so that,

$$\begin{aligned}
m^{1/2}T^{-1}G_k &= \left(m^{-1}T^{-2}\widehat{\omega}_4\right)^{-1/2}\left(T^{-2}C_k-\frac{k}{T}T^{-2}C_T\right)\\
&\to r\left(r-1\right)\sqrt{3/2}.
\end{aligned}$$

Then, $T^{-1/2}G_k \approx O_p\left(\left(T/m\right)^{1/2}\right)$. ■

10 Tables

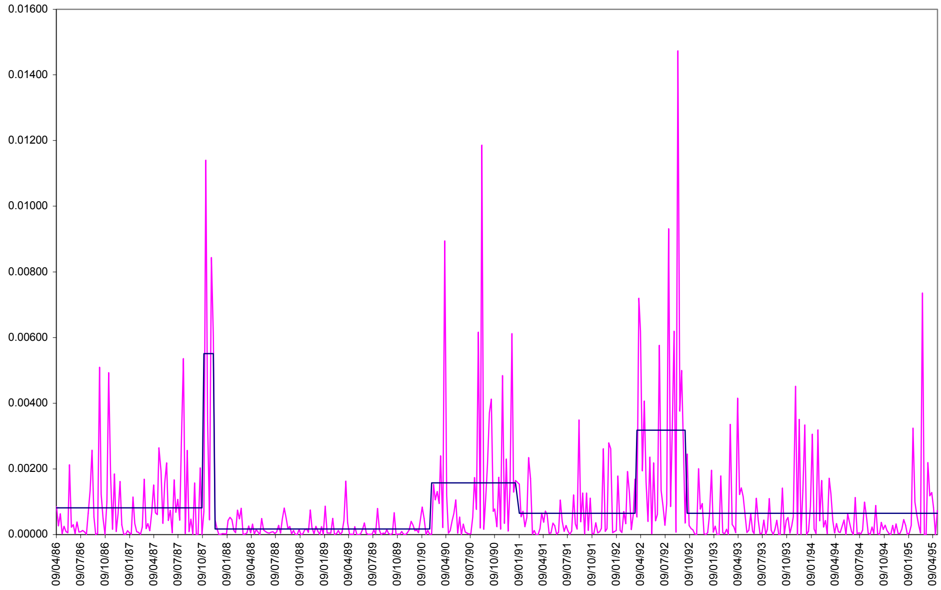

Figure 1: Squared returns of Nikkei Index and detected changes in the unconditional variance using the Inclan-Tiao test.

Table 1: Critical values for κ_1 and κ_2

	κ_1				κ_2			
$\alpha \backslash T$	100	200	500	1000	100	200	500	1000
0.9	1.148	1.167	1.195	1.200	1.170	1.177	1.192	1.197
0.95	1.268	1.300	1.328	1.330	1.269	1.294	1.317	1.329
0.975	1.383	1.420	1.453	1.447	1.352	1.395	1.428	1.442
0.99	1.515	1.547	1.592	1.592	1.448	1.508	1.557	1.586

Note: computed using 50,000 replications of $\varepsilon_t \sim iidN(0,1)$, $t = 1, ..., T$.

Table 2: Response surfaces for the 5% quantiles of the tests

	IT	κ_1	κ_2
$p_1 = 0$	1.359167 (771.8)	1.363934 (846.1)	1.405828 (75.31)
$p_2 = -0.5$	-0.737020 (-22.75871)	-0.942936 (-30.64)	-3.317278 (-4.24)
$p_3 = -1$	-0.691556 (-6.03)	0.500405 (4.70)	31.22133 (3.68)
$p_4 = -2$			-1672.206 (-5.66)
$p_5 = -3$			52870.53 (8.92)
$p_6 = -4$			-411015.0 (-9.64)
R^2	0.996566	0.995914	0.998772
$\widehat{\sigma}_v$	0.003659	0.003492	0.013052
$\max_T \lvert\widehat{v}_{i,T}\rvert$	0.01202	0.00847	0.04374

Note: $q_{i,T}^{0.05} = \sum_{j=1}^{m} \theta_{i,p_j}^{0.05} T^{p_j} + v_{i,T}$, where $q_{i,T}^{0.05}$ is the 5%-quantile, based on 50,000 replications, of test $i = \{IT, \kappa_1, \kappa_2\}$ for a sample size T. 63 different sample sizes were considered. White's heteroskedasticity-consistent t-ratios between brackets. For the κ_2 test we have used the quadratic spectral window with automatic bandwidth selection (Newey-West, 1994).

Table 3: Rejection frequencies for the tests. Non mesokurtic independent sequences

		$T = 100$			$T = 500$		
	kurtosis	IT	κ_1	κ_2	IT	κ_1	κ_2
Uniform	-1.2	0.0003	0.0570	0.0583	0.0003	0.0500	0.0530
Normal	0	0.0570	0.0567	0.0517	0.0527	0.0503	0.0537
Logistic	1.2	0.1660	0.0497	0.0450	0.1857	0.0473	0.0467
Laplace	3	0.3243	0.0397	0.0423	0.3830	0.0450	0.0470
Exponential	6	0.4597	0.0280	0.0277	0.6360	0.0343	0.0370
Lognormal	$\approx$110	0.8130	0.0240	0.0213	0.9700	0.0150	0.0153

Note: computed using 3,000 replications of $\varepsilon_t \sim iid$, $t = 1, ..., T$.

Table 4: Rejection frequencies for the tests. ARCH(1) processes

	ARCH(1): $\delta = 0.1$					
	$T = 100$			$T = 500$		
γ	IT	κ_1	κ_2	IT	κ_1	κ_2
0.1	0.083	0.083	0.036	0.105	0.095	0.054
0.3	0.256	0.172	0.039	0.346	0.203	0.040
0.5	0.489	0.296	0.035	0.692	0.338	0.044
0.7	0.643	0.359	0.036	0.902	0.426	0.033
0.9	0.765	0.393	0.024	0.963	0.480	0.022

Note: Computed using 1,000 replications of $\varepsilon_t = u_t\sqrt{h_t}$, where $u_t \sim iidN(0,1)$ and $h_t = \delta + \gamma\varepsilon_{t-1}^2$ and $h_0 = \delta/(1-\gamma))$.

Table 5: Rejection frequencies for the tests. IGARCH(1,1) processes

	IGARCH(1,1)					
	Panel A: $\delta = 0.1$					
	$T = 100$			$T = 500$		
γ	IT	κ_1	κ_2	IT	κ_1	κ_2
0.1	0.696	0.704	0.488	0.983	0.970	0.794
0.3	0.767	0.697	0.205	0.990	0.950	0.372
0.5	0.777	0.620	0.101	0.988	0.875	0.142
0.7	0.812	0.588	0.052	0.987	0.779	0.075
0.9	0.834	0.492	0.044	0.988	0.643	0.025
	Panel B: $\delta = 0$					
0.1	0.583	0.614	0.427	0.998	0.998	0.958
0.3	0.979	0.963	0.578	1.000	1.000	0.838
0.5	1.000	0.971	0.336	1.000	0.991	0.378
0.7	1.000	0.933	0.150	1.000	0.939	0.143
0.9	1.000	0.799	0.060	1.000	0.782	0.039

Note: Computed using 1,000 replications of $\varepsilon_t = u_t\sqrt{h_t}$, where $u_t \sim iidN(0,1)$ and $h_t = \delta + \gamma\varepsilon_{t-1}^2 + \beta h_{t-1}$ with $\beta + \gamma = 1$ and starting values $h_0 = 1$ and $\varepsilon_0 = 0$.

Table 6: Power of the test when there is a change in the variance

	$T = 100$			$T = 500$		
θ	IT	κ_1	κ_2	IT	κ_1	κ_2
0.25	0.097	0.107	0.091	0.355	0.351	0.343
0.5	0.224	0.225	0.191	0.841	0.826	0.818
0.75	0.425	0.389	0.330	0.982	0.982	0.982
1	0.587	0.535	0.423	0.999	0.999	0.996
1.5	0.824	0.770	0.639	1.000	1.000	1.000

Note: Rejections of the null hypothesis. Computed using 1,000 replications of $\varepsilon_t \sim iidN(0,1)$ for $t = 1, ..., 0.5T$ and $\varepsilon_t \sim iidN(0, 1+\theta)$ for $t = 0.5T+1, ..., T$.

Table 7: Rejection frequencies for the ICSS procedure. Non-mesokurtic independent sequences

	n_0	n_1	n_2	n_3	n_4	$n_{>4}$
			ICSS(IT)			
Uniform	1.000	0.000	0.000	0.000	0.000	0.000
Normal	0.949	0.047	0.004	0.000	0.000	0.000
Logistic	0.835	0.107	0.047	0.008	0.001	0.002
Laplace	0.604	0.186	0.122	0.058	0.025	0.005
Exponential	0.428	0.161	0.183	0.127	0.065	0.036
Lognormal	0.037	0.091	0.125	0.413	0.197	0.137
			ICSS(κ_1)			
Uniform	0.958	0.036	0.006	0.000	0.000	0.000
Normal	0.946	0.047	0.007	0.000	0.000	0.000
Logistic	0.956	0.041	0.002	0.001	0.000	0.000
Laplace	0.955	0.043	0.002	0.000	0.000	0.000
Exponential	0.972	0.027	0.001	0.000	0.000	0.000
Lognormal	0.988	0.010	0.001	0.001	0.000	0.000
			ICSS(κ_2)			
Uniform	0.958	0.037	0.005	0.000	0.000	0.000
Normal	0.942	0.056	0.002	0.000	0.000	0.000
Logistic	0.953	0.044	0.003	0.000	0.000	0.000
Laplace	0.949	0.049	0.002	0.000	0.000	0.000
Exponential	0.968	0.030	0.002	0.000	0.000	0.000
Lognormal	0.985	0.014	0.001	0.000	0.000	0.000

Note: ICSS(i), $i = \{IT, \kappa_1, \kappa_2\}$ stands for the ICSS algorithm based on the i test; n_j, $j = \{0, 1, .., 4, > 4\}$ stands for the relative frequency of detecting j changes in variance. $T = 500$.

Table 8: Rejection frequencies for the ICSS procedure. ARCH(1) processes

γ	n_0	n_1	n_2	n_3	n_4	$n_{>4}$
			ICSS(IT)			
0.1	0.902	0.074	0.021	0.003	0.000	0.000
0.3	0.665	0.138	0.127	0.032	0.026	0.012
0.5	0.317	0.112	0.185	0.132	0.119	0.135
0.7	0.144	0.073	0.094	0.131	0.132	0.426
0.9	0.038	0.030	0.048	0.096	0.091	0.697
			ICSS(κ_1)			
0.1	0.904	0.073	0.021	0.002	0.000	0.000
0.3	0.789	0.128	0.063	0.014	0.004	0.002
0.5	0.677	0.154	0.104	0.042	0.017	0.006
0.7	0.583	0.148	0.129	0.065	0.047	0.028
0.9	0.464	0.145	0.197	0.078	0.066	0.050
			ICSS(κ_2)			
0.1	0.952	0.039	0.009	0.000	0.000	0.000
0.3	0.944	0.050	0.005	0.001	0.000	0.000
0.5	0.969	0.030	0.001	0.000	0.000	0.000
0.7	0.976	0.024	0.000	0.000	0.000	0.000
0.9	0.972	0.025	0.003	0.000	0.000	0.000

Note: See Table 7.

Table 9: Rejection frequencies for the ICSS procedure. IGARCH(1,1) processes

γ	n_0	n_1	n_2	n_3	n_4	$n_{>4}$
			ICSS(IT_T)			
0.1	0.022	0.085	0.159	0.239	0.235	0.260
0.3	0.014	0.010	0.039	0.065	0.095	0.777
0.5	0.010	0.006	0.027	0.041	0.060	0.856
0.7	0.012	0.010	0.033	0.058	0.057	0.830
0.9	0.013	0.013	0.032	0.054	0.064	0.824
			ICSS(κ_1)			
0.1	0.035	0.103	0.148	0.202	0.229	0.283
0.3	0.050	0.053	0.081	0.104	0.152	0.560
0.5	0.129	0.066	0.128	0.120	0.116	0.441
0.7	0.230	0.126	0.132	0.121	0.130	0.261
0.9	0.371	0.117	0.199	0.103	0.099	0.111
			ICSS(κ_2)			
0.1	0.229	0.271	0.219	0.159	0.088	0.034
0.3	0.625	0.206	0.119	0.042	0.007	0.001
0.5	0.858	0.100	0.035	0.006	0.001	0.000
0.7	0.925	0.062	0.013	0.000	0.000	0.000
0.9	0.964	0.035	0.001	0.000	0.000	0.000

Note: See Table 7.

Table 10: Power of the ICSS procedure when there is a change in the variance

	DGP 1			DGP 2		
θ	IT	κ_1	κ_2	IT	κ_1	κ_2
0.25	0.173	0.171	0.134	0.222	0.213	0.154
0.5	0.691	0.631	0.511	1.382	1.175	0.688
0.75	1.399	1.314	1.061	2.026	1.975	1.312
1	1.860	1.794	1.534	2.112	2.125	1.715
1.5	2.115	2.094	1.973	2.161	2.164	1.864

Note: Average number of breaks detected. Computed using 1,000 replications of DGP 1: $\varepsilon_t \sim iidN(0,1)$ for $t = 1, ..., 200$, $\varepsilon_t \sim iidN(0, 1+\theta)$ for $t = 201, ..., 400$, and $\varepsilon_t \sim iidN(0,1)$ for $t = 401, ..., 500$; DGP 2: $\varepsilon_t \sim iidN(0,1)$ for $t = 1, ..., 200$, $\varepsilon_t \sim iidN(0, 1+\theta)$ for $t = 201, ..., 400$, and $\varepsilon_t \sim iidN(0, (1+\theta)^{-1})$ for $t = 401, ..., 500$.

Table 11: Descriptive statistics

	FT100	Nikkei	S&P	Hang-Seng
Mean	0.00135	0.000169	0.002026	0.003259
Min	-0.17817	-0.10892	-0.16663	-0.34969
Max	0.09822	0.12139	0.06505	0.11046
std. dev.	0.02275	0.02940	0.02084	0.03765
Skewness	-1.54899	-0.51655	-1.45512	-2.31416
Kurtosis	15.8642	4.78076	12.3227	19.6888
$Q2(15)$	88.278	130.93	87.015	38.06
	(0.00)	(0.00)	(0.00)	(0.00)
$LM(2)$	103.69	34.29	65.09	32.577
	(0.00)	(0.009)	(0.00)	(0.00)
$LM(5)$	106.77	62.66	65.51	34.02
	(0.00)	(0.00)	(0.00)	(0.00)

Note: $Q2(15)$ stands for the Ljung-Box statistic on the squared of the series for 15 lags and $LM(j)$ for Engle's Lagrange multiplier test for ARCH(j) effects. p-values between brackets.

Table 12: Detected changes in variance with the ICSS algorithm

	AIL	ICSS(IT)	ICSS(κ_1)	ICSS(κ_2)
FT100	14-10-87 (80)			
	23-12-87 (90)			
Nikkei	17-6-87 (63)	14-10-87 (80)	14-10-87 (80)	
	18-11-87 (85)	25-11-87(86)	25-11-87(86)	
	14-2-90 (199)	14-2-90(199)	14-2-90(199)	
	23-01-91 (247)	9-1-91(245)	9-1-91(245)	
	25-3-92 (307)	25-3-92(307)	25-3-92(307)	
	30-9-92 (334)	30-9-92(334)	30-9-92(334)	
S&P	21-5-86(55)	21-5-86(55)	21-5-86(55)	
	7-10-87(127)	7-10-87(127)		
	4-11-87 (131)	4-11-87 (131)		
	10-8-88(171)		1-06-88(161)	
	1-8-90(274)		1-8-90(274)	
	13-2-91(302)		13-2-91(302)	
	22-4-92(364)	15-4-92 (363)	22-4-92(364)	
Hang-Seng	14-10-87(128)	14-10-87(128)		
	4-11-87(131)	4-11-87(131)		
	2-3-88(148)	17-2-88(146)		
	17-5-89(211)	17-5-89(211)		
	12-7-89(219)	12-7-89(219)		
	7-10-92(388)	7-10-92(388)		

Note: Dates of the detected changes in variance (position of the observation between brackets). AIL stands for the results of Aggarwal et al. (1999). ICSS(i), $i = \{IT, \kappa_1, \kappa_2\}$ stands for the ICSS algorithm based on the i test.