Does altruism affect the optimal fiscal policy?*

Carlos Garriga
FSU and CREB

Fernando Sánchez-Losada†
Universitat de Barcelona and CREB

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Abstract

This paper explores the optimal fiscal policy in economies with altruism. We characterize the optimal taxes in the transition path and in the steady state. We show that the first best policy can be attained independently of the altruism type if the government disposes of a complete set of fiscal instruments (public debt and proportional taxes on consumption, capital, labor and bequests), being the consumption tax not redundant. If the government can not use the consumption tax, then the way of how altruism exactly operates is decisive in setting optimal taxes. When dynastic altruism characterizes the economy, the optimal capital tax becomes zero in steady state if and only if the optimal bequest tax is zero. When joy-of-giving is the altruism motive, the optimal capital tax does not converge to zero and this happens independently of the bequest tax.

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†Contact address: Centre de Recerca en Economia del Benestar; Parc Científic de Barcelona; Baldiri Reixac, 4-6; 08028 Barcelona. Spain; Tel.: (+ 34) + 934034539; Fax: (+ 34) + 934021937; e-mail: fersan@eco.ub.es; cgarriga@pcb.ub.es .

1 Introduction

The empirical evidence shows that corporate profits, dividends and interest income are source of capital income that are taxed in most of the OECD countries. The average capital income tax rate is around 25%. The observed time series for capital income taxes contrasts with the standard view of modern dynamic fiscal policy that says that the optimal tax on capital income should be zero.

In an economy where individuals are infinite-lived Chamley (1986) has shown that capital income should not be taxed in the long run. This result is robust to changes in the set of assumptions, as introducing heterogeneous consumers (Judd, 1985), endogenous growth (Jones, Manuelli and Rossi, 1993), open economy (Razin and Sandka, 1995) or uncertainty (Chari, Christiano and Kehoe, 1994). Moreover, with stronger assumptions on preferences, the optimal fiscal policy along the transition path imply that capital income taxes are zero after a finite number of periods.

There is also an extensive literature on optimal fiscal policy using stylized two period overlapping generations models, see Pestieau (1974), Atkinson and Sandmo (1980) and Auerbach (1985). In this economies under certain assumptions on preferences it can be shown that the optimal capital income tax is zero in the long run.

Recently, several papers of Garriga (2000a, 2000b) and Erosa and Gervais (2000) have shown that the optimal fiscal policy in dynamic economies depend crucially in the individuals life span. Departures from the stylized two period model has dramatic implications in terms of the optimal fiscal policy imply. In a more general model, in the tradition of Auerbach and Kotlikoff (1987), we should not expect zero capital income taxes neither in the long run nor along the transition path. The general conditions for zero capital taxation are not satisfied for the type of utility functions commonly used in the macro and public finance literature. Moreover, the predictions of the optimal capital income tax in this type of economies is consistent with the empirical evidence for a plausible set of parameters.

This paper tries to combine both approaches, so we explore the optimal fiscal policy in economies with altruism, motivated by the different predictions due to the life span, T or ∞. We focus our analysis on the role of parental links across generations and on the way altruism operates from parents to children. There are several ways to introduce altruism in a model, we analyze two types, namely dynastic altruism and joy-of-giving. In the former, parents derive utility directly from the utility of their offsprings, as in Barro (1974). In the later, parents derive utility directly from giving bequests to their offsprings, as in Yaari (1965). In general we should expect different predictions in terms of the optimal fiscal policy, but as we show that this is not necessary the case.

These theoretical frameworks try to capture the empirical evidence on the importance of altruism. Thus, as confirmed by the studies of Kotlikoff and Summers (1981, 1986) and McGarry and Schoeni (1995), physical bequests play an important role in the economy since 80% of wealth is transferred across generations. Even though models with dynastic altruism can not account for the observed wealth distribution. De Nardi (2000) succeeds in accounting for the observed wealth and income distribution and shows that the intergenerational linkages are important to explain the upper tail of the wealth distribution. Therefore we consider that it is important to have deeper understanding in terms of studying the optimal fiscal policy in environments that are consistent with the empirical evidence.

We use a standard general equilibrium overlapping generations economy with production. For simplicity we assume that individuals live for 2 periods and work only the first periods, all the result will not depend on this assumption. There is no uncertainty and the measure of individuals in each cohort is constant. Individuals supply their labor services and assets in competitive markets. Then, individuals receive a competitive wage and the rental rate, on the second period individuals might choose to transfer a fraction of wealth to their heard. We assume that this decision is also taxed, that allow us to study the optimal determination of bequest taxation from a normative point of view. In this environment individuals cannot die indebted. The rest of the model is very standard, government expenditure is financed with proportional distortionary taxes and government debt. Firms are neoclassical and finally factor markets are competitive.

In this economy there exists a benevolent government that chooses the optimal fiscal policy π, to maximize the welfare of all current and future generations. This choice faces two constraints: first, the government budget constraint must be balanced in present value and second, the resulting allocation constitutes a competitive equilibrium with taxes. In order to characterize the government problem we follow the *primal approach* as discussed in Lucas and Stokey (1983) and Chari and Kehoe (1999). Another shortcut is the existence of a commitment technology that allows the government in period zero to choose a policy π and never deviate.

The optimal fiscal policy in altruistic economies diverges from the infinite or the finite life span consumers economies. In particular we find that if the government has access to a complete set of distortionary taxes, the economy can achieve a first best allocation independently of the altruism type. This set consists of public debt and proportional taxes on consumption, capital, labor and bequests. In contrast with previous results in the literature, in economies with altruims consumption taxes are not redundant and play a crucial role in order to achieve an Pareto efficient competitive equilibrium using distortionary taxes.

By contrast if the government has a restricted set of instruments and cannot use proportional taxes on consumption, then the way of how altruism operates is decisive in determing the optimal fiscal policy.

In a dynastic altruism economy the optimal capital tax becomes zero in steady state if and only if the optimal bequest tax is zero and if the utility function satisfies certain conditions. Nevertheless, preferences that imply zero capital taxes in an infinite-lived consumers economy, do not necessarily imply the same result in the presence of altruism. The negative relation between capital income and bequest taxes implies that the government uses the capital tax to offset the effect of bequest taxation in order to achieve the socially optimal capital.

In an economy with joy-of-giving altruism, the optimal capital tax does not converge to zero in the long run and along the transition path. In this environment there not exists an explicit relationship between capital income and bequest taxes.

The consumption tax becomes the key element since it allows the government to link the coexistent generations and decentralize a first best allocation. The government can attain the first best by making both capital and bequest taxes zero and, in addition, taxing labor and consumption at the same rate but with opposite sign.

In the next section we present the basic model, define the competitive equilibrium and solve for the optimal fiscal policy for the dynastic altruism. The joy-of-giving altruism optimal policy is discussed in section 3, and finally, section 4 concludes.

2 Dynastic Altruism

2.1 Competitive Equilibrium

We construct a two overlapping generation economy with constant population, whose mass is normalized to 1. There are three agents in the economy: consumers, firms and government.

A continuum of firms produce aggregate output y_t, according to a production function $F(k_t, l_t)$ where k_t and l_t are the capital and labor employed by the firm, respectively. The technology $F(\cdot)$ is assumed to be constant returns to scale, strictly concave, C^2, and satisfies the Inada conditions. At each period capital depreciates at a constant positive rate δ. As markets are competitive, production factors are paid its marginal product:

$$r_t = F_{k_t} - \delta, \tag{1}$$

$$w_t = F_{l_t}, \tag{2}$$

where r_t denotes the net return of capital investment and w_t denotes the wage rate. The economy resource constraint is

$$c_{1t} + c_{2t} + k_{t+1} - (1-\delta)k_t + G_t = F(k_t, l_t), \tag{3}$$

where c_{1t} and c_{2t} denote the consumption of a representative individual when young and old in period t, respectively, G_t denotes government consumption and $k_{t+1} - (1-\delta)k_t$ denotes gross investment.

In the economy, individuals live for two periods. Young generations are endowed with one unit of time which they allocate between labor market activities l_t and leisure $(1 - l_t)$. The fraction of time devoted to work is rewarded as labor income $w_t l_t$. They also might receive a physical bequest b_t from their parents. Then, they decide consumption c_{1t} and asset holdings a_{t+1}. When individuals become old, they allocate the return from savings between consumption c_{2t+1} and bequests to their offspring b_{t+1}. Each young agent in period t solves the following problem:

$$\max_{\{c_{1t}, c_{2t+1}, a_{t+1}, l_t, b_{t+1}\}} \qquad V_t = U\left(c_{1t}, l_t\right) + \rho U\left(c_{2t+1}\right) + \beta V_{t+1} \tag{4}$$

$$s.t. \qquad \left(1+\tau_t^c\right) c_{1t} + a_{t+1} = \left(1-\tau_t^l\right) w_t l_t + b_t, \tag{5}$$

$$\left(1+\tau_{t+1}^c\right) c_{2t+1} + \left(1+\tau_{t+1}^b\right) b_{t+1} = a_{t+1}\left(1 + r_{t+1}\left(1-\tau_{t+1}^k\right)\right), \tag{6}$$

$$c_{1t}, c_{2t+1}, b_{t+1} \geq 0, \qquad l_t \in (0,1),$$

where V_{t+1} is the utility of their offspring, $\rho > 0$ is the subjective discount factor, β is the altruism factor, and τ_t^c, τ_t^b, τ_t^k and τ_t^l denote consumption, bequests, capital and labor income proportional taxes, respectively. In order to ensure that V_t is bounded from above, we assume that $\beta \in (0,1)$. The period utility function $U(\cdot)$ is strictly concave, C^2, and satisfies the usual Inada conditions. The bequest tax τ_t^b is paid by the old generations in the economy, otherwise it would be equivalent to a lump-sum tax.

At $t = 0$, an old generation owns the initial capital stock and level of government debt, and solves the following problem:

$$\max_{\{c_{20}, b_0\}} \qquad \rho U\left(c_{20}\right) + \beta V_1 \tag{7}$$

$$s.t. \qquad (1+\tau_0^c)\, c_{20} + \left(1+\tau_0^b\right) b_0 = (1+r_0(1-\tau_0^k))(k_0+d_0), \qquad (8)$$

where k_0 and d_0 are the initial level of capital stock and government debt, respectively. The government budget constraint is

$$G_t + R_t d_t - d_{t+1} = \tau_t^c(c_{1t}+c_{2t}) + \tau_t^b b_t + \tau_t^k r_t a_t + \tau_t^l w_t l_t, \qquad (9)$$

where d_t denotes government debt, which return is $R_t = 1 + r_t(1-\tau_t^k)$. Note that capital income arising from holding either physical capital or public debt is taxed at the same rate τ_t^k. The amount of government debt is bounded by a large positive constant to ensure that the government budget constraint is satisfied in present value.

The aggregate level of asset holdings equals the stock of physical capital and the level of government debt at $t+1$:

$$a_{t+1} = k_{t+1} + d_{t+1}, \qquad (10)$$

Next we define a competitive equilibrium in this economy:

Definition 1 *Given a fiscal policy* $\pi = \{\tau_t^c, \tau_t^b, \tau_t^k, \tau_t^l, d_t\}_{t=0}^{\infty}$ *and a sequence of public expenditure* $\{G_t\}_{t=0}^{\infty}$*, a competitive equilibrium is a sequence of individual allocations* $\{c_{1t}, c_{2t}, a_{t+1}, b_t, l_t\}_{t=0}^{\infty}$*, production plans* $\{k_t, l_t\}_{t=0}^{\infty}$*, and prices* $\{r_t, w_t, R_t\}_{t=0}^{\infty}$*, such that:* 1) *The household problem is solved,* 2) *Firms maximize profits,* 3) *Markets clear,* 4) *Government budget constraint holds,* 5) *Feasibility is satisfied.*

Given the assumptions of concavity and monotonicity on the functional forms, the first-order conditions are sufficient to characterize an interior solution. Let λ_t and ϕ_{t+1} be the Lagrange multipliers associated to the budget constraint (5) and (6), respectively. The first-order conditions from the individual maximization problem are

$$[c_{1t}] \qquad U_{c_{1t}} - \lambda_t (1+\tau_t^c) = 0, \qquad (11a)$$
$$[c_{2t+1}] \qquad \rho U_{c_{2t+1}} - \phi_{t+1}\left(1+\tau_{t+1}^c\right) = 0, \qquad (11b)$$
$$[a_{t+1}] \qquad -\lambda_t + \phi_{t+1} R_{t+1} = 0, \qquad (11c)$$
$$[l_t] \qquad U_{l_t} + \lambda_t \left(1-\tau_t^l\right) w_t = 0, \qquad (11d)$$

where U_x denotes the marginal utility with respect to x, where x denotes c_{1t}, c_{2t+1} and l_t. Using the envelope theorem, the optimal decision with respect to b_{t+1} is

$$b_{t+1}\left[\frac{\beta U_{c_{1t+1}}}{(1+\tau_{t+1}^c)} - \phi_{t+1}\left(1+\tau_{t+1}^b\right)\right] = 0. \qquad (12)$$

Regardless of the operativeness of the bequest motive, combining the first-order conditions, we obtain the standard Euler equation and a static decision for consumption and labor,

$$\frac{U_{c_{1t}}}{\rho U_{c_{2t+1}}} = \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)} R_{t+1}, \tag{13}$$

$$-\frac{U_{c_{1t}}}{U_{l_t}} = \frac{(1+\tau_t^c)}{\left(1-\tau_t^l\right) w_t}. \tag{14}$$

When bequests are operative, i.e. $b_t > 0\ \forall t$, we derive two additional expressions

$$\frac{\beta U_{c_{1t+1}}}{\rho U_{c_{2t+1}}} = \left(1+\tau_{t+1}^b\right), \tag{15}$$

$$\frac{U_{c_{1t}}}{\beta U_{c_{1t+1}}} = \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)} \frac{R_{t+1}}{\left(1+\tau_{t+1}^b\right)}. \tag{16}$$

The threshold level of the altruism factor $\overline{\beta}$ above (below) which the bequest motive is (is not) operative depends on the fiscal policy as it has been pointed out by Caballé (1998). We assume that the altruism factor is high enough so bequests are always operative. Next we discuss the government problem.

2.2 Government problem

The government in this economy chooses a fiscal policy π in order to maximize society's welfare subject to some constraints. These constraints imply that the government budget constraint has to be satisfied in present value and, second, that the optimal policy must be consistent with the competitive equilibrium. This is the so-called *Ramsey equilibrium*.

We assume that the government has access to a commitment technology. We model this behavior as follows: the government first chooses a fiscal policy for all future dates and then agents, taking prices and taxes as given, choose allocations.

In this economy the government values all individuals according to an utilitarian objective function. The relative weight that the government places between present and future generations is captured by the government discount rate β. A necessary assumption for the existence of the Ramsey equilibrium is that the individual altruistic factor and the government discount rate coincide. Formally:

$$W(\cdot) = \beta^{-1}\rho U(c_{20}) + \sum_{t=0}^{\infty} \beta^t \left[U(c_{1t}, l_t) + \rho U(c_{2t+1})\right].$$

Rearranging terms we can rewrite the government objective function as

$$W(\cdot) = \sum_{t=0}^{\infty} \beta^t \left[U\left(c_{1t}, l_t\right) + \rho \beta^{-1} U\left(c_{2t}\right) \right]. \tag{17}$$

In order to solve the government problem we use the primal approach of optimal taxation proposed by Atkinson and Stiglitz (1980). This approach is based on characterizing the set of allocations that the government can implement given a fiscal policy π. The set of Ramsey allocations is described by the following set of constraints: the resource constraint (RC) and an implementability constraint (IC). The implementability constraint is the households budget constraint, after substituting in first-order conditions of the consumers problem and relative prices. The implementability constraints take into account that changes in the tax policy will affect agents decisions, and therefore allocations and prices. Thus, the government problem is to maximize its objective function over the set of implementable allocations. Then we can back out policies and prices from the allocations.

Definition 2 *The set of implementable allocations is characterized by the resource constraint,*

$$c_{1t} + c_{2t} + k_{t+1} - (1-\delta) k_t + G_t = F\left(k_t, l_t\right) \quad \forall t \geq 0,$$

an implementability constraint for the newborn generations,

$$c_{1t} U_{c_{1t}} + l_t U_{l_t} + \rho c_{2t+1} U_{c_{2t+1}} + \frac{\beta b_{t+1} U_{c_{1t+1}}}{\left(1+\tau_{t+1}^c\right)} = \frac{b_t U_{c_{1t}}}{\left(1+\tau_{c_t}\right)} \quad \forall t \geq 0, \tag{18}$$

and an implementability constraint for the initial generation at $t = 0$,

$$\rho c_{20} U_{c_{20}} + \frac{\beta b_0 U_{c_{10}}}{\left(1+\tau_0^c\right)} = \frac{\rho U_{c_{20}} \left[1 + \left(1-\tau_0^k\right)\left(F_{K_0} - \delta\right)\right]\left(k_0 + d_0\right)}{\left(1+\tau_0^c\right)}. \tag{19}$$

Next we describe the construction of the set of implementable allocations, and then we solve the government problem.

Proposition 1 *The allocations in a competitive equilibrium* $\{c_{1t}, c_{2t}, b_t, k_{t+1}, l_t\}_{t=0}^{\infty}$ *satisfy the set of implementable allocations. Moreover, if an allocation is implementable, then we can construct a fiscal policy* $\pi = \{\tau_t^c, \tau_t^k, \tau_t^l, \tau_t^b, d_{t+1}\}_{t=0}^{\infty}$ *and competitive prices* $\{r_t, w_t, R_t\}_{t=0}^{\infty}$*, such that the allocation together with prices and the policy* π *constitute a competitive equilibrium.*

Proof. We start by proving the first part of the proposition. Any competitive equilibrium allocation has to satisfy the resource constraint. The IC for the newborn generations can be derived as follows. Multiplying (11a) by c_{1t}, (11b) by c_{2t+1}, (11d) by l_t and (12) by b_{t+1}, and adding up these four equations we have

$$c_{1t}U_{c_{1t}} + \rho c_{2t+1}U_{c_{2t+1}} + l_t U_{l_t} + \frac{\beta b_{t+1}U_{c_{1t+1}}}{(1+\tau^c_{t+1})} = \lambda_t \left[(1+\tau^c_t)\, c_{1t} - \left(1-\tau^l_t\right) w_t l_t\right] +$$

$$\phi_{t+1}\left[\left(1+\tau^c_{t+1}\right) c_{2t+1} + \left(1+\tau^b_{t+1}\right) b_{t+1}\right]. \qquad (20)$$

Now substituting in the right hand side of (20) for (5) and (6), and using (11c) and (11a) we derive the IC for newborn generations. The initial old agent at $t = 0$ has a different IC because she is endowed with the initial stock of capital and debt. It can be derived using the same procedure.

Now we prove the second part of the proposition. Given an implementable allocation $\{c_{1t}, c_{2t}, b_t, k_{t+1}, l_t\}_{t=0}^{\infty}$, the competitive prices can be back out using firms first order conditions (1) and (2). The fiscal policy $\pi = \{\tau^c_t, \tau^b_t, \tau^k_t, \tau^l_t, d_t\}_{t=0}^{\infty}$ is recovered from the households first-order conditions (13), (14) and (15). Substituting $U_{c_{1t}}$, $U_{c_{2t+1}}$ and U_{l_t} in the implementability constraint we obtain the consumer budget constraint. The debt level is found from the market clearing condition in the capital markets (10). Finally, given the tax on capital τ^k_t and the net interest rate r_t, by arbitrage we find the return on government debt. If the resource constraint and the consumers budget constraints are satisfied, then the government budget constraint is also satisfied. ■

We assume that the initial tax on capital τ^k_0, consumption τ^c_0, and bequests τ^b_0, are inherited by the government. The optimal policy can be derived by maximizing the government objective function over the set of implementable allocations.

Definition 3 *Given a sequence of public expenditure $\{G_t\}_{t=0}^{\infty}$ and the initial taxes and debt at $t = 0$, the optimal fiscal policy $\pi = \{\tau^c_t, \tau^k_t, \tau^l_t, \tau^b_t, d_{t+1}\}_{t=0}^{\infty}$ is derived by solving the following Ramsey allocation problem:*

$$\max_{\{c_{1t}, c_{2t}, l_t, b_t, k_{t+1}, \tau^c_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left[U(c_{1t}, l_t) + \rho\beta^{-1} U(c_{2t})\right]$$

$$s.t. \quad c_{1t} + c_{2t} + k_{t+1} - (1-\delta)\, k_t + G_t = F(k_t, l_t) \quad \forall t \geq 0,$$

$$c_{1t}U_{c_{1t}} + l_t U_{l_t} + \rho c_{2t+1}U_{c_{2t+1}} + \frac{\beta b_{t+1}U_{c_{1t+1}}}{\left(1+\tau_{c_{t+1}}\right)} = \frac{b_t U_{c_{1t}}}{(1+\tau_{c_t})} \quad \forall t \geq 0,$$

$$c_{20}U_{c_{20}} + \frac{\beta b_0 U_{c_{10}}}{(1+\tau_0^c)} = \frac{\rho U_{c_{20}}\left[1+\left(1-\tau_0^k\right)(F_{k_0}-\delta)\right](k_0+d_0)}{(1+\tau_0^c)},$$

$$c_{1t}, c_{2t}, b_t \geq 0 \qquad l_t \in (0,1), \qquad k_0 \textit{ and } d_0 \textit{ given.}$$

Note that in this optimization program the government can choose allocations and consumption taxes $\{\tau_t^c\}_{t=1}^{\infty}$. This is due to the fact of the existence of positive bequests, which link present and future generations in the economy. If bequests were zero we would be in a standard overlapping generations model in the tradition of Diamond (1965), where consumption taxes are redundant, as it has been pointed out by Garriga (2000a) and Erosa and Gervais (2000). Given that consumption taxes are not redundant, we can state the next proposition:

Proposition 2 *If the government can choose optimally* $\{\tau_t^c\}_{t=1}^{\infty}$*, then the economy achieves a first-best allocation from period* $t \geq 1$ *onwards.*

Proof. To prove this result we just have to consider the implementability constraint of a newborn generation and its first-order conditions. The optimal consumption taxes $\{\tau_t^c\}_{t=1}^{\infty}$ are as follows:

$$1+\tau_{t+1}^c = \frac{\beta b_{t+1}U_{c1t+1}}{\frac{U_{c1t}b_t}{1+\tau_t^c} - (c_{1t}U_{c1t} + l_t U_{lt} + \rho c_{2t+1}U_{c2t+1})}, \tag{21}$$

where the initial consumption tax τ_0^c is given. Then, capital, labor and bequest taxes are chosen so that individual decisions are not distorted. That implies, from $(13)-(15)$, setting labor taxes as $\{\tau_t^l\}_{t=1}^{\infty} = -\{\tau_t^c\}_{t=1}^{\infty}$, bequest taxes as $\{\tau_t^b\}_{t=1}^{\infty} = 0$, and capital taxes as:

$$\{\tau_t^k\}_{t=1}^{\infty} = \left\{1 - \frac{1}{r_{t+1}}\left[\left(\frac{1+\tau_{t+1}^c}{1+\tau_t^c}\right)(1+r_{t+1}) - 1\right]\right\}_{t=1}^{\infty}. \tag{22}$$

At $t=0$, the optimal labor tax is obtained by decentralizing the solution of the Ramsey allocation problem. ■

Note that in the long run the capital tax becomes zero. As we have seen, the optimal policy depends crucially on the set of instruments available to the government. This result contrasts with the standard redundancy of consumption taxes in infinitely-lived consumers models, as in Chamley (1986), or in standard overlapping generations models, as in Atkinson and Sandmo (1980).

Now we want to characterize the optimal fiscal policy in the absence of consumption taxes. In this case the new Ramsey allocation problem is given by

$$\max_{\{c_{1t},c_{2t},l_t,b_t,k_{t+1}\}_{t=0}^{\infty}} \qquad \sum_{t=0}^{\infty}\beta^t\left[U(c_{1t},l_t) + \rho\beta^{-1}U(c_{2t})\right]$$

$$s.t. \quad c_{1t} + c_{2t} + k_{t+1} - (1-\delta) k_t + G_t = F(k_t, l_t) \quad \forall t \geq 0,$$

$$(c_{1t} - b_t)U_{c_{1t}} + l_t U_{l_t} + \rho c_{2t+1} U_{c_{2t+1}} + \beta b_{t+1} U_{c_{1t+1}} = 0 \quad \forall t \geq 0,$$

$$\rho c_{20} U_{c_{20}} + \beta b_0 U_{c_{10}} = \rho U_{c_{20}} \left[1 + \left(1 - \tau_0^k\right)(F_{k_0} - \delta)\right](k_0 + d_0),$$

$$c_{1t}, c_{2t}, b_t \geq 0 \qquad l_t \in (0,1), \qquad k_0 \text{ and } d_0 \text{ given.}$$

In order to derive a solution to the previous problem, we redefine the objective function by introducing the implementability constraint of each generation on it. For a newborn generation the period utility is given by

$$W(c_{1t}, l_t, c_{2t+1}) = U(c_{1t}, l_t) + \rho U(c_{2t+1}) + \tag{23}$$

$$\eta_t \left((c_{1t} - b_t)U_{c_{1t}} + l_t U_{l_t} + \rho c_{2t+1} U_{c_{2t+1}} + \beta b_{t+1} U_{c_{1t+1}}\right),$$

where η_t is the Lagrange multiplier associated to the implementability constraint of a generation born at time t. Let α_t be the Lagrange multiplier of the resource constraint. Then, the first-order conditions for $t \geq 0$ are

$$[c_{1t}] \qquad \beta^t W_{c_{1t}} - \alpha_t = 0, \tag{24a}$$

$$[c_{2t}] \qquad \beta^{t-1} \rho W_{c_{2t}} - \alpha_t = 0, \tag{24b}$$

$$[l_t] \qquad \beta^t W_{l_t} + \alpha_t F_{l_t} = 0, \tag{24c}$$

$$[b_t] \qquad \eta_t - \eta_{t-1} = 0, \tag{24d}$$

$$[k_{t+1}] \qquad -\alpha_t + \alpha_{t+1}\left(F_{k_{t+1}} - \delta + 1\right) = 0, \tag{24e}$$

and a transversality condition for the capital stock

$$\lim_{t \to \infty} \alpha_t k_{t+1} = 0.$$

It is important to note that given the nature of this problem, the first-order conditions together with the transversality condition are just necessary but not sufficient; this will depend on the properties of the implementability constraint that might fail to be convex. A discussion of this problem can be found in Lucas and Stokey (1983). Rearranging terms, we obtain

$$\frac{W_{l_t}}{W_{c_{1t}}} = -F_{l_t}, \tag{25}$$

$$W_{c_{1t}} = \rho W_{c_{2t+1}}(1 - \delta + F_{K_{t+1}}), \tag{26}$$

$$W_{c_{1t}} = \frac{\rho}{\beta} W_{c_{2t}}, \tag{27}$$

and combining (26) and (27) we have

$$W_{c_{1t}} = \beta W_{c_{1t+1}}(1 - \delta + F_{K_{t+1}}), \tag{28}$$

where the partial derivatives with respect to c_{1t}, l_t and c_{2t}, combined with $(24d)$, are

$$W_{c_{1t}} = (1+\eta_t)U_{c_{1t}} + \eta_t\left(c_{1t}U_{c_{1t}c_{1t}} + l_tU_{l_tc_{1t}}\right), \tag{29}$$

$$W_{l_t} = (1+\eta_t)U_{l_t} + \eta_t\left(l_tU_{l_tl_t} + c_{1t}U_{c_{1t}l_t}\right), \tag{30}$$

$$W_{c_{2t}} = (1+\eta_{t-1})U_{c_{2t}} + \eta_{t-1}c_{2t}U_{c_{2t}c_{2t}}. \tag{31}$$

and at time $t = 0$ are

$$W_{c_{20}} = (1+\eta_{-1})U_{c_{20}} + \eta_{-1}\left(c_{20}U_{c_{20}c_{20}} - \frac{s_{-1}\left(1+r_0\left(1-\tau_0^k\right)\right)U_{c_{20}c_{20}}}{1+\tau_0^c}\right), \tag{32}$$

$$W_{l_0} = (1+\eta_0)U_{l_0} + \eta_0\left[c_{10}U_{c_{10}l_0} + l_0U_{l_0l_0} - \frac{\rho U_{c_0^{-1}}s_{-1}F_{k_0l_0}\left(1-\tau_0^k\right)}{1+\tau_0^c}\right]. \tag{33}$$

In order to derive the optimal fiscal policy we have to combine the first-order conditions of the Ramsey allocation problem together with the consumer and the firm's first-order conditions. The optimal taxes from $t > 0$ are

$$\tau_t^l = 1 - \frac{W_{c_{1t}}}{W_{l_t}}\frac{U_{l_t}}{U_{c_{1t}}}, \tag{34}$$

$$\tau_t^b = \frac{W_{c_{1t}}}{W_{c_{2t}}}\frac{U_{c_{2t}}}{U_{c_{1t}}} - 1, \tag{35}$$

$$\tau_{t+1}^k = 1 - \frac{1}{r_{t+1}}\left[(1+r_{t+1})\left(1+\tau_{t+1}^b\right)\frac{W_{c_{1t+1}}}{W_{c_{1t}}}\frac{U_{c_{1t}}}{U_{c_{1t+1}}} - 1\right]. \tag{36}$$

In this economy bequest taxation and capital income taxation are closely related. This is a non standard result in the literature, which usually abstracts from bequest taxation. Next we focus our analysis on the optimal capital income tax and more precisely we explore the necessary conditions on individual preferences that ensure zero capital taxation.[1]

In general we should not expect capital taxes to be zero in the long run. The government, by choosing taxes on bequests, is affecting to different margins of the consumer problem: the intertemporal savings decision and the bequest decision. Given that in general bequest taxes are not zero, we should not expect taxes on capital income to be zero. Therefore, capital taxes are

[1] Chamley (1986) shows that, in a representative consumer model, if the solution to the Ramsey problem converges to a steady state, then the optimal tax rate on capital is zero in the long run. For certain utility functions, a much stronger result can be established, that is, the optimal tax on capital returns is zero after only a few periods. For a detailed explanation see Chari and Kehoe (1999).

chosen to offset the distortionary effect of bequest taxes on intertemporal decisions.

The model requires stronger assumptions on preferences in order to have zero capital taxation. Next we show the conditions that preferences need to satisfy to not distort capital accumulation decisions from period 1 onwards.

Proposition 3 *If the utility function satisfies the following conditions:*

$$\frac{c_{1t}U_{c_{1t}c_{1t}} + l_t U_{l_t c_{1t}}}{U_{c_{1t}}} = \frac{c_{2t}U_{c_{2t}c_{2t}}}{U_{c_{2t}}}, \tag{37}$$

$$\frac{c_{1t}U_{c_{1t}c_{1t}} + l_t U_{l_t c_{1t}}}{U_{c_{1t}}} = \frac{c_{1t+1}U_{c_{1t+1}c_{1t+1}} + l_{t+1} U_{l_{t+1} c_{1t+1}}}{U_{c_{1t+1}}}, \tag{38}$$

then, the optimal policy implies setting $\tau_t^b = \tau_t^k = 0$ *from* $t > 1$.

Condition (37) ensures that $\tau_t^b = 0$, but it is not sufficient to ensure that $\tau_t^k = 0$. The next three examples consider different types of preferences that imply zero capital taxes in the long run in infinitely-lived consumers economy, but may or may not violate the conditions of the previous proposition.

Example 1 *The following class of preferences satisfies conditions* (37) *and* (38) *and therefore the optimal taxes on capital are zero:*

$$U(c,l) = \frac{c^{1-\sigma} - 1}{1-\sigma} + h(l). \tag{39}$$

where $\sigma \geq 0$.

Example 2 *The following class of preferences violates condition* (37) *and satisfies* (38) *and therefore capital taxes are different from zero, even in steady state:*

$$U(c,l) = \frac{\left[c^{\gamma} l^{-\gamma}\right]^{1-\sigma}}{1-\sigma}, \tag{40}$$

where $\gamma \in (0,1)$ *and* $\sigma \geq -1$.

Example 3 *The following class of preferences violates condition* (37) *and* (38) *and therefore capital taxes are different from zero, even in steady state:*

$$U(c,l) = \frac{\left[c^{\gamma}(1-l)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma}, \tag{41}$$

where $\gamma \in (0,1)$ *and* $\sigma \geq 0$.

The positiveness of capital income taxes depends crucially on the existence of bequest taxes. These taxes are necessary to decentralize the economy, as it has been shown in Proposition 1. If the government cannot use bequest taxation the Ramsey allocation problem needs an additional intergenerational constraint to ensure that the solution of the competitive equilibrium coincides with the solution of the government problem, i.e.

$$\frac{U_{c_{1t}}}{U_{c_{2t}}} = \frac{\rho}{\beta}.$$

3 Joy-of-Giving Altruism

In this section we explore if the previous results are robust to changes in the way altruism is modeled. Among the different types of altruism we choose joy-of-giving, since we want to keep the analysis as simple as possible.[2] The joy-of-giving altruism implies that individuals derive utility from giving bequests to their children, but they do not derive it directly from their children happiness. The newborn generation solves:

$$\max_{\{c_{1t}, c_{2t+1}, a_{t+1}, l_t, b_{t+1}\}} \quad U\left(c_{1t}, l_t\right) + \rho U\left(c_{2t+1}, b_{t+1}\right), \tag{42}$$

$$s.t. \qquad \left(1+\tau_t^c\right) c_{1t} + a_{t+1} = \left(1-\tau_t^l\right) w_t l_t + b_t, \tag{43}$$

$$\left(1+\tau_{t+1}^c\right) c_{2t+1} + \left(1+\tau_{t+1}^b\right) b_{t+1} = a_{t+1}\left(1+r_{t+1}\left(1-\tau_{t+1}^k\right)\right), \tag{44}$$

$$c_{1t}, c_{2t+1}, b_{t+1} \geq 0, \qquad l_t \in (0,1),$$

where b_t enters explicitly in the utility function, which satisfies the standard assumptions. Firms and government are introduced as in the previous section. The solution of this optimization problem gives the same first-order conditions with respect to c_{1t}, c_{2t+1}, a_{t+1}, l_t, but a new condition with respect to b_{t+1}:

$$\rho U_{b_{t+1}} - \phi_{t+1}\left(1+\tau_{t+1}^b\right) = 0. \tag{45}$$

Combining (11b) together with (45) we obtain:

$$\frac{U_{c_{2t+1}}}{U_{b_{t+1}}} = \frac{\left(1+\tau_{t+1}^c\right)}{\left(1+\tau_{t+1}^b\right)}. \tag{46}$$

[2] We abstract from including two sided altruism because the analysis with respect the optimal policy and the non redundancy of consumption taxes carries over.

The initial old generation at $t = 0$ solves the problem:

$$\max_{\{c_{20}, b_0\}} \qquad \rho U\left(c_{20}, b_0\right) \tag{47}$$

$$st. \qquad \left(1+\tau_0^c\right) c_{20} + \left(1+\tau_0^b\right) b_0 = \left(1+r_0(1-\tau_0^k)\right)(k_0 + d_0). \tag{48}$$

In this economy the optimal fiscal policy is obtained by solving the corresponding Ramsey allocation problem:

Definition 4 *Given $\{G_t\}_{t=0}^{\infty}$ and the initial taxes and debt at $t = 0$, the optimal fiscal policy $\pi = \{\tau_t^c, \tau_t^k, \tau_t^l, \tau_t^b, d_{t+1}\}_{t=0}^{\infty}$ is derived by solving:*

$$\max_{\{c_{1t}, c_{2t}, l_t, b_t, k_{t+1}, \tau_{t+1}^c\}_{t=0}^{\infty}} \qquad \sum_{t=0}^{\infty} \beta^t \left[U\left(c_{1t}, l_t\right) + \rho\beta^{-1} U\left(c_{2t}, b_t\right)\right]$$

$$s.t. \qquad c_{1t} + c_{2t} + k_{t+1} - (1-\delta) k_t + G_t = F\left(k_t, l_t\right) \quad \forall t \geq 0,$$

$$c_{1t} U_{c_{1t}} + l_t U_{l_t} + \rho\left(c_{2t+1} U_{c_{2t+1}} + b_{t+1} U_{b_{t+1}}\right) = \frac{b_t U_{c_{1t}}}{\left(1+\tau_t^c\right)} \quad \forall t \geq 0, \tag{49}$$

$$c_{20} U_{c_{20}} + b_0 U_{b_0} = \frac{U_{c_{20}}\left[1+\left(1-\tau_0^k\right)\left(F_{k_0} - \delta\right)\right]\left(k_0 + d_0\right)}{1+\tau_0^c}, \tag{50}$$

$$c_{1t}, c_{2t}, b_t \geq 0 \qquad l_t \in (0,1), \qquad k_0 \text{ and } d_0 \text{ given.}$$

It is important to mention that we have modified the government objective function to introduce the bequest motive. The associated implementability constraints differ from the dynastic altruism case described in the previous section.

Given that consumption taxes are not redundant, the government can choose these taxes in order to mimic lump-sum taxation from period 1 onwards. This result is stated in the next proposition:

Proposition 4 *If the government can choose $\{\tau_t^c\}_{t=1}^{\infty}$, then the economy achieves a first-best allocation from period $t \geq 1$ onwards.*

Proof. The optimal sequence of consumption taxes $\{\tau_t^c\}_{t=1}^{\infty}$ is derived directly from the implementability constraint for the newborn generations in the economy:

$$1+\tau_t^c = \frac{b_t U_{c_{1t}}}{c_{1t} U_{c_{1t}} + l_t U_{l_t} + \rho\left(c_{2t+1} U_{c_{2t+1}} + b_{t+1} U_{b_{t+1}}\right)}, \tag{51}$$

then, labor, capital and bequest taxes $\{\tau_t^l, \tau_t^k, \tau_t^b\}_{t=1}^{\infty}$ are set, similarly as with dynastic altruism, to not distort individual decisions. ■

When consumption taxes are available the type of altruism is irrelevant and the economy can achieve a first-best allocation from period 1 onwards. This result contrasts with the case of infinitely-lived consumers economy where consumption taxes are redundant. Nevertheless, if the government can choose consumption taxes at time $t = 0$, then the economy can achieve a Pareto efficient allocation. In contrast, in altruistic economies (dynastic or joy-of-giving) even though we have assumed that consumption taxes at $t = 0$ are given by the government, the economy can achieve a first-best allocation.

Now we want to characterize the optimal policy in environments where the government cannot use consumption taxes. In this case, the government problem is

$$\max_{\{c_{1t},c_{2t},l_t,b_t,k_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left[U\left(c_{1t}, l_t\right) + \rho\beta^{-1} U\left(c_{2t}, b_t\right) \right]$$

$$s.t. \qquad c_{1t} + c_{2t} + k_{t+1} - (1-\delta)\, k_t + G_t = F\left(k_t, l_t\right) \quad \forall t \geq 0,$$

$$c_{1t}U_{c_{1t}} + l_t U_{l_t} + \rho \left(c_{2t+1}U_{c_{2t+1}} + b_{t+1}U_{b_{t+1}} \right) \leq b_t U_{c_{1t}} \quad \forall t \geq 0, \tag{52}$$

$$c_{20}U_{c_{20}} + b_0 U_{b_0} \leq U_{c_{20}} \left[1 + \left(1 - \tau_0^k\right)\left(F_{k_0} - \delta\right) \right] (k_0 + d_0), \tag{53}$$

$$c_{1t}, c_{2t}, b_t \geq 0 \qquad l_t \in (0,1), \qquad k_0 \text{ and } d_0 \text{ given.}$$

If we introduce the implementability constraints in the government objective function, as in the previous section, and solve the Ramsey allocation problem, the associated first-order conditions for $t > 0$ are

$$\frac{Z_{l_t}}{Z_{c_{1t}}} = -F_{l_t}, \tag{54}$$

$$Z_{c_{1t}} = \rho Z_{c_{2t+1}} (1 - \delta + F_{k_{t+1}}), \tag{55}$$

$$Z_{c_{1t}} = \frac{\beta}{\rho} Z_{c_{2t}}, \tag{56}$$

and with respect b_t,

$$\rho Z_{b_t} = -\eta_t \beta U_{c_{1t}}, \tag{57}$$

where η_t denotes the Lagrange multiplier of the implementability constraint of a newborn generation and:

$$Z_{c_{1t}} = (1+\eta_t) U_{c_{1t}} + \eta_t \left(\left(c_{1t} - b_t\right) U_{c_{1t}c_{1t}} + l_t U_{l_t c_{1t}} \right), \tag{58}$$

$$Z_{l_t} = (1+\eta_t) U_{l_t} + \eta_t \left(l_t U_{l_t l_t} + \left(c_{1t} - b_t\right) U_{c_{1t} l_t} \right), \tag{59}$$

$$Z_{c_{2t}} = (1+\eta_{t-1})U_{c_{2t}} + \eta_{t-1}\left(c_{2t}U_{c_{2t}c_{2t}} + b_t U_{c_{2t}b_t}\right), \quad (60)$$

$$Z_{b_t} = (1+\eta_{t-1})U_{b_t} + \eta_{t-1}\left(b_t U_{b_t b_t} + c_{2t}U_{c_{2t}b_t}\right). \quad (61)$$

In order to derive the optimal fiscal policy, we combine the first-order conditions of the competitive equilibrium with the Ramsey allocation problem. The optimal capital tax for $t > 0$ is

$$\tau_{t+1}^k = 1 - \frac{1}{r_{t+1}}\left[(1+r_{t+1})\frac{Z_{c_{2t+1}}}{Z_{c_{1t}}}\frac{U_{c_{1t}}}{U_{c_{2t+1}}} - 1\right]. \quad (62)$$

Note that in this economy the optimal tax on bequest and capital are different than in the model with dynastic altruism. In this case, bequest taxation does not distort asset holdings decisions because the individual decision problem with respect to bequest is static. In that sense the size of bequests only depends on the relative price of goods in the second period. Therefore taxes on bequest are equivalent to consumption taxes.

In general capital taxes are different from zero in the transition path and in the long run. The next proposition shows a necessary condition that preferences need to satisfy in order to have zero capital taxes.

Proposition 5 *If the utility function satisfies the following condition:*

$$\frac{(c_{1t}-b_t)U_{c_{1t}c_{1t}} + l_t U_{l_t c_{1t}}}{U_{c_{1t}}} = \frac{c_{2t+1}U_{c_{2t+1}c_{2t+1}} + b_{t+1}U_{c_{2t+1}b_{t+1}}}{U_{c_{2t+1}}}, \quad (63)$$

then $\tau_t^k = 0$.

Standard preferences commonly used in macro and public finance do not satisfy this condition. Therefore we should not expect to have zero capital taxes neither in the transition path nor in the long run. Hence the way altruism operates has important implications in terms of the optimal fiscal policy.

4 Conclusions

In this paper we have seen that the way of how altruism exactly operates is decisive in setting optimal taxes. We conclude that a first best can be attained independently of the altruism type if the government disposes of a complete set of fiscal instruments, that is, public debt and proportional taxes on consumption, capital, labor and bequests. The consumption tax becomes the key tax because it allows the government to link the coexistent generations and decentralize a first best allocation. The government, making capital and bequest taxes to zero and labor tax to the negative of consumption tax, turns a second best problem into a first best problem. Therefore, and in contrast with previous literature, the consumption tax is not redundant.

If we remove the consumption tax from the set of fiscal instruments that the government can use, then the way of how altruism exactly operates is decisive in setting optimal taxes. When dynastic altruism characterizes the economy, that is, parents derive utility from the welfare of their offsprings, the optimal capital tax becomes zero in steady state if and only if the optimal bequest tax is zero, which depends on the specific utility function. In fact, the government uses the capital tax to offset the effect of bequest taxation in order to achieve the socially optimal capital.

When joy-of-giving is the altruism motive, that is, parents derive utility directly from giving bequests to their offsprings, the optimal capital tax does not converge to zero and this happens independently of the bequest tax.

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