

**STAR and ANN models:  
Forecasting performance on the Spanish “Ibex-35” stock index**

**Jorge V. Pérez-Rodríguez**

Department of Quantitative Methods  
University of Las Palmas de Gran Canaria  
Campus de Tafira, Tafira Baja.  
E-35017, Las Palmas. Spain  
Phone:+34 928 458222 y Fax: +34 928-458225 / 1829  
(Email: [jorge@empresariales.ulpgc.es](mailto:jorge@empresariales.ulpgc.es))

**Salvador Torra**

Department of Econometrics, Statistics and Spanish Economy  
University of Barcelona  
(Email: [storra@eco.ub.es](mailto:storra@eco.ub.es) )

and

**Julian Andrada -Félix**

Department of Quantitative Methods  
University of Las Palmas de Gran Canaria  
(Email: [julian@empresariales.ulpgc.es](mailto:julian@empresariales.ulpgc.es))

Corresponding author:

Dr. Jorge V. Pérez-Rodríguez  
Department of Quantitative Methods  
University of Las Palmas de Gran Canaria  
Campus de Tafira, Tafira Baja.  
E-35017, Las Palmas. Spain

**Abstract:**

This paper examines the out-of-sample forecast performance of smooth transition autoregressive (STAR) models and artificial neural networks (ANNs) when applied to daily returns on the Ibex-35 stock index, during the period from 30 December 1989 to 10 February 2000. The forecasts are evaluated with statistical criteria such as goodness of forecast, including tests of forecast encompassing, directional accuracy and the equality of mean squared prediction error; the relative forecast performance is assessed with economic criteria in a simple trading strategy including the impact of transaction costs on trading strategy profits. In terms of statistical criteria, the results show that different artificial neural network specifications forecast better than the AR model and smooth transition non-linear models. In terms of the economic criteria in the out-of-sample forecasts, we assess profitability and combine a simple trading strategy known as the filter technique by using a range filter percentage and trading costs. The results indicate a better fit for ANN models, in terms of the Sharpe risk-adjusted ratio. These results show there is a good chance of obtaining a more accurate fit and forecast of the daily stock index returns by using non-linear models, but that these are inherently complex and present a difficult economic interpretation.

**Keywords :** Non-linearities, statistical criteria, trading strategies.

**JEL classifications:** C22, C45, C52.

## 1. Introduction

Some researchers have questioned the hypothesis of Efficient Markets (HEM), i.e. that the random walk model is a reasonable description of asset price movement and that linear models successfully describe the evolution of such prices. For example, Hinich and Paterson (1985), Cochrane (1988), Fama and French (1988), Lo and McKinlay (1988), White (1988), Sheinkman and LeBaron (1989), Hsieh (1991), Granger (1992), Gençay (1996), Campbell, Lo and McKinlay (1997), De Lima (1998), Fernández, García and Sosvilla (1999) and García and Gençay (2000) have raised the question of whether the behaviour of asset returns is completely random; whether linear modelling techniques are appropriate to capture some of the complex models that chartists have observed in the evolution of asset prices and the market negotiation process; whether it is possible to identify and exploit the behaviour of asset returns over time; or whether the adjustments made in the market in response to price deviations and their theoretical value might not be proportional to the quantity by which prices deviate from their real value.

Theoretical and practical interest in non-linear time series models has increased rapidly in recent years. Various factors might account for non-linearity. On the one hand, we could admit the possibility that not all the agents simultaneously receive all the information; there may be important differences in targets and in negotiation time; or those agents with more complex algorithms might be able to make better use of the available information<sup>1</sup>. However, there are several reasons why non-linear modelling is not easy. First, because there exist a great number of options, i.e. bilinear models, ARCH and its extensions, smooth transition autoregressive models (STAR), artificial neural networks (ANN), wavelets and even chaotic dynamics. Second, because the flexibility inherent in its use can create spurious fits [Granger and Teräsvirta (1993)]; and third, because when considering a long period of time there can appear the problem of structural change and the existence of more outliers, which makes model estimation difficult [De Lima (1998)].

---

<sup>1</sup> For example, Granger (1992) argues that if we spread the time horizon, use seasonally adjusted data, give a suitable treatment to exceptional events and outliers and, in particular, consider non-linearity, we can achieve better returns. However, if there is no rule about profits and no profits are made over a long period, then the weak hypothesis of Efficient Market (WHEM) should not be rejected.

Due to their variety and flexibility, one class of regime switching models and ANN models has become popular in the class of non-linear models. The regime switches in economic time series can be described by STAR models. These models imply the existence of two distinct regimes, with potentially different dynamic properties, but with a smooth transition between regimes. On the other hand, ANN is considered to be a universal approximator in a wide variety of non-linear patterns, including regime switches and other non-linearities. Both models are examined in this study. The purpose of this article is to evaluate their adequacy and validity and to compare the forecasting performance of different STAR and ANN models in predicting Ibex-35 Spanish stock index returns<sup>2</sup>. In this sense, the work in this paper is empirical, and we do not attempt to explain the results obtained, or those claimed by other researchers, on theoretical grounds.

The out-of sample one-step-ahead forecasts from different models are evaluated using *statistical* criteria such as mean squared prediction error (MSPE), tests for forecast encompassing [Chong and Hendry (1986)]<sup>3</sup>, equality of accuracy of competing forecasts or MSPE of competing models [Diebold and Mariano (1995)] and directional accuracy [DA, Pesaran and Timmermann (1992)]. We examine whether out-of-sample forecasts generated by the non-linear models are more accurate and preferable to out-of-sample forecasts generated by linear ARMA models for stock index returns. We also analyse whether non-linear ANNs really are superior to linear and STAR models in practice, assessing the relative forecast performance with *economic* criteria. For example, we use the return forecasts from the different linear and non-linear models in a simple trading strategy and compare pay-offs to determine if ANNs are useful forecasting tools for an investor. As shown by Leitch and Tanner (1991) and Satchell and Timmermann (1995), the use of statistical or economic criteria can lead to very different outcomes. The correlation between MSPE and trading profits, for example, is usually quite small. The performance of a particular model in terms of DA is often a better indicator of its performance in a trading strategy. However, given that some papers find that neural networks do not perform much

---

<sup>2</sup> We analyze one of the official indexes of the Madrid Stock Market: the Ibex35, an index composed of the 35 most liquid values listed in the Computer Assisted Trading System (CATS). This index was designed to be used as a reference value in the trade of derivatives products, i.e. options and futures. This continuous system was introduced onto the Madrid Exchange Market in December 1989.

better than linear and STAR models in terms of DA, we would not find it surprising if it turned out that ANNs do not offer significantly higher trading profits. Finally, it would also be useful to examine the impact of transaction costs on the profits of trading strategies.

This paper is structured as follows. Section 2 describes the main characteristics of STAR and ANN non-linear models. In Section 3, we describe the Ibex35 data and some statistical properties. Section 4 shows non-linear model estimates. In Section 5, we examine the predictive capacity of some non-linear models over a long period, and in Section 6, we examine the trading strategy profits. Finally, Section 7 summarises the most important conclusions of this study.

## 2. Regime switching models and ANNs for stock index returns

In this section we briefly explain regime switching models such as STAR and a class of flexible non-linear models inspired by the way in which the human brain processes information. Let us consider an asset which provides a daily return equal to  $r_t$ ,  $t = 1, \dots, T$ . Consider the asset market as an information processing system. The information set constantly changes, and the processing of market information produces a fitting of prices towards the perceived market value. The market is considered to form an expectation for the next period, depending on current information, which could be written mathematically as:  $E_{t-1}[r_t] = f(\mathbf{y}_{t-1})$  where  $\mathbf{y}_t$  is the information set during period  $t$  and the  $f(\cdot)$  function could be either linear or characterised by complex non-linear functions. Some authors argue that the asset market has the capacity to be a non-linear dynamic system. In this sense, we could say that the return during the period  $t$  is equal to:

$$r_t = f(\mathbf{y}_{t-1}) + \mathbf{e}_t \quad [1]$$

where  $\mathbf{e}_t$  is a prediction error.

In the following sub-sections, we assume that  $f(\mathbf{y}_{t-1})$  can be modelled by  $p$ -order AR in a non-linear way. We also assume that lagged returns are needed in the conditional mean specification, because autocorrelation in stock returns can appear because of non-

---

<sup>3</sup> A set of forecasts is said to encompass a competing set if the latter should optimally receive a zero weight in a composite predictor that is a weighted average of the two individual predictors.

synchronous trading effects. In this sense, we describe the STAR and ANN models that are compared in this paper.

### 3.1. Specification and estimation of STAR models for stock index returns

Non-linear time series models have become very popular in recent years. Regime switching models are very popular in the class of non-linear models and they are an alternative way to investigate potential non-linearities and cyclical behaviour in stock returns. Estimates based on non-linear models suggest that stock price growth rates are characterised by asymmetric cycles in most countries, with the speed of transition between expansion and contraction regimes being relatively slow. The regime switching models we consider here are known as smooth transition regression (STAR), and they are a flexible family of non-linear time series models that have also been used for modelling economic data. STAR models have been described by Teräsvirta, Tjøstheim and Granger (1994).

This paper evaluates the statistical adjustment and the forecast performance of different STAR models using the Ibex-35 index of stock returns. A simple first-order STAR model with two regimes is defined as follows:

$$r_t = \mathbf{f}_{10} + \sum_{i=1}^p \mathbf{f}_{1i} r_{t-i} + \left[ \mathbf{f}_{20} + \sum_{i=1}^p \mathbf{f}_{2i} r_{t-i} \right] F_{t,d}(s_t; \mathbf{g}, c) + \mathbf{e}_t \quad [2]$$

where  $r_t$  are the returns,  $\mathbf{f}_{ij}$ , ( $i=1,2, j=0,1,2\dots p$ ) are the unknown parameters that correspond to each of the two regimes.  $F_{t,d}(s_t; \mathbf{g}, c)$  is the transition function, assumed to be twice differentiable and bounded between 0 and 1,  $\mathbf{g}$  is the transition rate or smoothness parameter,  $c$  is the threshold value which represents the change from one regime to another, and  $d$  is the number of lags of transition variable. This function introduces regime switching and non-linearity into the parameters of the model. Although there are few theoretical results regarding the stationarity of the STAR model, a sufficient condition is  $\mathbf{f}_{ij} < 1, \forall i, j$ . The transition variable,  $s_t$ , is usually (but not always) defined as a linear

combination of the lagged values of  $r_t$ , as:  $s_t = \sum_{i=1}^d \mathbf{a}_i r_{t-i}$ .

Regarding the choice of transition function, the two most widely used in the literature are the first-order logistic function:

$$F_{t,d}(s_t; \mathbf{g}, c) = \{1 + \exp[-\mathbf{g}(s_t - c)]\}^{-1}, \mathbf{g} > 0, \quad [3]$$

in which case the model is called logistic STAR or LSTAR( $p; d$ ); and by the first-order exponential function, for which:

$$F_{t,d}(s_t; \mathbf{g}, c) = \{1 - \exp[-\mathbf{g}(s_t - c)^2]\}, \mathbf{g} > 0, \quad [4]$$

and in this case, the model is called exponential STAR or ESTAR( $p; d$ ). In both cases, the transition variable can be any variable in the information set  $\mathcal{Y}_{t-1}$ . In order to use this model effectively, it is important to choose the appropriate transition function and threshold variable. There exist many LM-type tests to determine the appropriate choice of  $F_{t,d}(s_t; \mathbf{g}, c)$  and  $s_t$ . However, LSTAR and ESTAR models describe different types of dynamic behaviour. The LSTAR model allows the expansion and contraction regimes to have different dynamics, with a smooth transition from one to another. On the contrary, the ESTAR model suggests that two regimes have similar dynamics, while the behaviour in the transition period (middle regime) may be different. Both models characterise asymmetric cycles.

Such models are often estimated by non-linear least squares (NLS) or by maximum likelihood estimations (MLE). If  $\mathbf{e}_t$  is normal, NLS is equivalent to MLE (but not generally), otherwise it can be interpreted as QMLE. Under suitable regularity conditions, NLS is consistent and asymptotic normal. After many iterations we would probably reach the optimal value of the target function.

### ***3.2. Specification and estimation of ANN models for stock index returns***

Though not without their critics, ANNs have come into wide use in recent years, due to the advantages such models offer analysts and forecasters in the financial markets<sup>4</sup>. In particular, non-parametric and non-linear models can be trained to map past values of a time series for purposes of classification or function estimation, and allow us to depict non-

---

<sup>4</sup> For a detailed discussion of ANNs and their econometric applications, see Kuan and White (1994).

linear complex relationships automatically; they are universal approximators; they describe various forms of regime switching, and thus different asymmetric effects, which leads us to suggest that some subperiods are more predictable than others; finally, they are good predictors<sup>5</sup> [see Swanson and White (1997)]. Perhaps, the ANN methodology is preferred to other non-linear models because it is non-parametric.

This technique consists of modelling in a non-linear fashion the relationships between variables to construct a forecast. An ANN is a collection of transfer functions which relate the dependent variable,  $r_t$ , to certain vectors of explanatory variables,  $R$ , which can even be functions of other explanatory variables. In this sense, ANNs are a class of non-linear regression models and in particular mechanisms for non-parametric statistical inference. Two basic aspects characterize them: a parametric specification or network topology, and estimation mechanism or network training. These representations nest many familiar statistical models, such as linear and non-linear regressions, classification (i.e. logit and probit), latent variable models (MIMIC), principal component analysis and time series analysis (ARMA, GARCH).

Three ANN models are examined in this paper. The lagged stock returns are taken as explanatory variables, because we assume that a forecasting relationship for  $r_t$  can be derived from the information revealed by  $p$  inputs,  $R = (1, r_{t-1}, r_{t-2}, \dots, r_{t-p})'$ , including a constant term. Thus, an ANN model for  $r_t$  can be taken as an extension of a basic linear regression. Like the STAR model, the ANN model can describe regime switches in economic time series, at least when these are confined to the intercepts.

The ANN models are the multilayer perceptron model (MLP), jump connection nets (JCN) and a partial recurrent network by Elman (1990). Such networks are capable of rich dynamic behaviour. MLP and JCN networks are referred to in the literature as *feedforward*

---

<sup>5</sup> Their application in economics is mainly in management. For example, in the areas of cross-sectional data, bankruptcy prediction [Tam and Kiang (1992)], the ratings of corporate bonds [Surkan and Singleton (1990), Moody and Utans (1995)], in the area of time series prediction, the study of asset returns [White (1988)], and decision-related topics [Sharda and Patil (1992) and Hill, Márquez, O'Connor and Remus (1994), among others. In general, almost every study has analysed the predictive capacity of networks by comparing several models, both linear and non-linear. The results obtained have shown the moderate advantage of ANN prediction against any of the linear ARIMA and non-linear GARCH time-series models analysed.

networks, while Elman is designated as a *recurrent network*, because it exhibits memory and context sensitivity.

The first model that we built was a multilayer perceptron model (MLP) with a single hidden layer, and  $q$  hidden units. This is the most commonly found neural model in the specialized literature. In general, a non-linear regression model which represents the MLP( $p; q$ ) has the following form for a single hidden layer network:

$$r_t = \mathbf{b}_0 + \sum_{j=1}^q \mathbf{b}_j g \left( \sum_{i=1}^p \mathbf{f}_{ij} r_{t-i} + \mathbf{f}_{0j} \right) + \mathbf{e}_t \quad [5]$$

where  $r_t$  is the return in  $t$  or system output<sup>6</sup>; the parameter vector is  $\mathbf{q} = (\mathbf{b}', \mathbf{f}')$ , where  $\mathbf{b} = (\mathbf{b}_1, \dots, \mathbf{b}_q)'$  and  $\mathbf{f} = (\mathbf{f}_{1j}, \dots, \mathbf{f}_{pj})'$ ,  $j=1, \dots, q$ , brings together all the network weights, with  $\mathbf{b}_j$  representing the weights from the hidden to the output unit and  $\mathbf{f}_{ij}$  the weights from the input layer to the hidden unit  $j$ ;  $g(\cdot)$  can take several functional forms, such as the threshold function, which produces binary ( $\pm 1$ ) or (0/1) output, or the sigmoid function, which produces an output between 0 and 1<sup>7</sup>. This function determines the connections between nodes of the hidden layer, and it is used as the hidden-unit activation function to enhance the non-linearity of the model; and  $\mathbf{e}_t$  is a residual *i.i.d.*.

---

<sup>6</sup> Hornik, Stinchcombe and White (1990) showed that the ANNs of the type defined in Eq.[5] are universal approximators in a wide variety of function spaces of practical interest. We specified one hidden layer on the basis that single hidden layer MLPs possess the universal approximation property, namely they can approximate any nonlinear function to an arbitrary degree of accuracy with a suitable number.

<sup>7</sup> Function  $g(\cdot)$  is sigmoid if  $g: R \rightarrow [0,1]$ ;  $g(a) \rightarrow 0$  when  $a \rightarrow -\infty$ ;  $g(a) \rightarrow 1$  when  $a \rightarrow \infty$ . For example,  $g$  can be the logistic activation cumulative distribution function:  $g(a) = [1 + \exp(-a)]^{-1}$ . It could also be a bipolar function:  $h(a) = 2g(a) - 1$ . Or it could be defined by the hyperbolic tangent function:  $\tanh(a) = [\exp(a) - \exp(-a)] / [\exp(a) + \exp(-a)]$ . There are some heuristic rules for the selection of the activation function. For example, Klimasauskas (1991) suggests logistic activation functions for classification problems and hyperbolic tangent functions if the problem involves learning about deviations from the average, such as the forecasting problem. However, it is not clear whether different activation functions have a greater effect on the performance of the networks [see Zhang *et al.* (1998)].

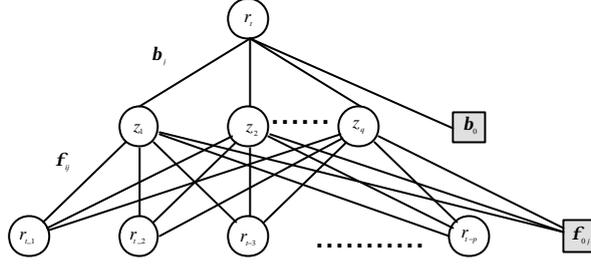


Figure 1. Hidden layer network for stock index returns.

The network interpretation of Eq. [5] is as follows (see Figure 1). The explanatory variables (or input units) defined in  $R$  send signals to each of the hidden units,  $z_j$ , that represent the output vectors of hidden units. The signal from the  $i$ -th input unit to the  $j$ -th hidden unit is weighted, denoted by  $f_{ij}$ , before it reaches the hidden unit number  $i$ . All signals arriving at the hidden units are first summed and then converted to a hidden unit activation by the operation of the hidden unit activation function  $g(\cdot)$  that transforms the signal into a value between 0 and 1. The next layer operates similarly with connections sent to the dependent variable (or output unit). As before, these signals are attenuated or amplified by weights  $b_j$  and summed.

The second model that we use is a network with direct connections between the inputs and outputs, called jump connection nets (JCN). According to Kuan and White (1994), the parametric specification for the output of the model adds the  $p$ -order AR to the MLP network. In this sense, the ANN with a single hidden layer has a linear component augmented by non-linear terms, and it is written as JCN( $p;q$ ) by:

$$r_t = \sum_{i=1}^p \mathbf{a}_i r_{t-i} + \mathbf{b}_0 + \sum_{j=1}^q \mathbf{b}_j g\left(\sum_{i=1}^p \mathbf{f}_{ij} r_{t-i} + \mathbf{f}_{0j}\right) + \mathbf{e}_t \quad [6]$$

where  $\mathbf{a}_1, \dots, \mathbf{a}_p$  are direct input-output weights (see Figure 2). Eq.[6] nests the linear model

because it includes the term  $\sum_{i=1}^p \mathbf{a}_i r_{t-i}$  as a linear autoregressive component.

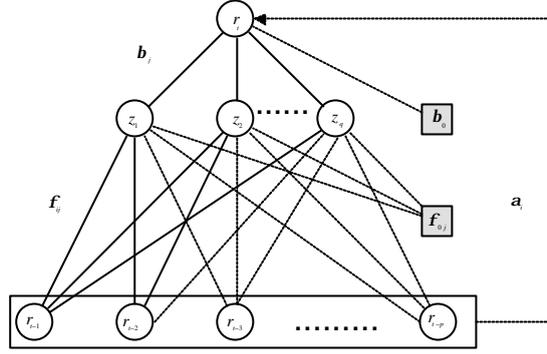


Figure 2. Augmented hidden layer network for stock index returns.

The network interpretation of Eq. [6] is similar to that of Eq. [5], but with one added aspect. Also, signals are sent directly from all the explanatory variables to the dependent variable with weights  $\mathbf{a}_i$ . The latter signals effectively constitute the linear part of this JCN model<sup>8</sup>. This model nests the linear model within the JCN, and ensures that the JCN will perform in-sample at least as well as the linear model.

Finally, the third model we use is a partially recurrent network, as proposed by Elman (1990). This has the ability to recognize and, sometimes, to reproduce sequences. This type of ANN is somewhat more complex than the unidirectional ANNs defined by Eq. [5] and by Eq. [6]. In the specific case of a recurrent Elman( $p,q$ ) type network, this is characterized by a dynamic structure where the hidden layer output feeds back into the hidden layer with a time delay. This model can take the form in the single hidden layer as:

$$\begin{aligned}
 r_t &= \mathbf{b}_0 + \sum_{j=1}^q \mathbf{b}_j z_{j,t} + \mathbf{e}_t \\
 z_{j,t} &= g \left( \sum_{i=1}^p \mathbf{f}_{ij} r_{t-i} + \mathbf{f}_{0j} + \mathbf{d}_{ij} z_{j,t-1} \right)
 \end{aligned}
 \tag{7}$$

where  $z_j$  is the output vector of the hidden units, and  $\mathbf{d}_{ij}$  are the weights between the hidden units evaluated in  $t$  and  $t-1$ . In econometric terms, a model of the form Eq. [7] can be viewed as a non-linear dynamic latent variable model [see Kuan and White (1994)]. Elman

<sup>8</sup> In its most complex version, the topology allows us to introduce one or more hidden layers between the output and the inputs. The main advantage of this model is its capacity to act as an approximation of non-linear complex relationships. Its main disadvantage is its static nature, which is overcome by other topologies that incorporate the dynamics of input-output relationships with time.

has introduced an architecture called the simple recurrent network where the input layer can be considered to be divided into two parts, true input units  $R$  and context units,  $z_j$ . The feedback between them is represented schematically in Figure 3. The context units simply hold a copy of the activations of the hidden nodes from the previous time step,  $z_{j,t-1}$  by recursive substitution<sup>9</sup>. The network interpolation of Eq. [7] is similar to Eq. [5], but adds the possibility that hidden units can be connected with lagged hidden units by the weights  $d_{ij}$ , which introduce a recursive update.

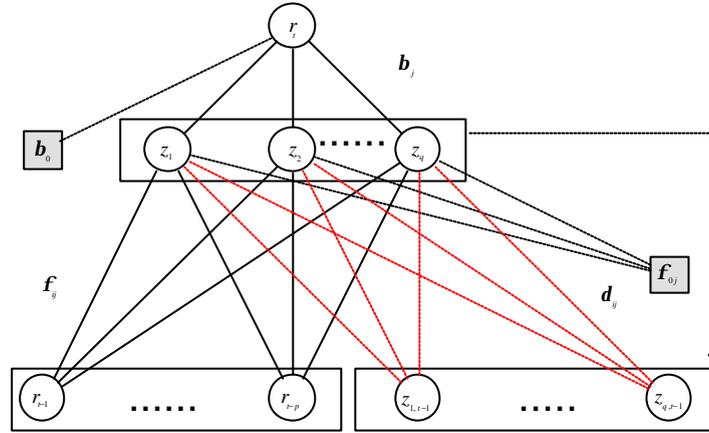


Figure 3. Elman network for stock index returns.

The expressions of Eq. [5], [6] or [7], and the flexibility of specifications defined in footnote 7, show that when  $q$  takes a large enough value, the ANN model can approximate any arbitrarily close function [see Kuan and White (1994)]. The most widely used estimation method (or so-called learning rule) of the neural network is error backpropagation. Backpropagation is a recursive gradient descent method that mimics a learning behaviour. In this method, the weights of the signals are updated. Using the first set of observations, at the initial stage the method does a forward and backward pass through the network, initially computes the weights, and determines the value of the error

<sup>9</sup> These types of network have certain features which make them especially suitable for modelling time series because through the feedback, network output depends on the initial value and the entire history of system inputs. These networks are capable of rich dynamic behaviour, exhibiting memory and context sensitivity by the presence of internal feedbacks [see Gençay (1997)].

function, recomputes the weights, and redetermines the value of the error the target values of the output variable. At the next stage, it uses the second set of observations, and so on. This estimation procedure is characterized by the recursive process. The learning algorithm converges and thus the process stops when the value of the error function is lower than a predetermined convergence criterion.

More specifically, the network weight vector  $\mathbf{q}$  is chosen to minimize the sum of the squared-error loss:

$$\min_{\mathbf{q}} \sum_{t=1}^T [r_t - \hat{r}_t]^2,$$

where  $T$  is the sample size, and  $\hat{r}_t$  is the calculated output value from Eq.[5], Eq.[6] and Eq.[7]. Then the iterative step of the gradient descent algorithm takes  $\mathbf{q}$  to  $\mathbf{q} + \Delta\mathbf{q}$ , and

$$\Delta\mathbf{q} = -\mathbf{h}\nabla f(R_t, \hat{\mathbf{q}}_t)\mathbf{e}_t,$$

where  $\mathbf{h}$  is the “learning rate”;  $\nabla f(R_t, \hat{\mathbf{q}}_t)$  is the gradient of  $f(R_t, \hat{\mathbf{q}}_t)$  with respect to  $\mathbf{q}$  (a column vector of parameters); and  $\mathbf{e}_t = r_t - \hat{r}_t$  is the “network error” between the computed output and the target return value,  $r_t$ . For recurrent networks, the network output depends on  $\mathbf{q}$  directly and indirectly through the presence of lagged hidden-unit activations. For this reason, the model can be estimated by the recurrent backpropagation algorithm and by the recurrent Newton algorithm [see Gençay (1997) for details].

### 3. Data and preliminary statistics

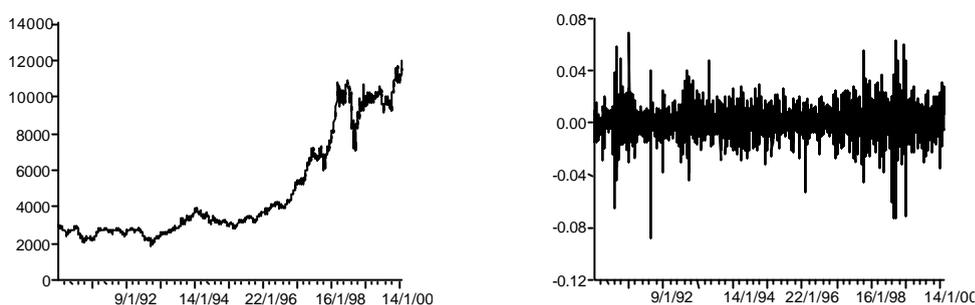
This study uses the daily closing prices of the Spanish Ibex-35 stock index, from 30 December 1989 to 10 February 2000, with a total of 2520 observations. The Ibex-35 index ( $I_t$ ) comprises the 35 most liquid values negotiated in the continuous system which during the control period had the highest trading volume in cash pesetas. The Ibex-35 is a composite index which is highly representative and is fitted by capitalisation and dividends of the assets included, but not by expansions in capital. The series is transformed into logarithms to compute continuous returns, according to the following expression:

$$r_t = \log\left(\frac{I_t}{I_{t-1}}\right), \text{ where } \log \text{ is the natural logarithm.}$$

Figure 4 shows the evolution of the index and its daily returns. Its sharp increase since 1996 is due to the downturn in risk-free interest rates and the sequential move by investors towards the stock market. The period of special interest for the evolution of prices and returns is the Asiatic crisis of 27-29 October 1997, when returns fell abruptly.

**Figure 4.** Time evolution of daily closing Ibex35 and returns.

(i) Closing prices (ii) Returns



We study some statistical properties of the Ibex35 index, shown in Tables AI.1, AI.2 and AI.3 (see Appendix I). Table AI.1 reports the augmented Dickey-Fuller (ADF) and Phillips and Perron (PP) statistics for non-stationarity for the logarithm index and returns. The statistics indicate that  $\log I_t$  is non-stationary and that the index of returns is stationary. Another statistical test for the null hypothesis of stationarity, Kwiatkowski, Phillips, Schmidt and Shin (KPSS), obtains the same results. Table AI.2 reports the variance-ratio test. The results show the existence of negative autocorrelations or mean reversion (for values between  $2 \leq q \leq 45$  days). This test rejects the null hypothesis of random walk at the 10% significance level. Finally, the Table AI.3 reports the Brock, Dechert and Scheinkman (BDS) test. This test indicates the presence of non-linearities and, therefore, of complex models in the data<sup>10</sup>. The main conclusion is that Ibex35 stock index returns may be predicted using non-linear models.

---

<sup>10</sup> In order to avoid possible rejections of the null hypothesis due to non-stationarity the BDS test is commonly applied to the estimated residuals of the ARIMA process. The asymptotic distribution of BDS is not affected when linear filters are applied to data. Table AI-3 also shows “shuffled” residuals, i.e., recreated randomly as if they were “sample” data without replacement. We use this technique following Scheinkman and LeBaron (1989) in order to reinforce the results, so that in this case we should not reject the null

## 4. Non-linear model estimates

The BDS statistic reveals considerable evidence of non-linearity, and the variance ratio test shows that mean reversion exists. In this section, we analyse the non-linear model estimates from the STAR and ANN models, which we consider to represent some stylised facts of the short-term dynamics of stock index returns. The fitted period is 30 December 1989 to 30 April 1999 ( $T=2320$  observations). We did not include GARCH models in the set of forecasting models because these models parameterise the conditional variance, whereas the object to be forecast is the stock index returns, not their volatility. However, we tested for any omitted ARCH non-linearity.

### 4.1. STAR Models

This section investigates empirical issues regarding STAR models with Gaussian errors. In this paper we do not distinguish between regimes of low and high volatility, because our aim is to analyse stock index returns, not their volatility. Therefore, we evaluated different models that show regime switching, for example, Eq. [2] with the ESTAR and LSTAR function<sup>11</sup>. The modelling procedure for building STAR models is carried out in three stages [see Granger and Teräsvirta (1993, pp.113-124), Teräsvirta (1994), and Eitrheim and Teräsvirta (1996)]. The first stage is to specify a linear  $AR(p)$  model. We estimated different AR models and chose  $p$  on the basis of the AIC, SBIC and Ljung-Box (LB) statistics for autocorrelation. The AR model has a relatively short order. We chose  $p=2$  on the basis of AIC and SBIC equal to -5.92 and  $LB(1)=0.0$  ( $P$ -value is 1.0),  $LB(5)=-0.001$  ( $P$ -value is 0.98) and  $LB(10)=0.039$  ( $P$ -value is 0.78), which indicates that the  $AR(2)$  model has white noise residuals. The second stage is to test the linearity against STAR models,

---

hypothesis of the i.i.d. linear process. Thus, we will be able to prove that there is a non-linear structure in the original data which has been removed by the “shuffling”. In both situations, we use  $m=2$  to 8 and a value for  $\epsilon$  between  $0.5\sigma$  and  $2\sigma$ , using  $\sigma=0.1088$ . The results show that there are non-linear structures in data in the logarithm of the Ibex35 index, since the tests applied to residuals and to “shuffled” residuals show the rejection of the null hypothesis in the first case and its non-rejection in the second case.

<sup>11</sup> Also, STAR models have been estimated assuming conditional heteroscedasticity or GARCH errors, but the results are worse than those obtained without considering such an assumption. For this reason, we have not shown it. Lundbergh and Teräsvirta (1998) made an extensive study of STAR models with GARCH errors.

for different values of the delay parameter  $d$ , using the linear model specified at the first stage. This stage tests the parameter constancy, such as testing whether STAR is more appropriate than a single AR model. Therefore, we tested whether non-linear functions of lagged regressor variables contribute significantly to the fit (after correction for a linear AR part), using  $s_t = r_{t-d}$ . The linearity test is based on the auxiliary regression:

$$r_t = \mathbf{f}_{10} + \sum_{i=1}^p \mathbf{f}_{1i} r_{t-i} + \sum_{j=1}^p \mathbf{b}_{1j} r_{t-j} r_{t-d} + \sum_{j=1}^p \mathbf{b}_{2j} r_{t-j} r_{t-d}^2 + \sum_{j=1}^p \mathbf{b}_{3j} r_{t-j} r_{t-d}^3 + v_t$$

To specify the value of the delay parameter  $d$ , the estimation of the auxiliary regression is carried out for a wide range of values,  $1 \leq d \leq D$ , given uncertainty about the most appropriate value of  $d$ . The null hypothesis is  $\mathbf{b}_{ij} = 0, \forall i, j$ . The  $F$ -test values for the significance of the regressor added to the linear AR regressions can be used to test the null hypothesis of linearity. We can obtain a first impression of the  $d$  value by looking at the relative value of the  $F$ -test statistics, that is, the  $d$  for which the corresponding  $P$ -value is smallest may be selected, and this corresponds to the largest  $R^2$  of the regression model. In carrying out linearity tests, we considered values for the delay parameter over the range  $1 \leq d \leq 12$  (Table 1). The  $d$  value selected is 6, because it has the lowest  $P$ -value. The linearity is rejected at the 5% level of significance because the minimum  $P$ -value is 0.000001.

**Table 1.**  $P$ -values for linearity test and sequential procedure.

Delay	<i>Linearity test</i>	<i>Choosing between ESTAR and LSTAR</i>		
	$H_0 : \mathbf{b}_1 = \mathbf{b}_2 = \mathbf{b}_3 = 0$	$H_{01} : \mathbf{b}_3 = 0$	$H_{02} : \mathbf{b}_2 = 0 / \mathbf{b}_3 = 0$	$H_{03} : \mathbf{b}_1 = 0 / \mathbf{b}_2 = \mathbf{b}_3 = 0$
1	0.000017	0.05234	0.000003	0.7227
2	0.003785	0.6991	0.000221	0.4296
3	0.000155	0.3504	0.00009	0.04344
4	0.001467	0.001320	0.2169	0.07367
5	0.21942	0.89108	0.8022	0.02238
6	0.000001 <sup>a</sup>	0.000004 <sup>b</sup>	0.03413	0.02762
7	0.00608	0.004892	0.04378	0.5614
8	0.00194	0.05794	0.000930	0.5462
9	0.0005	0.05888	0.02621	0.00355
10	0.0004	0.60798	0.3756	0.000017
11	0.1787	0.02942	0.8538	0.4638
12	0.0116	0.8671	0.2582	0.0012

*Note:* **a** indicates lowest  $P$ -value for the null hypothesis of linearity over the interval  $0 \leq d \leq 12$ . **b** indicates lowest  $P$ -value when  $d=6$ .

The third stage is to choose between ESTAR and LSTAR models where linearity is rejected. Teräsvirta (1994) suggests applying the following sequence of nested tests: (i) test whether all fourth-order terms are insignificant,  $\mathbf{b}_{3j} = 0, \forall j$ ; (ii) conditional on all fourth-order terms being zero, test the joint significance of all third-order terms  $\mathbf{b}_{2j} = 0/\mathbf{b}_{3j} = 0, \forall j$ , and (iii) conditional on all third and fourth-order terms being zero, test the significance of the second-order terms,  $\mathbf{b}_{1j} = 0/\mathbf{b}_{3j} = \mathbf{b}_{2j} = 0, \forall j$ . If the test in (i) does not reject the null hypothesis, we choose the LSTAR model. If we accept (i) and reject (ii), we choose the ESTAR model. Finally, accepting the null hypothesis in (i) and (ii), but rejecting (iii), we can choose an LSTAR model. We used  $P$ -values for the  $F$ -tests and made the choice of the STAR model on the basis of the lowest  $P$ -value. The  $P$ -values obtained were (i) 0.00004, (ii) 0.0341 and (iii) 0.02762. Thus, we chose to fit an LSTAR model (Table 1).

The next step is to estimate the parameters in the STAR models. Table 1 summarises the estimation results, including the ESTAR model. We used the MLE and BFGS numerical algorithms, which satisfy various regularity conditions (such as stationary, ergodicity, consistency and asymptotic normality). We now comment on some specific aspects of the two models, although the model selected was LSTAR in terms of the sequential procedure suggested by Teräsvirta (1994)<sup>12</sup>. The estimated coefficients are lower than unity,  $\hat{\mathbf{f}}_{ij} < 1, \forall i, j$ . The ML estimations of the STAR model parameters of the two regimes are similar. The  $t$ -statistics, reported in Table 2, are adjusted for heteroskedasticity using White heteroskedasticity-consistent standard errors to assess the significance of the parameter estimates.

With respect to the smoothness parameter ( $\mathbf{g}$ ), this is always positive, small in the case of ESTAR and large in the case of LSTAR. The LSTAR estimation suggests that regime shifts or transitions between the regimes are smooth.

---

<sup>12</sup> In general, the LSTAR and ESTAR models have the same number of parameters, and the comparison of their log-likelihoods may be meaningful. In this sense, the results show that both estimations are possible but, statistically, LSTAR(2;3) seems to fit better than ESTAR(2;3), in terms of the log-likelihood value. However, similar likelihood values might suggest that these models are likely to produce a similar forecast performance.

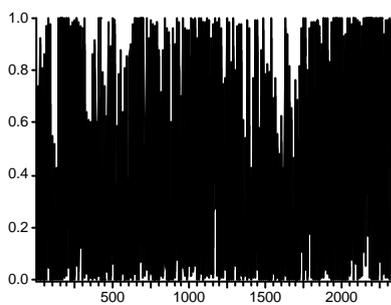
**Table 2.** MLE (BFGS) for STAR models. Period from 30-12-1989 to 30-04-1999. T=2350.

	$f_{10}$	$f_{11}$	$f_{12}$	$f_{20}$	$f_{21}$	$f_{22}$	$g$	$c$	LogL
<b>ESTAR</b>	0.059 (1.26)	0.1801 (15.5)	-0.015 (-1.10)	-0.033 (0.32)	-0.1214 (-4.99)	-0.049 (1.76)	1.869 (2.11)	0.00159 (0.59)	-1685.0
<b>LSTAR</b>	0.044 (1.72)	0.1265 (13.01)	-0.015 (-1.29)	0.0435 (0.81)	-0.0423 (-1.77)	-0.1754 (5.74)	7.345 (1.75)	0.01214 (7.84)	-1683.1

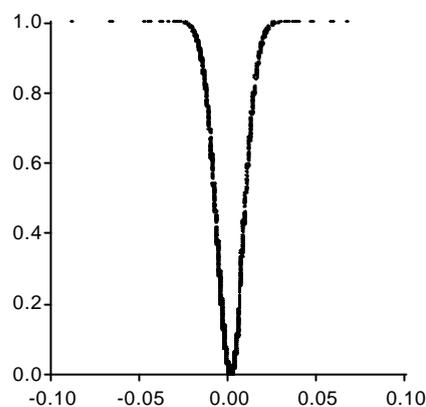
*Note:* The  $t$ -Student values for the null hypothesis that the parameter is equal to zero are given in parentheses. These values are calculated using White heteroskedastic-consistent standard errors.

The regime shift or threshold parameter ( $c$ ) indicates the halfway point between the expansion and contraction regimes. This is positive and statistically significant at the 5% level of significance in the ESTAR model and at 10% in the LSTAR model. Both models are in the range of the transition variable  $s_t = r_{t-6}$  (which varies between about -0.06 and 0.06). The transition is slow at the values for  $r_t$  of  $\hat{c}$ , with transition probabilities  $\hat{F}_{t,6}(s_t; \mathbf{g}, \hat{c})$  switching from 0 to 1 at this point. The two regimes can be described as follows: when  $F = 0$ , which we might refer to as the lower regime in the LSTAR model and the middle regime in the ESTAR model, the mean process for  $r_t$  is an AR(2) with complex roots (i.e. for the ESTAR model  $0.09 \pm 0.08i$ , having a modulus of 0.12; and for the LSTAR model it is  $-0.006 \pm 0.22i$  with a modulus equal to 0.22). When  $F = 1$ , we might refer to it as the upper or expansion regime in the LSTAR model and the outer regime (expansion and contraction regime) in the ESTAR model. In this case, the mean process for  $r_t$  is also an AR(2) with complex roots (i.e. for the ESTAR model  $0.06 \pm 0.10i$  having a modulus of 0.12; and for the LSTAR model it is  $-0.021 \pm 0.42i$  with a modulus equal to 0.42). So, if  $r_{t-6}$  exceeds 0.00159 in the ESTAR model and 0.01242 in the LSTAR model,  $\hat{F}_{t,6}(s_t; \mathbf{g}, \hat{c})$  can take values close to one, but with different dynamic properties. Figure 5a shows the graph of  $\hat{F}_{t,6}(s_t; \mathbf{g}, \hat{c})$  versus time in days, and Figure 5b displays  $\hat{F}_{t,6}(s_t; \mathbf{g}, \hat{c})$  versus  $r_{t-6}$  for the ESTAR model. In this model, high values for the transition probabilities imply that stock index returns are either in an expansion or in a contraction regime (outer regime). From Figure 5b we observe that the behaviour of stock index returns in the transition period or middle regime is different, but that the two regimes have similar dynamics. In this sense, we cannot identify expansionary and contractionary phases, but we can distinguish between the outer regime and the middle regime. Figures 6a and 6b show

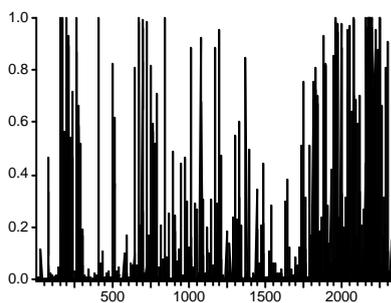
the same relationships when the model is LSTAR. In this model, the cyclical behaviour of stock returns can be inferred from the estimates of transition probabilities. When the  $\hat{F}_{t,6}(s_t; \mathbf{g}, \hat{c})$  transition probabilities are greater than 0.5, the stock market could be considered to be in an expansion regime. Figure 6a shows the probability of an expansion regime  $\hat{F}_{t,6}(s_t; \mathbf{g}, \hat{c})$ . We can clearly observe the periods of high and low returns and, from Figure 6b, we see that the transition between high and low returns (expansion and contraction regimes) is reasonably smooth, although there are not many data points for which  $r_{t-6}$  exceeds  $\hat{c}$ .



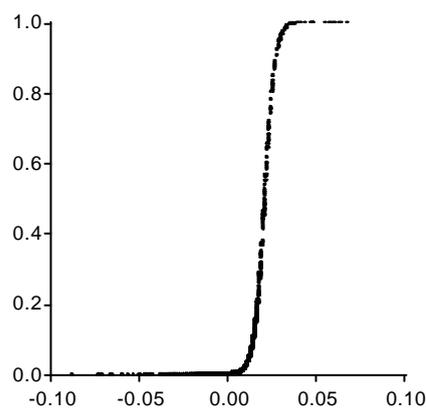
**Figure 5a.**  $\hat{F}_{t,6}(s_t; \mathbf{g}, \hat{c})$  ESTAR versus time.



**Figure 5b.**  $\hat{F}_{t,6}(s_t; \mathbf{g}, \hat{c})$  ESTAR vs.  $r_{t-6}$ .



**Figure 6a.**  $\hat{F}_{t,6}(s_t; \mathbf{g}, \hat{c})$  LSTAR versus time.



**Figure 6b.**  $\hat{F}_{t,6}(s_t; \mathbf{g}, \hat{c})$  LSTAR vs.  $r_{t-6}$ .

To evaluate the within-sample performance of the estimated STAR models, we used some misspecification tests, only for the LSTAR model. We did not use the Ljung-Box test for serial correlation because simulation studies suggest that  $\mathbf{c}^2$  asymptotic distribution may not be valid. Methods of testing the adequacy of fitted STAR models are discussed in Eitrheim and Teräsvirta (1996). These authors contribute to the evaluation stage of a proposed specification, estimation and evaluation of these models. To determine whether such a model is adequate, we first tested the hypothesis of no error autocorrelation or serial independence, but we did not reject the null hypothesis at the 5% level of significance. Secondly, to test against general *neglected* non-linearity or remaining linearity, second and third-order terms of the form  $r_{t-i}r_{t-j}$  for  $i=1,\dots,p$  and  $j=i,\dots,p$ , and  $r_{t-i}r_{t-j}r_{t-k}$  for  $k=j,\dots,p$  may be added to the LSTAR model and tested for significance. Doing so for the fitted LSTAR(2;3) model leads to a statistic that is significant at any level ( $P$ -value equal to 0.00), which confirms the possibility of additive non-linearity, although a rejection as such in general does not give much orientation as to what to do next [see Eitrheim and Teräsvirta (1996)]. In this sense, if the non-linearity is manifests in the conditional variance, then we would expect to find significant ARCH effects. Using the Lagrange multiplier test for ARCH effects, we obtained a  $P$ -value that suggested the presence of this type of non-linearity (i.e. ARCH(1) Lagrange Multiplier test with  $P$ -value equal to 0.00, ARCH(5) equal to 0.00 and ARCH(10) equal to 0.00).

Another important assumption is test parameter constancy. Eitrheim and Teräsvirta (1996) postulate a parametric alternative parameter constancy in STAR models, which explicitly allows the parameters to change smoothly over time. These tests are monotonic parameter change, a symmetric non-monotonic change and a more flexible test that allows monotonically and non-monotonically changing parameters. For these tests, the  $P$ -values are 0.5335, 0.9521 and 0.3564, respectively. In none of the three cases do we reject the null hypothesis at the 5% level of significance.

Thus, the validity of the LSTAR model for stock returns depends on the existence of remaining non-linearity and ARCH errors. We treat these empirical facts by reducing the magnitudes of extreme observations and outliers, and explore the estimation of the MLE for GARCH and STAR-type models by using two highly flexible non-linear models, namely STAR-GARCH and STAR-Smooth Transition GARCH [see Lundbergh and Teräsvirta

(1998)]. These authors have extended the STAR model by incorporating the concept of smooth transition into the GARCH component (STGARCH). This model is non-linear not only in the conditional mean, but also in the conditional variance. Moreover,  $\mathbf{e}_t$  is assumed to follow a GARCH(1,1) process that is useful for capturing volatility clustering, while the threshold variables are useful if the data exhibit regime switching behaviour for varying stock returns and  $\mathbf{e}_t$ . In the case of STGARCH-type models, we consider  $H(p_t; \mathbf{x}, e)$  as a transition function which satisfies the same conditions as  $F_{t,d}(s_t; \mathbf{g}, c)$ . We assume there exist two regimes with the transition variable  $p_t = \mathbf{e}_{t-1}$ ;  $\mathbf{x}$  is the transition rate;  $e$  is the threshold value, and regarding the choice of transition function, we employ the first-order logistic function. The following implications follow from the estimates in the LSTAR-GARCH(1,1) and LSTAR-STGARCH(1,1) models. First, the MLE is extremely sensitive to the choice of initial values. Second, convergence is achieved after very iterations. In the case of LSTAR-GARCH(1,1), it is easily achieved but the transition variable selected for the mean process is  $r_{t-2}$ . The estimated coefficients  $\mathbf{g} = 31.4$  and  $\hat{c} = 0.03$  are significant at the 5% level of significance, but the AR(2) parameters are not significant at any level. The GARCH coefficients meet the sufficient conditions for strict positive conditional variance ( $\hat{\omega} = 0.003$ ,  $\hat{\alpha}_1 = 0.11$  and  $\hat{\beta}_1 = 0.82$ , respectively). However, the results for the LSTAR-STGARCH(1,1) model appear to be worse after such adjustments.

We conclude that the estimated models based on adjusted data perform similarly and do not improve on the within-sample estimates in Table 2 for stock index returns. Although the effects of misspecification of non-linear models are generally unknown, it is difficult to draw firm conclusions about the effects of outliers and remaining non-linearity, because there are difficulties in fitting the LSTAR-STGARCH model for two regimes.

#### 4.2. *Artificial neural network models*

In this section we employ the technique of ANN estimation to obtain out-of-sample forecasts. An important feature of ANNs is that they are non-parametric models. We do not want to treat the ANN as a “black box”, in the sense that no analysis of the characteristics

and properties of the estimated networks is performed and no explanation is given as to why these models perform quite well in the forecasting exercise.

The specific types of ANN estimated in this study are  $MLP(p,q)$ ,  $JCN(p,q)$  and  $Elman(p,q)$  as discussed in Section 2. The architecture of these models includes one hidden layer and various hidden units or elements of the single hidden layer ( $q$ ). The output variable is the daily stock return. The input variables selected in the input layer include lagged stock index returns,  $p$  (the number of lags in the autoregressive part), and are scaled assuming a uniform distribution within the interval  $[-1;1]$ . The  $p$ -order lagged returns are calculated by sequential validation, and so we estimate ANN models with different values of  $p$  and  $q$ . The rank of the terms employed is  $p,q=1,\dots,5$ . The inclusion of these lags is based on the evidence in Section 4.1 that suggests lagged returns are needed in the conditional mean specification, while autocorrelation in the stock index returns can appear because of non-synchronous trading effects. Moreover, a link was introduced between the input variables and the output variable. As there is no reliable method of specifying the optimal number of hidden layers, we specified one hidden layer. This choice was made because many studies that carry out sensitivity analysis to determine the optimal number of hidden layers have found that one hidden layer is generally effective in capturing non-linear structures [see Adya and Collopy (1998) for an overview]. The hidden unit activation function  $g(\cdot)$  is the hyperbolic tangent function [see footnote 7], because it produces a better fit. We did not choose  $p$ ,  $q$  and  $g(\cdot)$  a priori.

The ANN models were trained over a training period (i.e. training sample) using 1500 training cycles and crossvalidation. The training set was used to estimate the neural network weights. To improve on the in-sample fitting performance of the ANN models, the estimated set of weights was used as a set of initial values for training. We used cross-validation strategy in training to avoid overfitting (good in-sample, but poor out-sample performance). The training phase of the ANN was performed with 1855 observations, whereas in the “test” phase 463 observations were used, both sizes being randomly determined. The two hundred final observations were set aside to make predictions. The decrease in the error rate in the training and “test” phases was then tested. The output was compared to the sample of original values of the output by comparing the root mean squared error (RMSE). We observed as the RMSE declines over successive training (i.e.,

$n$ ). When RMSE reaches a minimum and then starts increasing, this indicates that overfitting may occur. On the basis of the estimated weights from  $n$ -th training over the training period, out-of-sample forecasts were generated for subsequent “test” periods.

**Table 3.** MSE and MAE statistics of the ANN models with a single hidden layer during the training phase (period from 30-12-1989 to 17-06-97) and the “test” phase (period from 18-06-1999 to 30-04-99).

		MLP( $p,q$ )				JCN( $p,q$ )				Elman( $p,q$ )			
		Training		Test		Training		Test		Training		Test	
$p$	$q$	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
1	1	15.04	3.735	17.39	3.852	1.220	0.805	3.313	1.373	15.18	3.753	17.53	3.869
	2	1.598	0.915	3.937	1.503	1.268	0.825	3.419	1.343	1.384	0.881	3.470	1.388
	3	1.220	0.804	3.309	1.372	1.602	0.923	4.090	1.566	1.210	0.803	3.287	1.357
	4	1.194	0.800	3.219	1.328	1.183	0.793	3.218	1.334	1.204	0.799	3.249	1.356
	5	1.241	0.811	3.371	1.392	1.185	0.795	3.213	1.326	1.186	0.795	3.208	1.332
2	1	1.895	1.078	3.873	1.458	1.250	0.821	3.398	1.340	2.167	1.178	4.130	1.523
	2	1.307	0.830	3.726	1.440	1.198	0.800	3.195	1.322	3.195	1.247	9.849	2.193
	3	1.209	0.805	3.333	1.335	2.539	1.198	5.864	1.764	<b>1.186</b>	<b>0.794</b>	<b>3.193</b>	<b>1.332</b>
	4	<b>1.119</b>	<b>0.798</b>	<b>3.202</b>	<b>1.318</b>	<b>1.218</b>	<b>0.803</b>	<b>3.235</b>	<b>1.322</b>	1.190	0.797	3.205	1.325
	5	1.323	0.838	3.708	1.439	1.184	0.794	3.237	1.337	1.183	0.793	3.227	1.332
3	1	2.752	1.391	4.936	1.785	1.477	0.891	4.144	1.516	2.395	1.273	4.573	1.698
	2	1.653	0.996	3.772	1.498	1.541	0.905	3.838	1.447	1.772	1.045	3.897	1.532
	3	1.182	0.793	3.232	1.332	1.185	0.794	3.214	1.338	1.183	0.796	3.183	1.324
	4	1.350	0.846	3.858	1.461	1.213	0.803	3.345	1.365	1.260	0.818	3.590	1.411
	5	1.180	0.792	3.205	1.327	1.182	0.792	3.201	1.326	1.424	0.870	3.595	1.371
5	1	1.190	0.796	3.249	1.344	2.304	1.193	5.020	1.683	1.181	0.793	3.224	1.330
	2	2.620	1.352	4.759	1.752	1.570	0.914	4.258	1.548	1.945	1.114	4.060	1.579
	3	1.413	0.869	3.845	1.452	1.280	0.831	3.546	1.397	1.188	0.796	3.314	1.353
	4	1.594	0.917	4.497	1.544	1.538	0.907	4.257	1.512	1.292	0.836	3.666	1.409
	5	1.422	0.861	3.987	1.490	1.178	0.793	3.225	1.330	1.196	0.798	3.209	1.322

*Note:* Bold type denotes the MAE and MSE in the training and test phases which correspond to the best MSE in the out-of-sample phase.

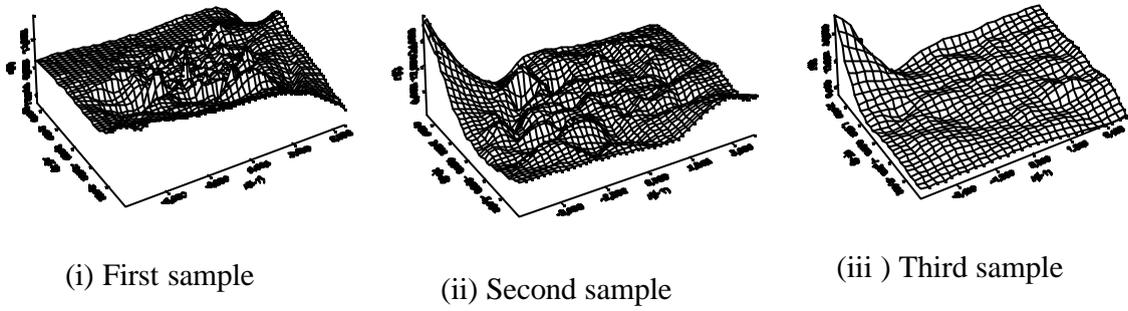
Table 3 shows the final results in the training and “test” phases in the last iteration for mean squared error (MSE) and mean absolute error (MAE) statistics. The estimates of network patterns present the following aspects. In terms of the minimum MSE in the out-of-sample phase, the best adjusted model holds two-explanatory variables,  $p=2$  (i.e. the one-period and two-period lagged stock index returns,  $r_{t-1}$  and  $r_{t-2}$ , as ESTAR and LSTAR estimated models in Section 4.1), and  $q=4$  hidden units in the single hidden layer in Eq. [5] and Eq. [6], and  $q=3$  in Eq. [7]. We can write these models as MLP(2,4), JCN(2,4) and

Elman (2,3) artificial neural networks. For these selected models, the MLP model has a lower MSE and MAE than the JCN and Elman models in the training phase (within-sample). If we compare the ANN results with the AR and STAR models, the ANN models fit the within-sample data better than the other models (i.e. regarding the MSE and MAE statistics, AR(2) has 1.526 and 1.098; ESTAR(2;3) has 1.173 and 0.791; and LSTAR(2;3) has 1.171 and 0.788).

We do not report the estimated weights from training the ANN model given in Eq.[5], Eq.[6] and Eq.[7] for the training period. However, there are some similarities regarding the magnitudes and signs of the weights that appear in all these models, such as  $\hat{\mathbf{b}}_0, \hat{\mathbf{b}}_j, \hat{\mathbf{f}}_{0j}, \hat{\mathbf{f}}_{i,j}$ ,  $i=1, \dots, p, j=1, \dots, q$ .

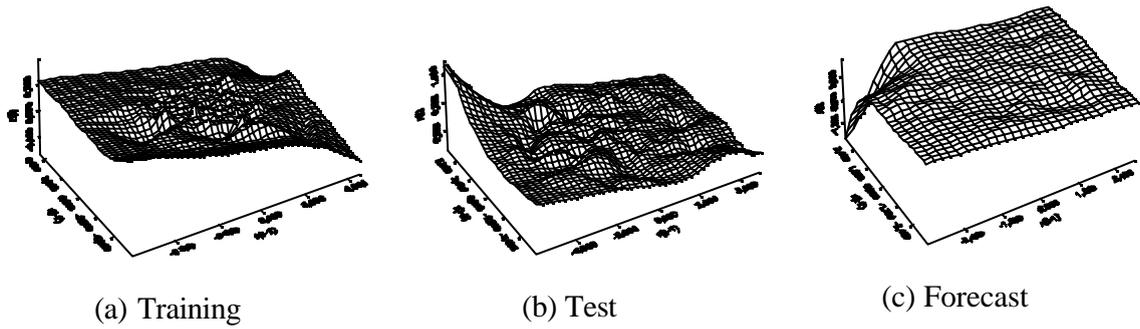
Let us consider what kind of non-linear relationships between the return and past returns are picked up by ANNs. Like Qi and Maddala (1999), to visualize what relationship between returns and the underlying predicting variables has been captured by the neural network, we report the results of sensitivity analysis and compare it with the observed returns. As an illustrative graph of possible non-linearity, let us consider Figure 7, which plots the observed returns ( $r_t$ ) against the one-period lagged return ( $r_{t-1}$ ) and the two-period lagged return ( $r_{t-2}$ ) in three samples: (i) the first sample is similar to the training set; (ii) the second is similar to the “test” phase; and (iii) the third is equivalent to the forecast phase. Figure 8 contains various groups of graphs (Figures 8a, 8b and 8c), which show the estimated returns in the training, “test” and forecast phases from the neural network for MLP (Figure 8a), JCN (Figure 8b) and Elman (Figure 8c) against the observed  $r_{t-1}$  and  $r_{t-2}$ . In Figures 8a, 8b and 8c, case (c) plots the simulated stock return ( $r_t$ ) from the neural network for MLP, JCN and Elman in the forecast phase against  $r_{t-1}$  and  $r_{t-2}$ . From these graphs, we can observe a complex non-linear relationship between returns and lagged returns, showing that this series displays a cyclical behaviour around points that shift over time when these shifts are endogenous, i.e., caused by past observations on  $r_t$  themselves, which can be viewed as a typical feature of non-linear time series. MLP and Elman ANNs perform better than JCN.

**Figure 7.** Observed returns.

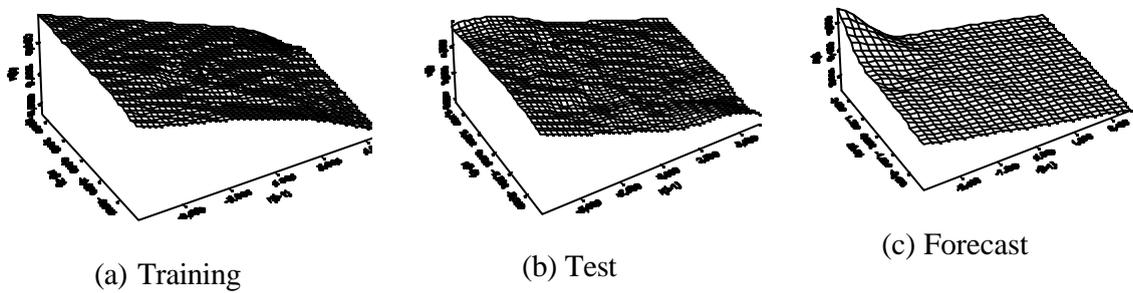


**Figure 8.** Sensitivity analysis

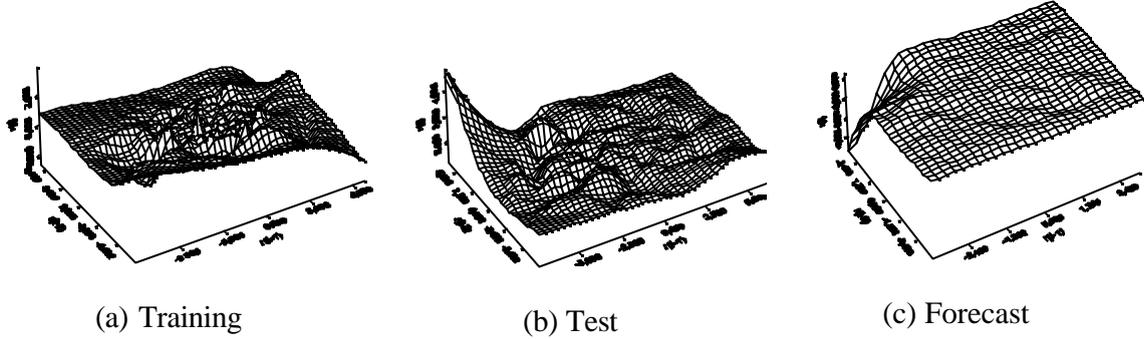
*Figure 8a.* MLP(2,4) estimated returns and lagged observed returns.



*Figure 8b.* JCN(2,4) estimated returns and lagged observed returns.



*Figure 8c.* Elman(2,3) estimated returns and lagged observed returns.



The better fit of the neural network model reported above is not surprising given its universal approximation property.

## 5. Statistical assessment of the out-of-sample forecast

This section focuses on the out-of-sample forecasting ability of the STAR and ANN models in terms of statistical accuracy. The randomly selected prediction period corresponds to the last 200 periods of the sample. This forecast period was from 3 May 1999 to 10 February 2000. One-step-ahead forecasts were generated from all models.

It is generally impossible to specify a forecast evaluation criterion that is universally acceptable. In order to assess the predictive ability of the different models, we use various statistics of prediction accuracy. The measures of accuracy used in this paper are based on  $h=1, \dots, H$  prediction periods for  $r_h$ , called  $\hat{f}_h$ . Although ANN is expected to have a superior in-sample performance, since it nests the AR linear model and STAR model, there is no guarantee that it will predominate in the out-of-sample period. The relationship between stock returns and lagged stock returns was investigated by comparing the predictions of AR and non-linear models that can be used for return prediction.

The forecast evaluation was made between the results from the following models for stock index returns: AR(2), LSTAR(2;3), ESTAR(2;3), MLP(2,4), JCN(2,4), and Elman(2,3), strictly for the prediction period. We did not include GARCH models in the set of forecasting models because these models parameterise the conditional variance, whereas the object to be forecast is the stock index returns, not its volatility.

We compared the out-of-sample forecasts using two different testing approaches. First, we examined the forecast accuracy from all the estimated models by calculating the MAE, mean absolute percentage error (MAPE), RMSE,  $U$ -Theil and the proportion of times the signs of returns are correctly forecasted (Table 4, Panel A). In terms of classic forecast evaluation criteria, the best results are the lowest values. As indicated in this table, the MAE, RMSE and  $U$ -Theil of the forecasts from the ANN models are lower than those of the linear model, except in the Elman net for RMSE. In terms of MAPE, AR is better than the other models. However, the signs correctly estimated are slightly superior in ANNs, with 55% success in the MLP model. This result implies that the ANN-based forecasts are in general more accurate than those of the linear and STAR models.

Second, to examine the directional prediction of changes, the forecast encompassing and to analyse whether the difference between the RMSEs is statistically significant for our out-of-sample forecasts, we employed various tests of hypotheses, such as the Pesaran and Timmermann (*DA*, 1992) test, which was used as a directional prediction test of changes. Under the null hypothesis, the real and predicted values are independent. The distribution of the *DA* statistic is  $N(0,1)$ , and it has the following structure:

$$DA = [\text{var}(SR) - \text{var}(SRI)]^{-0.5} (SR - SRI), \quad \text{where} \quad SR = H^{-1} \sum_{h=1}^H I_i[y_h \cdot \hat{y}_h > 0] \quad \text{and}$$

$SRI = p_1 \hat{p}_1 + (1 - p_1)(1 - \hat{p}_1)$ , where *SRI* is the success ratio in the case of independence between the real and predicted values under the null hypothesis. The other elements

$$\text{are: } p_1 = H^{-1} \sum_{h=1}^H I_i[y_h > 0], \quad \hat{p}_1 = H^{-1} \sum_{h=1}^H I_i[\hat{y}_h > 0], \quad \text{var}(SR) = H^{-1} [SRI(1 - SRI)] \quad \text{and}$$

$$\text{var}(SRI) = H^{-2} [H(2\hat{p}_1 - 1)^2 p_1(1 - p_1) + (2p_1 - 1)^2 \hat{p}_1(1 - \hat{p}_1) + 4p_1\hat{p}_1(1 - p_1)(1 - \hat{p}_1)]. \quad \text{The}$$

results are reported in Table 4, Panel B. At the 5% significance level these results do not reject the null hypothesis that forecasts and realizations are independent, which indicates that independence is not rejected for all the linear and non-linear models analysed.

We employed the forecast encompassing testing approach for our out-of-sample forecasts. In forecast encompassing, the criterion is that the  $i$ -th model should be preferred to the  $j$ -th model if the former can explain what the latter cannot. Let  $(f_{it}, f_{jt})$  be two competing forecasts of stock returns. When  $f_{it}$  encompasses  $f_{jt}$ , Chong and Hendry (1986)

and Clements and Hendry (1993) refer to this concept as “forecast conditionally efficiency”. The encompassing test of Chong and Hendry (1986) explores the encompassing forecast. To illustrate this, let  $e_i$  be the forecast error for model  $i$  and  $e_j$  the forecast error from model  $j$  ( $i, j = \text{AR, ESTAR, LSTAR, MLP, JCN and Elman}$ ). Given forecasts from these models, we can test the null hypothesis that neither model encompasses the other by running two regressions: the first involves regressing by ordinary least squares (OLS) the forecast error from the  $i$ -th model on the difference of forecasts between two models. Then, the equation to be estimated is:

$$e_{ih} = \mathbf{a}_1 + \mathbf{I}_1(e_{ih} - e_{jh}) + u_h,$$

thus obtaining the estimated coefficient  $\hat{\mathbf{I}}_1$ . The second involves the regression of the forecast error from the  $j$ -th model on the difference of forecasts:

$$e_{jh} = \mathbf{a}_2 + \mathbf{I}_2(e_{ih} - e_{jh}) + u_h$$

and obtaining the estimated coefficient  $\hat{\mathbf{I}}_2$ <sup>(13)</sup>.

The results appear in Table 4, Panel C. In this panel, we also consider the prediction error of the random walk,  $e_{rw}$ . In this case, we want to know if the prediction of the  $i$ -th model includes or is conditionally more efficient than the prediction of the random walk. This panel reports the  $t$ -statistics of the estimated coefficients and  $P$ -values in brackets. In this case, the standard regression-based statistic for testing the null hypothesis is corrected by using the Harvey, Leybourne and Newbold (1998) correction, because these authors have shown that the tests of forecast encompassing and equality of MSE (like the Diebold and Mariano test) are affected by the non-normality of forecast errors.

---

<sup>13</sup> Applying the standard regression-based test of the null hypothesis  $\mathbf{I}_1=0$  and  $\mathbf{I}_2=0$ , if  $\hat{\mathbf{I}}_1$  is not statistically significant and  $\hat{\mathbf{I}}_2$  is statistically significant, then we reject the null hypothesis that neither model encompasses the other in favour of the alternative hypothesis that the  $i$ -th model encompasses the  $j$ -th model. If  $\hat{\mathbf{I}}_1$  is significant and  $\hat{\mathbf{I}}_2$  is not significant, then the  $j$ -th model encompasses the  $i$ -th model. If both  $\hat{\mathbf{I}}_1$  and  $\hat{\mathbf{I}}_2$  are significant or if neither are significant, then we fail to reject the null hypothesis that neither model encompasses the other.

**Table 4.** Forecast evaluation. Statistical criteria for the linear and non-linear stock return models. Period from 3-05-1999 to 10-02-2000. H=200.

	RW	AR	ESTAR	LSTAR	MLP	JCN	Elman	
<b>Panel A: Goodness of forecast</b>								
MAE		0.9042	0.9104	0.9049	0.9000	0.9014	0.9095	
MAPE		112.77	116.31	115.22	115.99	116.90	118.93	
RMSE		1.1353	1.1417	1.1345	1.1309	1.1331	1.1410	
U-Theil	1.000	0.8821	0.8782	0.8750	0.8883	0.8779	0.8675	
Signs		0.505	0.515	0.515	0.555	0.534	0.525	
<b>Panel B: Pesaran and Timmermann test</b>								
DA		0.2066	-0.3837	-0.0292	0.7677	0.01798	0.2162	
		[0.42]	[0.65]	[0.51]	[0.22]	[0.49]	[0.41]	
<b>Panel C: Chong and Hendry test</b>								
<b>Upper triangular matrix: <math>\hat{I}_1</math> and P-value</b>								
<b>RW</b>	$\hat{I}_2$	--	0.48	0.14	0.53	0.86	0.57	0.31
			[0.40]	[0.79]	[0.33]	[0.28]	[0.37]	[0.50]
<b>AR</b>		0.52	--	-1.87	0.82	0.89	0.56	-0.45
			[0.0]	[0.10]	[0.50]	[0.24]	[0.24]	[0.58]
<b>ESTAR</b>		0.85	-2.87	--	1.88	1.38	0.81	0.75
			[0.0]	[0.01]	[0.07]	[0.05]	[0.09]	[0.38]
<b>LSTAR</b>		0.07	-0.18	0.88	--	0.80	0.52	-0.13
			[0.0]	[0.88]	[0.39]	[0.32]	[0.31]	[0.87]
<b>MLP</b>	-0.01	-0.10	0.38	-0.20	--	-0.15	0.07	
		[0.0]	[0.89]	[0.59]	[0.81]	[0.91]	[0.89]	
<b>JCN</b>	-0.05	-0.44	-0.19	-0.48	-1.15	--	0.33	
		[0.0]	[0.35]	[0.69]	[0.36]	[0.38]	[0.39]	
<b>Elman</b>	-0.11	-1.45	-0.25	-1.13	-0.93	-0.66	--	
		[0.0]	[0.08]	[0.77]	[0.16]	[0.09]	[0.09]	

Note: *P*-values appear between brackets. In the case of the Chong and Hendry test, the standard regression-based statistic for testing the null hypothesis is corrected by using the Harvey, Leybourne and Newbold (1998) expressions.

The upper triangular matrix of this panel shows *t*-statistics and *P*-values for  $\hat{I}_1$  and the lower triangular matrix shows *t*-statistics and *P*-values for  $\hat{I}_2$ . If the *P*-values of both estimated coefficients are lower than 5%, then the null hypothesis should be accepted (that neither model encompasses the other). If the *P*-value of  $\hat{I}_1$  is lower than 5% and the *P*-value of  $\hat{I}_2$  is higher than 5%, then the null hypothesis should be rejected in favour of the alternative hypothesis that the *i*-th model encompasses the *j*-th model. Finally, if the *P*-value of  $\hat{I}_1$  is higher than 5% and the *P*-value of  $\hat{I}_2$  is lower than 5%, the null hypothesis is rejected in favour of the alternative that the *j*-th model encompasses the *i*-th model. For example, as shown in Table3, Panel C, the null hypothesis is not rejected for all models. When the *i*-th model is equal to an ANN model and the *j*-th model is an AR model, this

means that the ANN model explains the forecast error of the linear model, whereas the linear model cannot explain the forecast error of the ANN. Also, comparing the random walk and the other models, we do not always reject the null hypothesis for  $\hat{I}_1$  at any significance level. In this sense, the prediction of the  $i$ -th model is conditionally more efficient than the prediction of the random walk.

Finally, we evaluated the equality of competitive forecasts by the Diebold and Mariano (DM, 1995) test, which examines whether the difference in the RMSE of the forecasts of the two models is statistically significant. Given two  $h$ -step-ahead predictors, and denoting the corresponding prediction errors by  $e_{1h}$  and  $e_{2h}$ , then the null hypothesis is tested that  $E[d_h] = 0$ ,  $d_h = f(e_{1h}, e_{2h})$  where  $d_h$  is a function of the prediction errors. These authors assume that the  $f(\cdot)$  case is of the type  $d_h = g(e_{1h}) - g(e_{2h})$ . For example, if  $d_h = e_{1h}^2 - e_{2h}^2$ , the null hypothesis is that the two forecasts have an equal mean squared error. The statistical test is based on the sample mean  $\bar{d}$ . It is  $S = [\hat{V}(\bar{d})]^{-1/2} \bar{d}$ , where  $\hat{V}(\bar{d})$  is a consistent estimator of the variance of the sample distribution of  $\bar{d}$ . The statistic has a standard normal asymptotic distribution under the null hypothesis. The consistent estimator is given by: 
$$\hat{V}(\bar{d}) = 2\mathbf{p} \hat{f}_d(0) = \sum_{t=-(k+1)}^{k-1} \mathbf{g}_d(\mathbf{t}), \quad \text{and} \quad \mathbf{g}_d(\mathbf{t}) = H^{-1} \sum_{h=|t|+1}^H (d_h - \bar{d})(d_{h-|t|} - \bar{d}).$$

However, Harvey, Leybourne and Newbold (1997, 1998 and 1999) have shown that the tests of forecast encompassing and the DM test are affected by non-normality of forecast errors and by the presence of ARCH effects. In particular, under these circumstances the tests are heavily oversized. Non-normality and ARCH are most likely to be important properties of the forecast errors in the present application to daily stock index returns. Hence, it would be useful to use the modified versions of the forecast evaluation tests developed by Harvey *et al.* (1997, 1998 and 1999), which were designed to alleviate the problem of size distortion.

Tables 5 and 6 show the corrected results of the Diebold and Mariano test ( $S$ ) for the assumptions that  $d_h = |e_{1h}| - |e_{2h}|$  and  $d_h = e_{1h}^2 - e_{2h}^2$ , respectively. We consider  $e_2$  forecast errors in columns and  $e_1$  in rows. The null hypothesis that  $d_h = 0$  is rejected at the 5% significance level in both cases. The sign of  $S$  is important. If  $S < 0$ , then  $\bar{d} < 0$ , and so the RMSE of model 1 is significantly smaller than that of the model 2 forecasts. On the

contrary, if  $\bar{d} > 0$ , then the RMSE of model 2 is significantly smaller than that of the model 1 forecasts. For example, if we use the random walk as  $e_2$  and use different forecast errors as  $e_1$  (for example, AR2, ESTAR, LSTAR, MLP, JCN and Elman), we observe that  $\bar{d} < 0$  in all cases. In this sense, the model considered as 1 always has a smaller RMSE than the random walk. Another interesting result is that the ANN models are preferred to the linear and STAR models, because the Diebold and Mariano test is positive. Also, the MLP model is preferred to the JCN and Elman models, because when we consider MLP as  $e_1$  and JCN and Elman as  $e_2$ , we have  $\bar{d} < 0$ .

**Table 5.** Diebold and Mariano test ( $\mathcal{S}$ ) with  $d_t = |e_{1t}| - |e_{2t}|$ , for the linear and non-linear model of returns. Period from 3-05-1999 to 10-02-2000. H=200.

		$e_2$						
		RW	AR2	ESTAR	LSTAR	MLP	JCN	Elman
$e_1$	RW	--	106.07	104.71	105.75	96.48	90.64	107.53
	AR2	-106.07	--	-23.87	-2.03	6.82	2.34	-16.74
	ESTAR	-104.71	23.87	--	14.77	15.67	9.01	2.17
	LSTAR	-105.75	2.03	-14.77	--	8.14	3.20	-10.90
	MLP	-96.48	-6.82	-15.67	-8.14	--	-5.57	-11.69
	JCN	-90.64	-2.34	-9.01	-3.20	5.57	--	-7.02
	Elman	-107.53	16.74	-2.17	10.90	11.69	7.02	--

Note: Critical distribution values N(0,1) are 1.645, 1.96, 2.576 at 10%, 5%, and 1%, respectively.

**Table 6.** Diebold and Mariano test ( $\mathcal{S}$ ) with  $d_t = e_{1t}^2 - e_{2t}^2$ , for the linear and non-linear model of returns. Period from 3-05-1999 to 10-02-2000. H=200.

		$e_2$						
		RW	AR2	ESTAR	LSTAR	MLP	JCN	Elman
$e_1$	RW	--	84.78	86.13	84.98	80.84	78.14	85.09
	AR2	-84.78	--	-18.67	3.97	7.41	1.48	-25.68
	ESTAR	-86.13	18.67	--	14.76	15.41	8.39	1.60
	LSTAR	-84.98	-3.97	-14.76	--	5.17	0.23	-19.60
	MLP	-80.84	-7.41	-15.41	-5.17	--	-8.81	-13.75
	JCN	-78.14	-1.48	-8.39	-0.23	8.81	--	-7.36
	Elman	-85.09	25.68	-1.60	19.60	13.75	7.36	--

Note: Critical distribution values N(0,1) are 1.645, 1.96, 2.576 at 10%, 5%, and 1%, respectively.

Thus, we conclude that in terms of classic forecast evaluation criteria, directional prediction tests and MSE, the ANN prediction slightly improves on the results of linear AR and STAR regime switching models.

## **6. Assessment the relative forecast performance with economic criteria in a simple trading strategy.**

We must consider why ANN models perform well in the forecasting exercise. In this Section, we assess the economic criteria. We could use the return forecasts from the different models in a simple trading strategy and compare the pay-offs to determine if ANNs are useful forecasting tools for an investor. As shown by Leitch and Tanner (1991) and by Satchell and Timmermann (1995), the use of statistical or economic criteria can lead to very different outcomes. In fact, the correlation between MSPE and trading profits, for example, is usually quite small, and the performance of a particular model in terms of DA is often a better indicator of its performance in a trading strategy. Given that the present paper finds that neural networks do not perform much better than linear and STAR models in terms of DA, we would not find it surprising if it turned out that ANNs do not offer significantly higher trading profits. Finally, it would also be useful to examine the impact of transaction costs on the profits of trading strategies.

As pointed out by Satchell and Timmermann (1995), standard forecasting criteria are not necessarily particularly well suited for assessments of economic value of predictions of a non-linear process.

In order to assess the economic significance of predictable patterns in the Ibox-35 series, it is necessary to consider explicitly how investors may exploit the computed local predictions as trading rules.

The trading rules considered in this paper are based on a simple market timing strategy, consisting of investing total funds in either the stock market or a risk-free security. The forecast from each predictor is used to classify each trading day into periods “in” (earning the market return) or “out” of the market (earning the risk-free rate of return security). The trading strategy specifies the position to be taken the following day, given the current position and the “buy” or “sell” signals generated by the different predictors. On the one hand, if the current state is “in” (i. e., holding the market) and the share prices are expected to fall on the basis of a sell signal generated by one particular predictor, then shares are sold and the proceeds from the sale invested in the risk-free security (earning the risk-free rate of return  $r_{ft}$ ). On the other hand, if the current state is “out” and the predictor indicates that

share market prices will increase in the near future, the rule returns a “buy” signal and then the risk-free security is sold and shares are bought (earning the market rate of return  $r_{ft}$ ). Finally, in the other two cases, the current state is preserved (Fernández, Sosvilla and Andrada, 2002).

The trading rule return over the predicted period of 1 to  $H$  can be calculated as:

$$r = \sum_{h=1}^H r_h \cdot I_{bh} + \sum_{h=1}^H r_{fh} \cdot I_{sh} + n \cdot \log\left(\frac{1-c}{1+c}\right)$$

where  $r_h$  is the market rate of return constructed over the closing price (or level of the Ibex-35 stock index,  $P_h$ ) on day  $h$ ;  $I_{bh}$  and  $I_{sh}$  are indicator variables equal to one when the predictor signals are to buy and sell, respectively, and zero otherwise, satisfying the relation  $I_{bh} \cdot I_{sh} = 0, \forall h \in [1, H]$ ;  $n$  is the number of transactions; and  $c$  denotes the one-way transaction costs (expressed as a fraction of the price). Regarding the transaction costs, results by Sweeny (1988) suggest that large institutional investors could achieve in the mid-1970s one-way transaction costs in the range of [0.1-0.2%]. Even though there have been substantial reductions in costs in recent decades, we initially used one-way transaction costs of 0.15%. We also investigated the robustness of the results with transaction costs of 0.25%.

In order to assess profitability, it is necessary to compare the return from the trading rule based on the predictors to an appropriate benchmark. To that end, we constructed a weighted average of the return from being long in the market and the return from holding no position in the market and thus earning the risk-free rate of return. The return on this risk-adjusted buy-and-hold strategy can be written as:

$$r_{bh} = \mathbf{b} \sum_{h=1}^H r_h + (1-\mathbf{b}) \sum_{h=1}^H r_{fh} + 2 \cdot \log\left(\frac{1-c}{1+c}\right)$$

where  $\mathbf{b}$  is the proportion of trading days that the rule is in the market.

In this paper we combine a simple and popular trading strategy known as the filter technique, originally analysed by Alexander (1961) and Fama and Blume (1966), with parametric and non-parametric forecasts, and compare the return obtained with this risk-adjusted buy-and-hold strategy. In the empirical implementation, we modified the simple rule by introducing a filter in order to reduce the number of false buy and sell signals, by

eliminating “whiplash” signals when one selected predictor at date  $t$  is around the closing price at  $t-1$ . This filtered rule will generate a buy (sell) signal at date  $t$  if the predictor is greater than (is less than) the closing price at  $t-1$  by a percentage  $\mathbf{d}$  of the standard deviation  $\mathbf{s}$  of the return time series from  $1$  to  $t-1$ . Therefore, if  $\hat{r}_h$  denotes the prediction for  $r_h$ :

- If  $\hat{r}_h > r_{h-1} + \mathbf{d} \cdot \mathbf{s}$  and we are out of the market, a buy signal is generated. If we are in the market, the trading rule suggests we should continue holding the market.
- If  $\hat{r}_h < r_{h-1} - \mathbf{d} \cdot \mathbf{s}$  and we are in the market, a sell signal is generated. If we are out of the market, we continue holding the risk-free security.

We used a range filter percentage 0:0.025:0.3, because higher filters generate no signals.

**Table 7.** Cost of 0.15%. Different filters.

Statistics	AR2	ESTAR	LSTAR	MLP	JCN	Elman
<b>Panel A: Filter 0.0*s</b>						
$r$	-0.00005	-0.00039	0.00009	0.00046	0.00005	-0.00002
$r_{bh}$	0.00059	0.00059	0.00058	0.00070	0.00068	0.00055
$RS_{r-a}$	-0.00714	-0.05518	0.01237	0.05190	0.00534	-0.00251
<b>Panel B: Filter 0.3*s</b>						
$r$	0.00096	0.00096	0.00096	0.00046	0.00123	0.00058
$r_{bh}$	0.00052	0.00052	0.00052	0.00021	0.00062	0.00076
$RS_{r-a}$	0.15567	0.15567	0.15567	0.34451	0.16237	0.06010

**Table 8.** Cost of 0.25%. Different filters.

Statistics	AR2	ESTAR	LSTAR	MLP	JCN	Elman
<b>Panel A: Filter 0.0*s</b>						
$r$	-0.00058	-0.00095	-0.00052	0.00003	-0.00036	-0.00054
$r_{bh}$	0.00058	0.00058	0.00057	0.00069	0.00067	0.00054
$RS_{r-a}$	-0.08084	-0.13313	-0.07477	0.00366	-0.04248	-0.08338
<b>Panel B: Filter 0.3*s</b>						
$r$	0.00095	0.00095	0.00095	0.00045	0.00122	0.00055
$r_{bh}$	0.00052	0.00052	0.00052	0.00020	0.00061	0.00075
$RS_{r-a}$	0.15403	0.15403	0.15403	0.33687	0.16104	0.05698

Given that individuals are generally risk averse, besides the excess return, we also considered a version of the Sharpe ratio (Sharpe, 1966). This is a risk-adjusted return measure given by:

$$RS_{r-a} = \frac{\bar{r}}{s_{r_b}^*}$$

where  $\bar{r}$  is the average return of the trading strategy and  $s_{r_b}^* = \mathbf{b} \cdot \mathbf{s}^*$  is the proportion of standard deviation of daily trading rule returns from being long in the market. As can be seen, the higher the Sharpe risk-adjusted ratio, the higher the mean net return and the lower the volatility returns from being long in the market.

The out-of-sample statistics with transaction costs of 0.15% and 0.25% are reported in Tables 7 and 8, respectively. In both cases we used the filter technique with extreme filters of 0.0% and 0.3% (Panels A and B, respectively)<sup>14</sup>.

As can be seen in Panel A of Table 7, we find non-negative mean returns for the out-of-sample period considered (except for the AR2, ESTAR, and ELMAN models). The MLP model of artificial neural networks yields higher mean returns than all other models. The results are similar in the Sharpe risk-adjusted ratio.

The introduction of the percentage band increases the spread between the number of buy and sell signals generated by each model. As can be seen in Panel B, we also found non-negative mean returns for the out-of-sample period considered, with all models. This panel reveals a high Sharpe risk-adjusted ratio from the MLP model, even though this gives the lower mean return. The results obtained show that the JCN model performs best.

These findings are similar to those given in Panel B of Table 8, but in Panel A we get non-negative mean returns and the Sharpe risk-adjusted ratio for the out-of-sample period was considered only for the MLP model.

In general, the filter techniques show that the MLP model improves on the mean returns from parametric specifications, which are common in financial market returns, in nine of the thirteen filters analysed. However, if we compare them with a risk-adjusted buy-and-hold strategy, we get a higher mean return for filters higher than 0.125% with both transaction costs.

## 7. Conclusions.

The present paper compares out-of-sample forecasts of daily returns for the Ibex-35 index, generated by six competing models, namely a linear AR model, the ESTAR and LSTAR smooth transition autoregressive models and three ANN models: MLP, JCN and Elman networks. We only considered the lagged returns as explanatory variables, because we wanted to analyse the dynamic characteristics of returns from the stock index. The comparison of out-of-sample forecasts was carried out on the basis of two approaches: different statistical criteria and by assessing the economic value of the predictors. In terms of the statistical criteria, we calculated various measures. First, we calculated the MAE, MAPE, RMSE, U-Theil and the number of corrected signs predicted. Second, we used the Pesaran and Timmermann test, and Chong and Hendry's forecast encompassing test. Third, we tested whether RMSE differences between the six competitors (and adding a random walk model) are statistically significant. In all cases, the results suggest that the out-of-sample ANN forecasts are more accurate than AR and STAR models. Furthermore, the ANN forecasts can explain the forecast errors of the former models. For this reason, we conclude that ANN models provide a better fit than the other models, and that MLP gives the best values.

In terms of the economic criteria in the out-of-sample forecasts, we assessed profitability by means of a simple trading strategy known as the filter technique, using a range filter percentage and trading costs. The results indicate a better fit for the ANN models, in terms of the Sharpe risk-adjusted ratio.

After assessing different statistical and economic criteria, we conclude that the return on the Ibex-35 index can be predicted by using ANN models.

---

<sup>14</sup> Complete results are available on request to the authors.

## References

- Adya, M. and Collopy, F. (1998). "How Effective are Neural Networks at Forecasting and Prediction?. A Review and Evaluation". *Journal of forecasting*, 17, 481-495.  
Reference:
- Alexander, S. (1961): "Price Movements in Speculative Markets: Trends or Random Walks". *Industrial Management Review*, 2, 7-26.
- Brock, W., Dechert, W. and Scheinkman, J. (1987): "A Test for Independence based on the Correlation Dimension". University of Wisconsin, Dept. of Economics. Revised in Brock, W., Dechert, W., Scheinkman, J. and LeBaron, B. (1996), *Econometric Reviews*, 15, 197-235.
- Campbell, J., Lo, A. and MacKinley, A. (1997): *The Econometric of Financial Markets*. Princeton University Press, Princeton, New Jersey.
- Chong, Y. and Hendry, D. (1986): "Econometric Evaluation of Linear Macroeconomic Models". *Review of Economic Studies*, 53, 671-690.
- Clements, M. and Hendry, D. (1993): "On the Limitations of Comparing Mean Square Forecast Errors". *Journal of Forecasting*, 12, 617-637.
- Clements, M. and Hendry, D. (1998): *Forecasting Economic Time Series*. Cambridge, UK: Cambridge University Press.
- Cochrane, J. (1988): "How Big is the Random Walk in GNP?". *Journal of Political Economy*, 96, 893-920.
- De Lima, P. (1998): "Non Linearities and Nonstationarities in Stock Return". *Journal of Business and Economic Statistics*, 16(2), 227-236.
- Diebold, F. and Mariano, R. (1995): "Comparing Predictive Accuracy". *Journal of Business & Economic Statistics*, 13(3), 253-263.
- Elman, J.L. (1990): "Finding Structure in Time". *Cognitive Science* 14, 179-211.
- Eitrheim, O and Teräsvirta, T. (1996): "Testing the Adequacy of Smooth Transition Autoregressive Models". *Journal of Econometrics*, 74, 59-75.
- Fama, E. and Blume, M. (1966): "Filters Rules and Stock Market Trading Profits". *Journal of Business*, 39, 226-241.
- Fama, E. and French, K. (1988): "Permanent and Temporary Components of Stock Prices". *Journal of Political Economy*, 96(2), 246-273.
- Fernández, F., Sosvilla, S. y García, D. (1997): "Using Nearest-Neighbour Predictors to Forecast the Spanish Stock Market". *Investigaciones Económicas*, 21, 75-91.
- Fernández, F., García, D. and Sosvilla, S. (1999): "Dancing with Bulls and Bears: Nearest-Neighbour Forecast for the Nikkei Index". *Japan and the World Economy*, 11, 395-413.
- Fernández, F., González, Ch. y Sosvilla, S. (2000): "On the Profitability of Technical Trading Rules based on Artificial Neural Networks: Evidence from the Madrid Stock Market". *Economic Letters*, 69(1), 89-94.  
Reference:
- Fernández, F., Sosvilla, S. y Andrada, J. (2002). "Technical analysis in foreign exchange Markets: Evidence from the EMS", forthcoming in *Applied Financial Economics*.
- García, R. and Gençay, R. (2000): "Pricing and Hedging Derivative Securities with Neural Networks and a Homogeneity Hint". *Journal of Econometrics*, 94, 93-115.
- Gençay, T. (1996): "Non-Linear Prediction of Security Return with Moving Average Rules". *Journal of Forecasting*, 15, 165-174.
- Gençay R. and Liu T. (1997): "Nonlinear Modeling and Prediction with Feedforward and Recurrent Networks". *Physica D*, 108, 119-134.
- Granger, C.W.J. (1992): "Forecasting Stock Market Proces: Lessons for Forecasters". *International Journal of Forecasting*, 8, 3-13.

- Granger, C.W.J. and Teräsvirta, T. (1993): *Modelling Nonlinear Economic Relationships*. Oxford, UK: Oxford University Press.
- Harvey, D., Leybourne, S. and Newbold, P. (1997): "Testing the Equality of Prediction Mean Squared Errors". *International Journal of Forecasting*, 13, 281-291.
- Harvey, D., Leybourne, S. and Newbold, P. (1998): "Tests for Forecast Encompassing". *Journal of Business & Economic Statistics*, 16, 254-259.
- Harvey, D., Leybourne, S. and Newbold, P. (1999): "Forecast Evaluation Tests in the Presence of ARCH". *Journal of Forecasting*, 18, 435-445.
- Hill, T., Marquez, L., O'Connor, L. and Remus, W. (1994): "Artificial Neural Network Models for Forecasting and Decision Making". *International Journal of Forecasting*, 10, 5-15.
- Hinich, M. and Paterson, D. (1985): "Evidence of Nonlinearity in Stock Returns". *Journal of Business and Economic Statistics*, 3, 69-77.
- Hornik, K., Stinchcombe, M. and White, H. (1990): "Universal Approximation of an Unknown Mapping and its Derivatives Using Multilayer Feedforward Networks". *Neural Networks*, 3, 551-560.
- Hsieh, D. (1991): "Chaos and Nonlinear Dynamics: Applications to Financial Markets". *Journal of Finance*, 46, 1839-1878.
- Kuan, C. and White, H. (1994): "Artificial Neural Networks: An Econometric Perspective". *Econometric Reviews*, 13, 1-91.
- Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Shin, Y. (1992): "Testing the Null Hypothesis of Stationary Against the Alternative of a Unit Root". *Journal of Econometrics*, 54, 159-178.
- Leitch, G. and Tanner, J. (1991): "Economic Forecast Evaluation: Profits versus the Conditional Error Measures". *American Economic Review*, 81, 580-590.
- Lo, A. and MacKinlay, A. (1988): "Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test". *Review of Financial Studies*, 1, 41-66.
- Lundbergh, S. and Teräsvirta, T. (1998): "Modelling Economic High-frequency Time Series with STAR-GARCH Models". Working Papers in Economic and Finance, n°. 291, Stockholm School of Economics.
- Moody, J. and Utans, J. (1995): "Architecture Selection Strategies for Neural Networks: Application to Bond Rating Prediction". In Refenes, A.-P.N. (ed.), *Neural Networks in the Capital Markets*, New York, Wiley.
- Pesaran, M.H. and Timmermann, P. (1992): "A Simple Nonparametric Test of Predictive Performance". *Journal of Business and Economic Statistics*, 10, 461-465.
- Pesaran, M. H. and Timmerman, A. (1995): "Predictability of Stock Returns: Robustness and Economic Significance", *Journal of Finance*, 50, 1201-1228.
- Qi, M. and Maddala, G.S. (1999): "Economic Factors and the Stock Market: A New Perspective". *Journal of Forecasting*, 18, 151-166.
- Satchell, S. and Timmermann, A. (1995): "An Assessment of the Economic Value of Non-linear Foreign Exchange Rate Forecasts". *Journal of Forecasting*, 14, 477-497.
- Schnader, M. and Stekler, H. (1990): "Evaluating Predictions of Change". *Journal of Business*, 63, 99-107.
- Sharda, R. and Patil, R. (1992): "Connectionist Approach to Time Series Prediction: An Empirical Test". *Journal of Intelligent Manufacturing*, 3, 317-323.
- Scheinkman, J. and LeBaron, B. (1989): "Nonlinear Dynamics and Stock Returns". *Journal of Business*, 62, 311-338.
- Sharpe, W. (1966): "Mutual Fund Performance". *Journal of Finance*, 39, 119-138.
- Surkan, A. and Singleton, J. (1990): "Neural Networks for Bond Rating Improved by Multiple Hidden Layers". International Joint Conference on Neural Networks, Vol.2, San Diego, CA (IEEE, New York), 157-162.

- Swanson, N. and White, H (1995): "A Model-Selection Approach to Assessing the Information in the Term Structure Using Linear Models and Artificial Neural Networks". *Journal of Business & Economic Statistics*, 13(3), 265-275.
- Swanson, N.R. and White, H. (1997): "Forecasting Economic Time Series Using Flexible Versus Fixed Specification and Linear Versus Nonlinear Econometric Models". *International Journal of Forecasting*, 13, 439-461.
- Sweeney, R. J. (1988): "Some New Filter Rule Tests: Methods and Results". *Journal of Financial and Quantitative Analysis*, 23, 285-300.
- Tam, K. and Kiang, M. (1992): "Managerial Applications of Neural Networks: The Case of Bank Failure Predictions". *Management Science*, 38, 926-947.
- Teräsvirta, T. (1994): "Specification, Estimation and Evaluation of Smooth Transition Autoregressive Models". *Journal of the American Statistical Association*, 89, 208-218.
- Teräsvirta, T., Tjøstheim, D. and Granger, C.W.J. (1994): "Aspects of Modelling Nonlinear Time Series". En *Handbook of Econometrics*, Vol. IV, Cap. 48, 2919-2957. Eds. R.F. Engle and D.L McFadden. Elsevier Science.
- White, H. (1987): "Some Asymptotic Results for Back-Propagation". Proceedings of IEEE First International Conference on Neural Networks, San Diego, New York: IEEE, Press, III, 261-266.
- White, H. (1988): "Economic Prediction Using Neural Networks: The Case of IBM Daily Stock Returns". Proceedings of the IEEE International Conference on Neural Networks, San Diego, California.
- Zhang, G.; Patuwo, E. and Hu, M.Y. (1998): "Forecasting with Artificial Neural Networks: The State of the Art". *International Journal of forecasting*, 14, 35-62.

**APPENDIX I**  
**UNIT ROOT, MEAN REVERSION AND LINEARITY TESTS**

**Table AI-1.** Conventional unit root tests. Period from 30-12-1989 to 10-02-2000.

Series	ADF(p=4)		PP(l=8)		KPSS(l=8)	
	Constant	Trend	Constant	Trend	Constant	Trend
Log I <sub>t</sub>	0.7974	-2.2433	-0.8302	-2.2154	24.517	5.0945
R <sub>t</sub>	-22.055 <sup>a</sup>	-22.147 <sup>a</sup>	-44.901 <sup>a</sup>	-44.948 <sup>a</sup>	0.4111 <sup>b</sup>	0.0475 <sup>b</sup>
1%	-3.4360	-3.9671	-3.4360	-3.9671	0.739	0.216
5%	-2.8632	-3.4142	-2.8632	-3.4142	0.463	0.146

*Note:* The final two lines show the critical values of the ADF and PP tests obtained by MacKinnon, and the critical values of the KPSS test. The number of lags  $p$  for the ADF test was decided on the basis of the information criterion of

Akaike, while the cut-off lag selected – to estimate the Newey-West variance – was  $l = \text{int} \left[ 4(T/100)^{2/9} \right]$ , where

$\text{int}$  is the integer of the number that does not exceed the argument. The returns derived from Ibx35 are denoted by **a** the rejection of the null hypothesis of the ADF and PP tests. **b** denotes the non-rejection of the null hypothesis of stationarity in the KPSS test.

**Table AI-2.** Values of the variance-ratio statistic for various  $q$  [VR( $q$ )]. Period from 30-12-1989 to 10-02-2000. T=2520 observations.

VR( $q$ )	Values of $q$ (days)								
	2	5	10	15	30	45	60	90	180
	0.9043	0.8839	0.8748	0.8233	0.7314	0.7177	0.7212	0.7364	0.8464
	(-4.80)	(-2.66)	(-1.86)	(-2.09)	(-2.19)	(-1.86)	(-1.58)	(-1.22)	(-0.50)

*Note:* The statistic value  $z(q)$  is given between brackets. The null hypothesis is that log I<sub>t</sub> is a random walk. The critical values of the N(0,1) distribution are 1.96 (5%) and 2.576 (10%).

**Table AI-3.** BDS test ( $V_{e,m}$ ) applied to the residuals of the ARIMA(2,1,0) model and “shuffled” residuals. Period from 30-12-1989 to 10-02-2000.

	$\varepsilon=0.5\sigma$	$\varepsilon=0.75\sigma$	$\varepsilon=\sigma$	$\varepsilon=1.25\sigma$	$\varepsilon=1.5\sigma$	$\varepsilon=1.75\sigma$	$\varepsilon=2\sigma$
$V_{e,m}$ Residuals							
m=2	7.8404	8.2212	8.8818	9.7587	10.881	11.943	12.950
m=3	11.177	11.474	12.013	12.805	13.811	14.755	15.647
m=4	15.024	14.847	14.978	15.471	16.117	16.745	17.369
m=5	18.654	17.903	17.429	17.483	17.773	18.136	18.565
m=6	24.188	22.037	20.520	19.759	19.420	19.364	19.523
m=7	30.773	26.367	23.708	21.955	10.885	20.343	20.204
m=8	39.572	31.219	27.211	24.280	22.371	21.307	20.843
$V_{e,m}$ “shuffled” residuals							
m=2	-0.4842	-0.7235	-1.0446	-1.0478	-1.0067	-0.8652	-0.4988
m=3	-0.3970	-0.0621	-0.9401	-0.9087	-0.8017	-0.6099	-0.2802
m=4	-0.0027	-0.2828	-0.5809	-0.4720	-0.3162	-0.1315	0.1551
m=5	0.1409	-0.0571	-0.2599	-0.1457	0.0006	0.1641	0.3998
m=6	0.0466	0.0494	-0.0091	0.2312	0.4054	0.5754	0.7738
m=7	-0.3157	-0.0593	-0.1203	0.2543	0.5042	0.7394	0.9674
m=8	-0.8812	-0.3359	-0.0064	0.4339	0.7234	0.9813	1.2159

*Note:* The critical values of the N(0,1) distribution are 1.645, 1.96 y 2.576 at 10%, 5%, and 1%, respectively.