Incentives among heteregeneous workers*

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Abstract

The Total Quality Control stream has shed the light on the impact of the quality of the work on the efficiency of the firm. The choice of quality has thus become a striking issue for the firms. We model a situation in which employees have to choose a certain level of quality of their work. Moreover agents are assumed to be heterogeneous with respect to their cost of providing a certain level of quality. In this paper, we analyze the impact of this heterogeneity on firms decisions of rewards when information either on the choice of quality and on the cost functions is private.

We show in a benchmark case that when there is no adverse selection problem, firms optimally choose a reward incentive scheme which leads to a unique equilibrium. This result is obtained without making any demanding assumption on the convexity of the problem. Introducing an asymmetry of information on agents' heterogeneity does not allow the firms to use the same contract. The agents have to be given incentives in order to avoid them mimicking the others. As a consequence a higher quality of work is obtained at equilibrium.

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1 Introduction

In this paper we are looking at a situation in which there exist asymmetries of information between competitive firms on the one hand, and their employees, on the other hand. These asymmetries of information, as put forward in the literature, induce inefficiencies. In our framework, employees have to choose the level of quality of their work, while the firms, which profits depend on this choice, do observe neither the effort made to choose a certain level of quality, nor the costs the agents have to pay to achieve this particular level of quality. We show that firms cannot get rid of the heterogeneity of agents with respect to the cost functions. Moreover the equilibrium is such that a lower level of quality of the work is achieved which is detrimental to social welfare. Our starting point is the Total Quality movement of the 1980-s, either from the United States with the Total Quality Control by Feigenbaum (1983), or from the Japan with the Company Wide Quality Control by Ishikawa. This movement answered to the willingness to improve the quality of the good or service, i.e. the ability of the good to fit the needs of the clients. More than improving the intrinsic quality of the product, the idea is to improve the organization itself in eliminating the costs of non-quality inherited from the Taylor model. The notion of quality is heterogeneous and encompasses many ideas and concepts. The main insight is to manage to control the different activities of the firm: contacts with the users (external customer), relationships within the firm (internal customer), computation of the costs, self-control. We want to focus here on the labor force component. By their motivation, their attention to work, their behavior in general, employees influence clearly the quality. As a consequence, the firm might have to develop incentives schemes to induce them to take decisions that achieve the optimal level of quality. But this level of quality that can be chosen by employees is subject to strong asymmetry of information. The level of quality is indeed difficult to observe.

We model a situation in which agents choose the quality of their work. This parameter of quality is not observable, nor verifiable. Increasing the quality improves the chance of the firm to be efficient. Agents are also heterogeneous with respect to their initial love for quality: agents value a priori more or less the quality in their organization. For a given agent the quality is supposed to have less and less value to him when increasing. Another way of understanding this possible heterogeneity among agents is to talk about a cost of providing a certain level of quality. Indeed, agents may have different abilities to achieve a certain level of quality. For the rest of the paper, we will consider for the ease of comprehension, the definition of heterogeneity with respect to the cost to provide a given level of quality.

We analyze the impact on the firm of this heterogeneity and non observability of the level of quality. The heterogeneity of the agents is a problem: it is not possible to find equilibrium such that the first best is achieved for each of the different employees. The distinction between the level of quality chosen and the valuation of the quality for the agent is important to better understand the impact of the adverse selection when types are endogenous. This formalization is in the spirit of the way the labor market literature has chosen to model the labor force in the explanation of the reduction in the working time (Calmfors and Hoel [1989], Askenazy [2001]). Indeed the idea is to split the stock of the labor into two different and independent variables: the time of working on the one hand, and the number of workers on the other hand. The firm is then more flexible to choose its internal organization: the reduction of the working time imposes constraint on the number of hours each employee can make in a day, but the counterpart is the relative flexibility (through bargaining) in choosing at what time in the day or in the year the work has to be done. Introducing an element of quality in the definition of the work gives to the firm another degree of freedom in its adaptation. We complicate a little bit the analysis by introducing a problem of information between the firm and the employees. Thus this can allow us to derive the consequences of this new degree of freedom, but taking into account the constraint arising when the firm cannot observe the level of quality chosen.

The problem of asymmetric information has been modelled by many authors. We bring to this literature is in modelling of the interaction between the adverse selection and the moral hazard problem. The two problems are not separable as many papers (like in Laffont-Tirole [1989]), and the idea is more to analyze the impact of the properties of the valuation or cost function on the equilibrium. We model on top of this a situation in which agents are risk averse , whereas the firm is not. The idea is here to take into account the fact that exchanges are profitable between the two types of agents since the firm is able to take all the risk. Moreover transfers between agents (the contract) can be interpreted as a wage, that we allow to be contingent to the realization of the outcome. The First Best may not have the same notion that in Rothshild Stiglitz [1976] because we model a situation in which the effort of quality is continuous. For this reason, the first best in the pure moral hazard case may not be in the full insurance line, i.e. a contract in which the payments are independent of the realization of the outcome. As a consequence, we might have the intuition that first best can be achieved without taking into account the adverse selection problem.

The paper is also related to the paper of Araujo and Moreira [1998] and to the work of Jullien, Salanié, Salanié [2000]. In the first paper the authors analyze a problem of insurance under asymmetric information model in which the single property condition does not hold. They find an equilibrium in which profits are positive . The authors of the second paper show that the power of incentives is always decreasing with risk aversion. Nevertheless in both papers, they have assumed that the problems of information are completely separable by considering exogenous heterogeneity of the aversion towards the risk. In our model, agents are all identical included the Von Neuman Morgenstern utility function except for their disutility to make the effort. We analyze in this model the impact of the properties of the cost function on the equilibrium characteristic. Besides, when the cost function is linear by parts, our model is isomorphic to the one of Chassagnon-Chiappori [1997]. When cost functions are C1 it is possible to classify the cases whether the preferences exhibit the single crossing properties.

Section 2 introduces the model. In section 3, we analyze a benchmark case in which firms know the costs function of their workers. Firms design contracts in which the reward of the workers are depending on the realization of the output. This is enough to achieve the first best level of quality of the work. The results are interesting since they are obtained without any demanding assumption on the convexity of the problem. Making the assumption that firms know the costs functions of providing a certain level of quality is obviously not realistic. In section 4 we thus relax this assumption. We show that the equilibrium contract obtained in section 3 is not more implementable. Firms suffer from this asymmetry of information on the way workers chose the quality, and as a consequence, a lower level of quality is achieved at the equilibrium. Finally, section 5 concludes.

2 The model

Quality of work affects the output of employees in the following way: they produce a random output of monetary value W_1 or W_2, $W_1 < W_2$, the occurence of the idiosyncratic risk depending on the effort chosen. We suppose basically that while the level of output is observable, the level of effort is not verifiable: it is then welfare improving for employers to share the risk with employees, so that employees remuneration (x_1, x_2) is output dependent.

2.1 Agents' risky output depending on effort

In this paper, employees are identical respect to their VNM utility function and to the technology of effort available: for identical level of effort, they expect identical random output. They only differ with respect to the disutility of effort.

We suppose that employees are risk averse and that their utility is the sum of the disutility of effort plus the expected utility of wage. All agents have the same VNM utility function u defined on $[0, +\infty[$, twice differentiable, strictly increasing and strictly concave.

Effort $e \in [0, 1]$ modifies the probability of the bad state of nature. the lowest effort induces a relatively high frequency q of bad state 1 while the highest, a lower frequency p. We suppose that agents can mix between zero and full effort. When an agent exerts any effort $e \in [0, 1]$, the probability of state 1 decreases and becomes $p(e) = e\ p + (1 - e)\, q$ (probability of state 2 is $1 - p(e)$). Then, the more effort agent choose, the greater the output and the expected output mean. The technology of effort $p(e)$ is the same for all employees.

Costs are modelled as additive disutility of effort. We suppose that there are different types in the economy, $T \in \{G, B\}$, characterized by their cost function $c_T(e)$.

Here we model a situation in which agents are not indifferent to the quality of their work, but in which high quality level is painful. Then it is natural to suppose that marginal cost of effort is increasing, negative for low level of effort (which means that for low quality level, employees are naturally induced to increase their effort) and positive for high level of effort. We denote by $\underline{e}_T$, the pivotal effort such that marginal cost is zero.

For sake of simplicity, we suppose in our paper that marginal cost is linearly increasing, however, in the last section we prove that our results are robust to modifications of this assumption. Then, effort cost is fully described, for each type, by two parameters, $\underline{e}_T$, the effort level such that marginal cost is zero, and c_T, the slope of the marginal cost function.

Assumption 1 *Cost is non observable, and cost function is type dependant, characterized by* $\underline{e}_T$ *and* c_T *such that*

$$c'_T(e) = c_T\ (e - \underline{e}_T) \tag{1}$$

Then, given a level of effort, utility is the difference between expected utility of outcome minus effort cost:

$$U_G(x_1, x_2, e) \quad = p(e)\ u(x_1) + (1 - p(e))\ u(x_2) - \frac{1}{2}\ c_G\,(e - \underline{e}_G)^2 \tag{2}$$

$$U_B(x_1, x_2, e) \quad = p(e)\ u(x_1) + (1 - p(e))\ u(x_2) - \frac{1}{2}\ c_B\,(e - \underline{e}_B)^2 \tag{3}$$

2.2 Choice of the Optimal level of effort by the workers

Non observability of the effort implies that incentives are made through the random level of wage (x_1, x_2). Employers propose a remuneration scheme (x_1, x_2) and employees choose their optimal level of effort.

The timing of our model is the following. At date 0, competing employers propose a remuneration scheme (x_1, x_2) and each agent choose an effort. In our framework, although effort cannot be monitored, it may be anticipated as we assume that agents behave rationally. At date 1, the realization of output is known and the employees are paid contingent on that realization.

Definition 1 *Anticipated effort of type* T *is the optimal effort chosen by type* T *employees when they get contract* $x = (x_1, x_2)$ *(*$T \in \{G, B\}$*). We denote it* $e_T(x)$*.*

The employees optimal effort is the solution of an arbitrage between a high level of effort increasing the expected utility of remuneration (particularly if $x_1 < x_2$) and the disutility of effort. Without loss of generality we restrict our attention to remuneration scheme such that $x_1 \leq x_2$. Then, under assumption 1, the anticipated effort of agent is either characterized by the equality between the marginal disutility of effort $c'_T(x)$ and the marginal expected utility due to effort either equal to one when the marginal cost is always lower to the marginal expected utility. In the first case, the employee optimal level of effort does satisfy equation

$$c'_T(e) = (q-p)\ (u(x_2) - u(x_1)) \tag{4}$$

which implies that whatever the case that

$$e_T(x) = \min\left(1, \underline{e}_T + \frac{1}{c_T}\ m(x_1, x_2)\right) \tag{5}$$

with notation $d(x_1, x_2) = (q-p)\,(u(x_2) - u(x_1))$

Last equations show that the higher the distance between x_2 and x_1 (measured through the difference $u(x_2) - u(x_1)$), the higher the incentive to make an effort.That measure associated to any contract can be related to what is called coverage in the insurance literature. More coverage induces to making more effort. Notice that the coverage measure that we provide here is type independent and implicitly defines the location in which effort is constant: In space $x_1 - -x_2$ the set of contract for which effort is constant for any level of effort, for any type is one of the curves of equation $d(x_1, x_2) = d$ $(d \in \mathbb{R}_+)$.

2.3 Indirect objective functions

It is natural in our framework to write the objective functions once the effort has been endogenized. We parametrize the objective functions of employees and employers as indirect utility and indirect profit obtained when this endogenous effort is taken in account. However, as we will see, these functions do not satisfy in general properties of convexity which makes them difficult to analyze.

Agent's utility depends both on the remuneration scheme they get and on the effort they choose.

Definition 2 *Anticipated utility function of type T agents is the utility that type T agents get with contract $x = (x_1, x_2)$ after they have chosen the optimal level of effort. We denote it $V_T(x)$:*

$$V_T(x) = p(e_T(x))\ u(x_1) + (1 - p(e_T(x)))\ u(x_2) - \frac{1}{2}\ c_T\ (e_T(x) - \underline{e}_T)^2 \tag{6}$$

The envelop theorem applies and it follows that the derivative vector of the anticipated utility function is the derivative vector of the utility function computed for the optimal level of effort:

Lemma 1 *For each type T, function $V_T(.)$ is differentiable and its derivative is*

$$\nabla_x V_T(x) = \nabla_x U_T(x_1, x_2, e_T(x)) = \Big(p(e_T(x))\ u'(x_1), (1 - p(e_T(x)))\ u'(x_2)\Big) \tag{7}$$

Proof. see Appendix. ■

Indifference curves are decreasing in space $x_1 - -x_2$. In the general case, it is not possible to say more without making strong assumptions on the VNM utility functions. For instance in the CARA case, we deduce from Jullien, Salanié and Salanié [2000] paper that indifference curves are convex. We will not follow this line of reasoning, and analyze the model even if there is no convexity ingredient for the indifference curves. We will show that interesting results can be derived even though such properties are not verified.

Employers payoffs depends upon the endogeneous level of effort of the agents. We define anticipated profit function all over the contract domain. Employers are assumed to be risk neutral.

Definition 3 *Anticipated profit function relative to type T is the employer expected profit when type T agent get contract x ($T \in \{G, B\}$). We denote it $\Pi_T(x)$:*

$$\Pi_T(x) = p(e_T(x))\ (W_1 - x_1) + (1 - p(e_T(x)))\,(W_2 - x_2) \tag{8}$$

This function is not linear, because of the non linearity of the effort $e_T(x)$. Moreover, it is not even sure that this function is decreasing with each coordinate. There are in fact many reasons for which the partial derivative relative to x_2 should be positive while the partial derivative relative to x_1 should be negative; that happens in particular when the effort is lower than 1. Indeed

$$\frac{\partial\,\Pi}{\partial x_2} = -(1 - p(e_T(x))) + \frac{\partial\ p(e)}{\partial\, x_2}[(W_1 - x_1) - (W_2 - x_2)] \tag{9}$$

The ambiguity comes from the fact that $\frac{\partial\ p(e)}{\partial\, x_2} = (p - q)\,\frac{\partial\ e(x)}{\partial\, x_2} < 0$ (from equation 5) and $(W_1 - W_2) - (x_1 - x_2) < 0$). In fact, increasing x_2 could increase the incentive to make an effort in such proportion that the gain due to probability change is greater than the loss due to the increase of the second coordinate.

Meanwhile the first partial derivative is never positive. In the case of an endogeneous effort less than 1, this derivative is negative as $\frac{\partial\, p(e)}{\partial\, x_2} > 0$

$$\frac{\partial\,\Pi}{\partial x_1} = -(1 - p(e_T(x))) + \frac{\partial\, p(e)}{\partial\, x_1}[(W_1 - x_1) - (W_2 - x_2)] \qquad (10)$$

Increasing x_1 lowers the incentive to make an effort and lowers the returns to employers.

3 Pure Moral Hazard Equilibrium Risk Allocations

In this section, we derive the standard properties of the contracts offered to employees at the symmetric information equilibrium, when types are observable and contractible while effort is not observable. In particular, we prove first that those contracts are located on the zero iso-profit line and then that they are unique. This benchmark case is called *"Pure Moral Hazard"* (PMH).

Competition induces employers to propose contracts which maximize employees utility among contracts making non negative profits. The difficulty in our analysis, with endogeneous effort, is that maximization problem is not convex: there is no convexity property holding either for the indifference curves or for the iso-profit curves.

We prove however that PMH implies zero profit, and then that the restriction of the indirect utility function restricted to the set of contracts making zero profit is quasi-concave. For that purpose, it is enough to prove that the determinant of the jacobian $(\nabla\, V_T(x), \nabla\, \Pi_T(x))$ is negative, null and positive when moving along the curve $\Pi_T(x) = 0$ by increasing the second coordinate[1]. The following analysis might seem tricky, but it is sufficient to get both existence and uniqueness of the PMH without introducing very restrictive conditions on the VNM function.

[1]Starting from a point x such that $\Pi_T(x) = 0$, consider a perturbation $(d\,x_1, d\,x_2)$ such that $x + dx$ remains on the zero profit curve, that is: $d\,x_1 = -\frac{\partial\,\Pi_T/\partial\,x_2}{\partial\,\Pi_T/\partial\,x_1}\, d\,x_2$. Then at the first order the variation of the indirect utility function is:

$$\nabla\, V_T(x) \cdot (d\,x_1, d\,x_2) = -\frac{d\,x_2}{\partial\,\Pi_T/\partial\,x_1}\left(\frac{\partial\, V_T}{\partial\,x_1}\,\frac{\partial\,\Pi_T}{\partial\,x_2} - \frac{\partial\, V_T}{\partial\,x_2}\,\frac{\partial\,\Pi_T}{\partial\,x_1}\right) \qquad (11)$$

which is positive null and negative when the determinant of the jacobian $(\nabla\, V_T(x), \nabla\, \Pi_T(x))$, is positive, null and negative when moving along the zero profit curve by increasing x_2 (record that $\partial\,\Pi_T/\partial\,x_1 < 0$.

3.1 Basic Properties of the Pure Moral Hazard Equilibrium

The set of contracts being compact, and the indirect utility and profit function being continuous, there exist a contract maximizing employees indirect utility without violating the profitability constraint of employers.

Lemma 2 *Given type T, let consider any contract $x^* = (x_1^*, x_2^*)$ traded at the equilibrium when types are observable and when effort is not verifiable. It must satisfy the following properties:*

1. $\Pi_T(x_1^*, x_2^*) = 0$

2. $x_1^* \leq x_2^*$

3. $e^* = e_T(x^*) \in]\underline{e}_T, 1[$

4. $p(e^*)\,(1-p(e^*))\left(\frac{1}{u'(x_1^*)} - \frac{1}{u'(x_2^*)}\right) + \frac{(q-p)^2}{c_T}\,((W_2-W_1)-(x_2-x_1)) = 0$

Proof. see Appendix. ■

The fact that the zero profit constraint is binding is natural: it comes from the monotonicity of the indirect utility function. Monotonicity of net payments with i is classical: when state 1 occurs, it is more likely to signal that low effort has been taken. Then moral hazard mechanism are built to encourage agents to effectively make some effort.

The third condition states that there is always incentives to increase the effort ($e_T(x^*) > \underline{e}_T$) and that any equilibrium contract should be chosen such that effort satisfies first order conditions 4 (as $e_T(x^*) < 1$). Similar conditions was verified in Chassagnon, Chiappori [1997] paper.

Last condition is exactly the colinearity between the derivative of the indirect utility function and the indirect profit function.

3.2 Uniqueness of the PMH

As mentioned at the beginning of the section, we show now that the sign of the determinant of the jacobian $(\nabla\, V_T(x), \nabla\, \Pi_T(x))$ is positive then negative along the zero profit line when x_2 is increasing. Thus there exists only one global maximum of the indirect utility function on that line, the unique PMH. We need only one condition to prove this result: parameters p and q are greater than $\frac{1}{2}$.

Proposition 3 *When $\frac{1}{2} \leq p < q$ there is a unique equilibrium allocation of risk between employers and employees when their cost function is observable but not their effort.*

Proof. see Appendix. ■

The condition $p, q > \frac{1}{2}$ is a severe restriction. In any case, the bad state of nature frequency is greater than the good one. Efforts effects are limited.

4 Incentives when the Cost of Effort is Private Information

In this paragraph, we study risk sharing between employees and employers when employees' types are not observed by the principal. We show first that in any case, the types are separated. Then, we study how the incentives for doing an effort are modified by asymmetric information. We show in particular that the equilibrium outcome of the market competition game is such that every type receives less coverage, which increases the quality of work.

4.1 Basic properties of menus of contracts

It is not sure that the incentives that we characterize in the preceding section could be proposed by the employers to the employees when their type is private information,. It is well known from the adverse selection literature that proposing jointly both PMH contract could lead some employees to deviate and choose the contract which is for the other type, generating loss for employers.

When that kind of informational problem arises, it is usual that employers propose pairs of contracts that satisfy revelation constraints: each type prefers the contract which is designed for him to the contract proposed to the other type.

Definition 4 *A pair of contracts (x_G, x_B) is a menu if the contracts are globally profitable when chosen respectively by type G and B and if they satisfy* *both type revelation constraints:*

$$\frac{1}{2}\ \Pi_G(x_G) + \frac{1}{2}\Pi_B(x_B)\ \geq 0 \tag{12}$$

$$V_G(x_G) \geq V_G(x_B) \tag{13}$$

$$V_B(x_B) \geq V_B(x_G) \tag{14}$$

The first property that we examine concerns the ranking of the effort made in any menu. It does not really depend on the specification of cost functions: the more the agent is covered, the less the effort level. This result has the same flavor as what was obtained in the insurance literature. However, here, it is much more surprising, as our model allow flexibility in the definition of employees' type.

Proposition 4 *Any menu* (x_G, x_B) *do satisfy following condition:*

$$(d(x_G) - d(x_B))\,(e_G(x_G) - e_B(x_B)) \geq 0 \tag{15}$$

That result should be interpreted as follows: second best menus are built such that the one making more effort is the one which remuneration is greater in state 2 and lower in state 1. In particular, it is not consistent with revelation constraint that one agent obtain a greater remuneration in both state of nature.

4.2 Second best separating Nash equilibrium

Competition among employers is modelled as a non cooperative game in which each competitor proposes a remuneration scheme. A firm strategy is to propose a remuneration scheme. This allows, at least theoretically for lump sum transfer between types. We suppose also that firm's numbers is exogenously fixed.

In standard adverse selection model, when equilibrium exists, winners maximize employees utility and make zero profit (see for instance Hahn [1978]).

In particular, lump sum transfer do not occur at the equilibrium, because it is always possible to break strategy by proposing a better remuneration scheme to the underprivileged type and making profits. The single crossing property makes easy to deal with such a breaking strategy without violating revelation constraints.

Here in the model with moral hazard, the single crossing property could be violated and the classical argument does not apply. However, we show that even in the standard case, lump sum transfers cannot occur at the equilibrium. The direct implication is that equilibrium is always unique.

Proposition 5 *When competing firms are allowed to propose menus, the unique proposition of equilibrium is the maximal menu in the class of transfer menus.*

4.3 How incentives should be modified at the second best

One important question is to understand how incentives should be modified when the PMH equilibrium cannot be achieved. We show here that in any case, when an incentive contract is modified at the second best, that results in a higher quality of work for all types.

Our result is twofold : it is true either when indifference curve cross at least once either when they could cross more than once . First case also called " single crossing case " happens when marginal cost are ranked. Second case corresponds to parameters such that marginal cost is lower and then greater for one type than for the other one when parameter $m(x_1, x_2)$ varies.

The single crossing case gives the intuition of the result in the case type G is doing lower effort than type B at the PMH. Suppose for instance that for any level of effort marginal cost of type B is greater than the one of type G. Then type G endogeneous effort is always greater than type B 's. Graphically type G MRS is always lower than type B MRS (in absolute terms). That implies that in any menu satisfying the revelation constraint, type G is proposed a greater income contingent on state 2 (the good one) and a lower outcome relative to state 1. In other words, $d(x_G) \geq d(x_B)$ for any menu. Apply now proposition 4, type G is induced to making more effort than type B in any menu satisfying the revelation constraint, namely, $e_G(x_G) \geq e_B(x_B)$. However, as we shall see, the second best allocation is such that type B gets the same contract as in the symmetric information case. In the (intuitive) case such that PMH verify $e_G(x_G^*) < e_B(x_B^*)$, the result follows then trivially: $d(\hat{x}_G) \geq d(x_B^*) \geq d(x_G^*)$.

Next proposition characterize fully the single crossing case equilibrium when it does exist.

Proposition 6 *When increasing effort is more costly to type B than to type G employee* $[c'_G(e) \leq c'_B(e)e_E(v_1]$

type B locally prefers state 1 remuneration. When an equilibrium on the adverse selection market does exists

1. *type B makes less effort at the second list and is more (less) remunerated in state 1 (state 2) than type G.*
2. *type B obtains the PMH;*
3. *type G obtains a contract more demanding effort than the PMH.*

Proof. see Appendix. ■

We generalize the results of proposition 6 in any case, particularly in the no single crossing case. What differ when single crossing is not satisfied is that in a subspace of contracts, type G prefers locally contingent on state 1 remuneration when in the complementary subspace, that is the other type who prefers contingent on state 1 remuneration. In that case, it could be possible that only type G obtains the PMH or *vice-versa*.

in fact, due to the quadratic cost assumption, marginal cost cannot cross more than once. Take the convention when they cross that $c_B > c_G$ (second derivative of the bad is greater): type B is locally more efficient for low level of effort and then type G is locally more efficient for high level ones. At the unique level e_0 such that both marginal cost equalize, endogeneous effort and profit function equalize. Also zero profit curves intersect for that level. Contract space is then divided in two parts separated by the curve $m(x) = c'(e_0)$. Below (resp. above) type G (resp. type B) locally prefers contingent on state 1 remuneration.

There are basically two situations. Either both PMH are on the same part of the space, and then, the model works as in the single crossing case, second best contracts being in the same subspace. At the contrary, when PMH are on different parts of the space, the analysis is slightly modified. However, the way incentives are modified is the same.

We can now state the main result of the paper.

Proposition 7 *When one type prefers the remuneration scheme of the other type at the PMH to his own remuneration scheme, then the other type get an inefficient remuneration scheme at the equilibrium, inducing him to doing a lower level of effort.*

Proof. see Appendix. ■

5 Concluding remarks

In this paper we have shown that it is not possible for the firms to get rid of the heterogeneity among workers with respect to the cost of achieving a certain level of quality. Firms will end up with a lower level of quality of the work, which may be detrimental to the final consumers, even though we do not model here the downstream market. This result is quite interesting since with our modelling, we are able to state directly that a lower level of effort will be made at the equilibrium, while not so obvious in the literature on the subject.

We are also able to characterize the unique equilibrium of the pure moral hazard benchmark case without making any assumption on the parameters of the model (preferences of the workers, profits of the firms...). For this reason, our paper improves the results of the existing literature on the subject (especially Jullien, Salanie, Salanie [2000]).

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Appendix

A.1 Proof of lemma 1

The theorem comes from the envelop theorem, $e(x)$ being the number such that $p(e)\ u(x_1) + (1 - p(e))\, u(x_2) - c(e)$ is maximum. Function V_T is $\mathcal{C}_1$ and in the computation of its derivative, the derivative terms relative to e cancel: it remains $(p(e)\ u'(x_1), (1 - p(e))\ u'(x_2))$

A.2 Proof of lemma 2

We suppose that a fix number of employers compete by proposing contracts. It is then a classical result that Nash equilibria of the employers game is a contract maximizing employees utility under a non negative profit constraint of employers. Let suppose then that $x^* = (x_1^*, x_2^*)$ is such a kind of contract. We prove now the four properties of the lemma for that particular contract.

1 - $\Pi_T(x_1^*, x_2^*) = 0$ This is the classical argument. If $\Pi_T(x^*) > 0$ then indirect profit is positive in an open neighborhood of x^*, and it is easy to build in that neighborhood another contract giving more utility to the agents by increasing slightly net payments in both states of the world, a contradiction.

2 - $x_1^* \leq x_2^*$ x^* cannot be such that $x_1^* > x_2^*$ because it is easy to find a full insurance contract which increases both profitability and indirect utility which would contradict the optimality of x^*.

Suppose by contradiction that $x_1^* > x_2^*$ and define $y = p(e^*)\ x_1^* + (1 - p(e^*))\ x_2^*$. Contract $y = (y, y)$ belongs to the 45 degree line, and it is easy to show that the associated effort is $\underline{e}_T$ and that $e^* < \underline{e}_T$ (or equivalently $p(e^*) > p(\underline{e}_T)$).

For such contract, $\Pi_T(y) > \Pi_T(x^*)$ as it is shown by next equation:

$$\begin{aligned}\Pi_T(y) - \Pi_T(x^*) &= (p(\underline{e}_T)\ W_1 + (1 - p(\underline{e}_T))\ W_2) - (p(e^*)\ W_1 + (1 - p(e^*))\ W_2) \\ &= (p(\underline{e}_T) - p(e^*))(W_1 - W_2) > 0\end{aligned} \tag{16}$$

Moreover, as $V_T(y) = u(p(\underline{e}_T)\ u(x_1) + (1 - p(\underline{e}_T)\ u(x_2))$ is greater than the number $u(p(e^*)\ u(x_1) + (1 - p(e^*)\ u(x_2))$ it comes easily with a Jansen inequality that $V_T(y) - V_T(x^*) \geq \frac{1}{2}\ c_T\ (e^* - \underline{e}_T)^2 > 0$, a contradiction.

3a - $e_T(x^*) > \underline{\mathbf{e}}_T$ We prove equivalently that $x_1^* < x_2^*$. Half of the work has already been done. It remains to prove that x^* do not belong to the 45 degree line. It would be the case if the derivative of the indirect profit function is parrallel to the derivative of the profit function without incentives. However, as already seen, the incentive part of this derivatives add a negative effect on the first coordinate and a positive effect on the second coordinate. Indirect profitability derivative is then never parrallel to the profitability derivative without incentive.

3b - $e_T(x^*) < 1$ We only consider the non generic case in which $e_T(x^*) = 1$ because $m(x_1, x_2) > c_T\ (1 - \underline{e}_T) > 0$ (see equation 5). Then in that case, moving around x^* has no incentive effect and $\frac{\partial\ p(e)}{\partial x_1} = \frac{\partial\ p(e)}{\partial x_2} = 0$. In that case, the derivative vector of the profit function is the one without incentives. However, it cannot be parrallel to the derivative vector of the utility function outside the 45 degree line.

4 - First order conditions We compute the determinant $\Delta(x_1^*, x_2^*)$ of the jacobian $(\nabla\ V_T(x), \nabla\ \Pi_T(x))$. For the sake of simplicity we remove the stars in that paragraph.

¿From the preceding paragraph, we know that we can use equation 5 to compute $\frac{\partial\ e}{\partial\ x_2}$ and $\frac{\partial\ e}{\partial\ x_1}$. It comes then, from equations 7, 9 and 10 that

$$\Delta(x_1, x_2) = \begin{vmatrix} p(e)\ u'(x_1) & -p(e_T) - \frac{(q-p)^2}{c_T}\ u'(x_1)\ M \\ 1 - p(e_T)\ u'(x_2) & -(1 - p(e_T)) + \frac{(q-p)^2}{c_T}\ u'(x_2)\ M \end{vmatrix} \tag{17}$$

with $M = (W_2 - W_1) - (x_2 - x_1) > 0$. Then, we find:

$$\frac{\Delta(x_1, x_2)}{u'(x_1)\ u'(x_2)} = p(e)\,(1 - p(e))\ \left(\frac{1}{u'(x_1)} - \frac{1}{u'(x_2)}\right) + \frac{(q-p)^2}{c_T}\ M \tag{18}$$

$\Delta(x_1, x_2) = 0$ gives the condition of the lemma.

A.3 Proof of proposition 3

We prove that along the zero profit line, the determinant $\Delta(x_1, x_2)$ is positive then negative when x_2 increases.

$\Delta(x, x) > 0$

At the intersection of the zero profit line and the 45 degree line, say at the point (x, x), this number is equal to

$$\frac{\Delta(x, x)}{u'(x)\ u'(x)} = \frac{(q-p)^2}{c_T}\ M > 0 \tag{19}$$

If $\Delta(x_1, x_2) < 0$, then it remains negative for higher values of x_2 Notice first that $\Delta(x_1, x_2) < 0$ implies necessarily that at that point $\frac{\partial\ \Pi_T}{\partial\ x_2}$ is necessarily negative (otherwise $\Delta(x_1, x_2)$ would be positive). That implies in turn that around that point, the zero profit line is locally decreasing.

Then, if we move along this zero profit curve around x by increasing x_2, x_1 will automatically decrease. Then, $d(x_1, x_2) = (q-p)\,(u(x_2) - u(x_1))$ increases and following equation 5 $e(x)$ increases also (which makes $p(e)$ lower).

As $p(e) > \frac{1}{2}$ we deduce that $p(e)\,(1 - p(e))$ increases.

Without ambiguity, $\left(\frac{1}{u'(x_1)} - \frac{1}{u'(x_2)}\right)$ decreases. Moreover, as this number is negative we deduce that

$$p(e)\,(1 - p(e))\ \left(\frac{1}{u'(x_1)} - \frac{1}{u'(x_2)}\right) \tag{20}$$

is decreasing. Now, without ambiguity, M is decreasing. Then,

$$\frac{\Delta(x_1, x_2)}{u'(x_1)\ u'(x_2)} \tag{21}$$

is decreasing, which achieve to prove that $\Delta(x_1, x_2) < 0$ remains negative for higher values of x_2.

A.4 Proof of proposition 4

Let suppose that (x_G, x_B) is a menu.

That result comes by comparing the difference in utility of and employees when they are proposed sequentiality (x_G and x_B all when they keep the same level of effort.

If $d(x_G) = d(x_B)$, equation 15 is trivial.

Let suppose now that $d(x_G) \neq d(x_B)$. For the sake of simplicity denote $e_{x_G} = e_G(x_G)$ and $e_{x_B} = e_B(x_B)$. Then, the fact that e_{x_G} (resp. e_{x_B} is not necessarelly the optimal effort made by type-G (resp. type-B) employees when their contract is x_B (resp. x_G) implies that

$$V_G(x_B) \geq U_G(x_B, e_G) \tag{22}$$

$$V_B(x_G) \geq U_B(x_G, e_B) \tag{23}$$

When we plug these inequations in the revelation constraints we get

$$V_G(x_G) \geq U_G(x_B, e_G) \tag{24}$$

$$V_B(x_G) \geq U_B(x_G, e_B) \tag{25}$$

and then, substracting those two inequations

$$V_G(x_G) - U_B(x_G, e_B) \geq U_G(x_B, e_G) - V_B(x_B) \tag{26}$$

Cost effort cancel. Then :

$$(P(e_G) - P(e_G)[u(x_{1G}) - u(x_{2G})] \geq (P(e_B) - P(e_G))[u(x_{1B}) - u(x_{2B})] \tag{27}$$

multiplying by $(q - p)$

$$(P(e_B) - P(e_G)(d(x_G) - d(x_B) \geq 0 \tag{28}$$

Which is equivalent to

$$(e_G - e_B)(d(x_G) - d(x_B)) \geq 0 \tag{29}$$

A.5 Proof of proposition 5

No equilibrium with positive transfers There are many necessary conditions in order that the menu (x_G, x_B) be an equilibrium with transfer.

1. Indifference curve are tengant at the contract corrresponding to a negative transfer of wealth.

2. The contract corresponding to a negative transfer of wealth does not generate positive profit if offered to both type of employes.

When the first condition is not satisfied, then it is easy to find in any neighborhood of the contract corresponding to a negative transfer of wealth another one more attractive for the type, less attractive for the other and making also positive profit.

When the second condition is not satisfied, then a breaking strategy would be to propose another pooling contract, in the neighborhood, more attractive for both agents.

However those two conditions are note compatible in our model. In fact

- Indifference curvers are only tengant when both agents choose the same level of effor [see lemme reflem:con], that is for such that $C'_B(e) = C'_G(e)$

- For all the contracts in which agents choose the same level of effort the firm's profit is the same. Then a firme cannot make positive profit when the contract is choosen by one type and non positive profit when the contract is choosen by both type.

In conclusion, positive transfert is not possible at the equlibrium.

Maximality of the equilibrium menu among the zero transfer menus Through the class of zero transfer menus, there is one maximal menu : that comes from the fact that from any set of menu, if you choose the best contract for both type, that makes a menu (see for instance Chassagnon 200 L). Each maximal contract is unique, because there is a bijective correspondance between contracts and the couples of numbers formend of their utility and their profit. Then, all the zero transfer menus, except that one, cannot be a Nash equilibrium.

A.6 Proof of proposition 6

The result concerning the modification of incentives comes from the quasi-concavity of V_B along the curve $\pi_G(x) = 0$ and from the profitability of x^*_B.

Wheb taken by type G employees ($\pi_G(x_{B^*}) \geq 0$). Before the main argument, the proof shows how the MRS are ranked and the menus basic property in the single crossing case.

Ranking of the MRS Equation **??** implies that type B employees optimal effort is always lower than type G one: $\forall\ x, e_B(x) \leq e_G(x)$ which implies at any point that $p(e_B(x)) > p(e_G(x))$. Finally type B locally prefers state 1 remuneration:

$$MRS_B(x) = \frac{p(e_B(x))}{1-p(e_B(x))}\ \frac{u'(x_1)}{u'(x_2)} \geq \frac{p(e_G(x))}{1-p(e_G(x))}\ \frac{u'(x_1)}{u'(x_2)} = MRS_G(x) \quad (30)$$

Basic property of the menus in the single crossing case: $m(x_G) \geq m(x_B)$ We use a simple graphic argument. Suppose that (x_G, x_B) is a menu. Draw type B and type G indirect indifference curves passing through contract x_G. Because of the MRS ranking, for $x_1 \geq x_{1G}$ type G indifference curve is over type B one and the revelation constraint imply that x_B must lie between those two indifference curves, on the SE of x_G. That means that the remuneration scheme of x_G is greater contingent on state 2 realization and lower contingent on state 1 realization.

That condition is equivalent to $m(x_G) \geq m(x_B)$. Then proposition 4 implies that $e_G(x_G) \geq e_B(x_B)$.

type B obtains the PMH and $m(x_G^{**}) \geq m(x_B^*)$ Type G employees are more performant that type B in the sense that for the same contract, they would choose a greater level of effort. Then, $e_G(x_B^*) > e_B(x_B^*)$. It follows from the condition $(W_2 - W_1) - (x_{2B}^* - x_{1B}^*) > 0$ that

$$\Pi_G(x_B^*) > 0 \quad (31)$$

which implies that (x_B^*, x_B^*) is a menu. Then

Quasiconcavity of $V_B(.)$ relative to d along the curve $\Pi_G(x) = 0$ We compute the determinant $\delta^{BG}(x_1, x_2)$ of the jacobian $(\nabla\ V_B(x), \nabla\ \Pi_G(x))$, and prove that along the zero profit line, the determinant $\Delta(x_1, x_2)$ is positive then negative when x_2 increases (equivalently when d increases).

$$\delta^{BG}(x_1, x_2) = \begin{vmatrix} p(e_B)\ u'(x_1) & -p(e_G) - \frac{(q-p)^2}{c_T}\ u'(x_1)\ M \\ 1-p(e_B)\ u'(x_2) & -(1-p(e_G)) + \frac{(q-p)^2}{c_T}\ u'(x_2)\ M \end{vmatrix} \quad (32)$$

with $M = (W_2 - W_1) - (x_2 - x_1) > 0$. Then, one can verify that the result depend on the expression $\Delta(x_1, x_2)$ computed above:

$$\frac{\delta^{BG}(x_1, x_2)}{u'(x_1)\ u'(x_2)} = \frac{\Delta(x_1, x_2)}{u'(x_1)\ u'(x_2)} + (p(e_G) - p(e_B))(p(e_G)\ u'(x_2) - (1 - p(e_G))\ u'(x_1)) \tag{33}$$

The discussion to prove that this number is positive then negative when x_2 increases is analogous to the one developped in proof of proposition 3 for $\Delta(x_1, x_2)$. We prove that if this number is negative, then, $\frac{\delta^{BG}(x_1,x_2)}{u'(x_1)\ u'(x_2)}$ is decreasing for higher values of x_2. That is true for $\frac{\Delta(x_1,x_2)}{u'(x_1)\ u'(x_2)}$. Now, it is also true for the expression $p(e_G)\ u'(x_2)$ as $p(e_G)$ and $u'(x_2)$ both decrease when x_2 increases (and also $-((1 - p(e_G))\ u'(x_1))$ decreases.

An argument similar to that developped in footnote achieves to prove that $V_B(.)$ is quasiconcave relative to m along the curve $\Pi_G(x) = 0$.

Type B obtains the PMH Notice that whatever the contract type B's endogeneous effort is lower than type G's one. Then when $M = (W_2 - W_1) - (x_2 - x_1) \geq 0$ $\Pi_B(x) \leq \Pi_G(x)$. That implies that $\pi_G(x_B^*) \geq 0$ and that (x_B^*, x_B^*) is a menu.

However, as we mentionned in proposition 5 if (x_B**, x_G**) is an equilibrium, it is optimal in the class of zero transfer menus. That implies in particular that menu dominetes (x_B*, x_B*) so that $B(x_B^{**} \geq V_B(x_B^*)$

However as $\pi_B(x_B^* \geq 0$ that implies automatically $x_B^{**} = x_B^*$.

Increasing the effort of type G Let denote x_G^{**} the second best contract of type G lying on the curve $\pi_G(x) = 0$

1. Define X the contract lying at the intersection of curve $d(x) = d(x_B^*)$ and curve $\pi_G(x) = 0$ As curve $d(x) = d(x_B^*)$ is increasing , either X dominates x_B^* either the contrary. However we already know that $\pi_G(x) = 0 \leq \pi_G(x_B^*$

2. If we suppose that (x_B^*, x_G^*) violate revelation contraints, it must be the case that $V_B(x_G^*) \geq V_B(x_G^*)$ because we know from inequation $\pi_G(x_B^*) \geq 0$ that $V_G(x_G^*) \geq V_G(x_B^*)$

3. As function V_B is quasiconcave on the curve $\Pi_G(x) = 0$ the two results above imply that for any contract x between X and x_G^*, $V_B(x) > V_B(x_B^*)$.

That implies that x_G^{**} is a contract such that:

$$d(x_G^{**}) \geq \max(d(x_G^*), d(X)) \quad \text{or} \quad d(x_G^{**}) \leq \min(d(x_G^*), d(X)) \tag{34}$$

However, we already know that $d(x_G^{**}) > d(x_B^*) = d(X)$. One should reject the second inequality and conclude that:

$$d(x_G^{**}) \geq \max(d(x_G^*), d(x_B^*)) \tag{35}$$

which implies $d(x_G^{**}) \geq d(x_G^*)$ and finally

$$e_G(x_G^{**}) \geq e_G(x_G^*) \tag{36}$$

A.7 Proof of theorem ??

Let suppose that an equilibrium exists and let denote it (x_G^{**}, x_B^{**}). Similar arguments to that developped for the single crossing case show that when one agent (type T) is such that the PMH contract could be proposed to pool all employees (that is $\Pi_{T'}(x_T^*) \geq 0$), then type T obtains the PMH and thype T' has to increases its effrt at the equilibrium.

The last situation to examine is when the two following inequality are verified:

$$\Pi_G(x_B^*) < 0 \tag{37}$$

$$\Pi_B(x_G^*) < 0 \tag{38}$$

In that case B and G employees are locally more efficient and in particular: $d(x_B^*) < c'(e_0) < d(x_G^*)$ (and $e_B^* < e_0 < e_G^*$). Denote by I the unique contract at the intersection of the two zero profit cruve of the two types (in particular $d(I) = c'(e_0)$). We prove in four steps that at the second best none of the type decrease their effort level.

All prefers their second best contracts to I: $V_G(x_G^{**}) \geq V_G(I)$ and $V_B(x_B^{**}) \geq V_B(I)$ By definition I belongs to $\Pi_G(x) = 0$ and to $\Pi_B(x) = 0$. Then (I, I) is a menu. By optimality of the second best, proved above, (x_G^{**}, x_B^{**}) Pareto dominates (I, I).

Incentives do not change the subspace in which agents get a contract: $d(x_G^{**}) \geq d(I) \geq d(x_B^{**})$ In other words, $d(I)$ cannot be between $d(x_T^*)$ and

$d(x_T^{**})$ for both types. Suppose the contrary. Then, preceding paragraph implies the following sequence:

$$V_T(x_T^*) \geq V_T(x_T^{**}) \geq V_T(I) \tag{39}$$

The conclusion follows from the quasiconcavity of V_T on the curve $\Pi_T(x) = 0$: I cannot be between x_T^* and x_T^{**} along the curve $\Pi_T(x) = 0$ (to which belongs x_T^* and x_T^{**} and I).

Type G second best effort is greater: $e_G(x_G^{**}) \geq e_G(x_G^*)$ If $x_G^{**} = x_G^*$ the result is trivial. Suppose then that those two contracts differ at the equilibrium.