Researching with Whom? *Stability and Manipulation*[*]

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Abstract

This paper explores the existence of stable research teams, when each agent's preferences depend on the set of researchers collaborating with her. We introduce a property over researchers' preferences, called tops responsiveness guaranteeing the existence of stable research teams configurations. We also provide a stable mechanism, induced by the so-called *tops covering algorithm*, which is strategy-proof when researchers preferences satisfy tops responsiveness. Furthermore, we also find out that, in this framework the tops covering mechanism is the *unique* strategy-proof mechanism that always selects stable allocations.

Keywords: Coalition Formation, Research Teams Configurations, Stability, Strategy-Proofness.

Journal of Economic Literature Classification Numbers: C71, C72, C78 and D71.

Conventions on Mathematical Symbols

- $\subset$ Inclusion: $S \subset S'$ indicates that for all $x \in S$ then $x \in S'$.
- $\subsetneq$ Strict Inclusion: $S \subsetneq S'$ indicates that for all $x \in S$ then $x \in S'$ and there is $y \in S'$ such that $y \notin S$.
- $\not\subset$ Negate inclusion: $S \not\subset S'$ indicates that there exists $x \in S$ such that $x \notin S'$.

1. Introduction

Economic agents usually cooperate to reach some objectives that they could not obtain by themselves. Taking this premise as a fact, it seems to be clear that agents' cooperation is founded on the own interest of each economic agent. We might give a further steep by saying that agents follow such a behavior not only by economic purposes but also for other (more generic) reasons.

Perhaps the main reason why agents try to find cooperation from others is related to the existence of some complementarities in their own abilities. A particular instance in which this assertion becomes as general as possible is related to some research activities. Let us observe that nowadays researchers are highly specialized and, often, their collaboration is necessary to reach any results. This fact is pointed out by the several papers published in any high-quality review (in any field) being co-authored.

The purpose of this paper is to present a theoretical framework explaining which collaborations will be done and which not. An insight that made this problem particularly interesting is that researchers' opinion on how to order the different research teams could vary from any agent to each other. For instance, some agents could prefer to be in a research group in which she is (considered) the leader, no matter the quality of other researchers; others could be looking for being in a multi-disciplinary group, whose components' scientific formation belongs to quite different (but complementary) research areas; others could prefer to be in research groups publishing many papers in scientific journals; etc.

Since there is no reason justifying a particular specification of agents' preferences, we will avoid as much as possible the use of restrictive conditions on agents' tastes. By this, we mean that each researcher preferences will depend only on the set of agents cooperating with her, and no other variable will influence her preferences.

The first question that we address in this paper is the study of which research groups will emerge from agents' cooperation. Since we want to predict agents' collective decisions in a cooperative framework, the equilibrium concept that we identify with such a prediction is stability. Our findings in this matter are positive: We identify a property whose fulfillment guarantees the existence of stable allocations. This condition, that we call *tops responsiveness*, tries to capture the idea of how each researcher believes that others could complement her.

The next question that we propose is how agents could coordinate to reach stable allocations. Let us observe that, when there are many researchers, it could be difficult (and some times highly costly) to manage their coordination. A way to avoid the need of such agents' interaction is to design mechanisms selecting stable allocations. By mechanism we mean automatic machinery whose input is a prescription of agents' characteristics (or preferences), being its output a partition of the set of researchers into research teams. In particular, we restrict attention to stable mechanisms, i.e., mechanism selecting stable allocations relative to agents' preferences. A further problem that could emerge when mechanisms are employed is related to agents' incentives to misreport their true characteristics. Therefore, if the employ of some mechanisms could be justified because they save coordination costs, we should not forget to avoid another source of costs related to researchers' manipulation when such a mechanism is employed. This is why we do not only concentrate on the design of mechanisms finding stable allocations but also on these mechanisms being immune to agents' strategic behavior. Related to this question we find not only the possibility of designing strategy-proof stable mechanisms, but also that, under tops responsiveness, the uniqueness of such a mechanism.

Therefore, what this paper proposes is:

(i) The existence of economic environments where the problem of finding stable research teams configurations have solutions, and

(ii) The existence (and uniqueness) of mechanisms selecting stable research teams configurations have solutions, provided that agents have no incentive to misreport their true characteristics.

2. Overview of the Literature

The phenomenon that we analyze in this paper was presented by Drèze and Greenberg [7], and called the hedonic aspect of coalition formation. By this we mean that each researcher mainly concentrates on which are the other agents being in the same group that she belongs to. Therefore, we will follow the formulation provided, independently, by Bogomolnaia and Jackson [5] and by Banerjee et al. [4]. This model can be seen as a generalization for the matching problems introduced by Gale and Shapley [8]. In fact, and using matching terminology, the model that we will introduce is known as the *many-to-many one-sided matching problem*.

Following the usual analysis in matching problems, we first concentrate on the study of the existence of stable allocations. Using our illustrative example of forming research groups, a stable allocation is a partition of the researchers' set in such a way that no set of researchers prefer to for a new group rather than to develop their activities following the recommendation of such an allocation.

The main general problem we face is the general existence of stable allocations. In fact, as Gale and Shapley [8] pointed out, it could be the case that no allocation was stable. Alcalde [1] analyzed this problem focusing on the case in which the research groups are constrained to have no more than two agents. This author proposed two kinds of conditions under which stable allocations always exist. The first one, called P-reciprocity, is established in terms of agents' preference profiles; whereas the second approach is stated in terms of each individual's preferences.

As we mentioned before, Banerjee et al. [4] proposed an extension of the *roommate problem* by Gale and Shapley [8] (or one-to-one one-sided matching problem) and provided a property to guarantee the existence of stable allocations. This condition can be seen as a generalization of Alcalde's [1] P-reciprocity. A second approach to this problem can be found in Alcalde and Romero-Medina [3]. These authors propose three properties, each of them guaranteeing the existence of stable allocations. The main difference between the approaches by Banerjee et al. [4] and by Alcalde and Romero-Medina [3] is that the conditions proposed in the first paper are related to preference profiles whereas the properties defined in the second one are established over agents' preferences. As Alcalde and Romero-Medina [3] pointed out, their approach has the advantage that it allows, in an easier way, an analysis of comparative statics. In this paper we follow the proposal by the formers and propose conditions over each researcher's preferences. In fact, tops responsiveness can be seen as a generalization of what Alcalde and Romero-Medina [3] called essentiality.

The second aspect that we explore in this paper is the existence of strategy-proof stable mechanisms. Following a tradition in matching problems, we shall refer the results by Roth [10] and Alcalde and Barberà [2] showing a general impossibility of finding non-manipulable mechanisms. These negative results were partially skipped by the study of some domain restrictions where strategy-proof stable mechanisms exist. In particular, Alcalde and Barberà [2] describe *Top Dominance*, a property whose fulfillment guarantees the existence of a unique mechanism satisfying the two properties above.

In a more general setting, the results by Gibbard [9] and Sattherwaite [12] also inform us about a general impossibility of designing strategy-proof stable mechanisms, except when agents' admissible preferences are restricted. The findings by Sönmez [14] are not much more optimistic: The only possibility to find strategy-proof stable mechanism, if any, is restricted to the case in which the set of stable outcomes is (essentially) a singleton.

The difficulties grow throughout our framework, if we want to propose strategy-proof mechanisms selecting stable allocations. First, a general impossibility result can be established because, without imposing any restriction on agents' characteristics, no stable mechanism can be designed. The reason is quite simple, as Gale and Shapley [8] pointed out, the set of stable allocations might be empty for some instances. On the other hand, when one imposes some of the restrictions known in the literature to guarantee the existence of stable researchers teams configurations, the existence of strategy-proof stable mechanisms becomes trivial. For instance, under the condition that Banerjee et al. [4] call *top coalition property*, since the core is single-valued (following Sönmez's [14] terminology) there is only one such a mechanism. A similar reasoning could be applied when agents' preferences are restricted to satisfy essentiality, a property proposed by Alcalde and Romero-Medina [3], since the set of stable allocations is always a singleton.

Given these antecedents, the question that we propose seems not to be quite trivial: There is some framework in which the (non-empty) set of stable allocations is not single-valued? And, assuming a positive answer, there is some strategy-proof mechanism that could be designed in such a case? This paper provides positive answers to both questions. More than that, we show that under tops responsiveness, there is a unique strategy-proof stable mechanism.

The way to prove our findings is quite constructive. To show the existence of stable research teams configurations, we introduce an iterative procedure yielding stable allocations. This algorithm, to be called the *tops covering algorithm*, is in spirit similar to Gale's *top trading cycle* described in Shapley and Scarf [13], or the algorithm designed by Cechlárová and Romero-Medina [6] for the roommate problem. Finally, and relative to the existence of a unique strategy-proof stable mechanism, we follow arguments similar to the employed in Alcalde and Barberà [2] for an impossibility result.

3. The Model

Let $R = \{r_1, \ldots, r_i, \ldots, r_n\}$ be the a finite set of researchers. A subset S of R is called a research group. Each researcher r_i is endowed with a complete preordering $\succsim_i$, defined over $\mathcal{A}^i = \{S \subset R : i \notin S\}$, which represents her preferences over all the possible research groups she can join. Let $\succ_i$ denote the strict preference derived from $\succsim_i$, i.e. $S \succ_i S'$ means that $S \succsim_i S'$ and not $S' \succsim_i S$. Similarly, let $\sim_i$ denote the indifference relationship induced by $\succsim_i$, i.e. $S \sim_i S'$ stands for

$S \succsim_i S'$ and $S' \succsim_i S$. A research group formation problem will be shortly described by a pair $\{R, \succsim\}$, where the agents' preferences profile $\succsim := (\succsim_1, \ldots, \succsim_i, \ldots, \succsim_n)$ is a description of each agent's preferences.

A solution for a research group formation problem, also called a *research teams configuration*, is a function

$$\tau : R \rightarrow 2^R$$

such that

(i) For each $r_i \in R$, $r_i \in \tau(r_i)$, and

(ii) for any two researchers r_i and r_j, $r_j \in \tau(r_i)$ if, and only if, $r_j \in \tau(r_i)$.

In fact, a research teams configuration can be viewed as a partition of the set of researchers.

We say that a research teams configuration τ is *stable* for $\{R, \succsim\}$ if there is no non-empty set of researchers, say S, such that $[S \setminus \{r_i\}] \succ_i \tau(r_i) \setminus \{r_i\}$ for each $r_i \in S$. A set satisfying the above property is said to *block* τ. Finally, we say that a research teams configuration τ is *individually rational* for $\{R, \succsim\}$ if there is no researcher blocking it, i.e. $\tau(i) \setminus \{r_i\} \succsim_i \emptyset$ for all r_i in R.

A research teams configuration rule Ψ is a function that selects, for each possible research group formation problem, say $\{R, \succsim\}$, a research teams configuration for it. We say that rule Ψ is stable if it selects a stable research teams configuration for each problem, i.e. for any problem $\{R, \succsim\}$, $\Psi[\{R, \succsim\}]$ is stable for $\{R, \succsim\}$.

As we mentioned in Section 2, it is well-known that in general there is no stable rules. This is because for some instances the set of stable research teams configurations might be empty. (See for instance the roommate problem, proposed in Gale and Shapley [8].) This is why the aim of some recent papers has been the study of economic environments in which stable research teams configurations always exist. Following this approach we introduce a new property guaranteeing the existence of stable research teams configuration. (See Theorem 4.4.) This condition, to be called *tops responsiveness*, is weaker than two domain restrictions introduced by Alcalde and Romero-Medina [3], namely *essentiality* and *union responsiveness*.

The idea underlying tops responsiveness is very simple, and is established in terms of each researcher's preferences. Just to explain it, let us consider a given researcher, say r_i. The first aspect that we require is that, for any fixed set of researchers, r_i's preferences have a unique maximal. This maximal can be understood as the set of researchers which (as a group) most complements r_i. Then, tops responsiveness introduces two considerations on how to compare two sets of researchers with whom to cooperate. The first one (Condition 1 in Definition 3.1) is that when both sets have different "best complement" to r_i, such a researcher will prefer to cooperate with the set whose best complement is preferred. The second one (Condition 2 in Definition 3.1) is referred to the case in which both research teams share the same "best complement" to r_i. In such a case, the set of researchers having less no-complements will be preferred by our researcher.

Let us propose a formal description for tops responsiveness.

Definition 3.1. *We say that preferences for researcher r_i, $\succsim_i$, satisfies tops responsiveness on $\mathcal{A}^i$, if for any set S in $\mathcal{A}^i$ there is a single maximal for $\succsim_i$, to be denoted by $Ch_i(S; \succsim_i)$, or simply $Ch_i(S)$ if there is no ambiguity on r_i's preferences;*[1] *and for any two sets S and S' in $\mathcal{A}^i$, the following conditions are fulfilled:*

1. *$Ch_i(S) \succ_i Ch_i(S')$ implies $S \succ_i S'$, and*
2. *If $Ch_i(S) = Ch_i(S')$, and $S \subsetneq S'$ then $S \succ_i S'$.*

Let $\mathcal{TR}$ denote the set of researchers' preference profiles in which each agent's preferences satisfy tops responsiveness.

Just to illustrate the conditions above, let us consider the following example,

Example 3.2. *Let consider 9 researchers belonging to three different specialities (a, b, and c). Therefore $R = \{a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3\}$. Let us imagine that each researchers' team can develop at most one research project. From c_3's point of view, a research project is successful only if it is developed by (at least) one researcher belonging to each category. Clearly, the quality of each project is related to the researchers participating in it. And, for a given project, the importance of her participation depends (positively) on her prestige among the researchers belonging to her speciality and (negatively) on the number of researchers investigating in such a project. We also consider that, for each speciality, researchers'*

[1] $Ch_i(S)$ denotes the choice of agent i in S under preferences $\succsim_i$. Thus, $Ch_i(S)$ is the set $S' \subset S$ such that $S' \succ_i S''$ for any other set $S'' \subset S$.

indices corresponds to researchers' prestige (from c_3's point of view). Finally, let us suppose that c_3's preferences are lexicographic on the quality of the project, i.e., she prefers to be in the project having the highest quality and, once the quality level is fixed, she takes into consideration the importance of her participation on it.

Given the description above, c_3 would prefer to join the research team $S = \{a_1, b_1\}$ rather than $S' = \{a_1, b_2\}$ this is because the project developed by $S \cup \{c_3\}$ will have best qualified that the project that $S' \cup \{c_3\}$ could develop. This is in concordance with what Condition 1 in Definition 3.1 establishes. On the other hand, and related to Condition 2, c_3 will prefer to join research team $S = \{a_1, b_1\}$ rather than $S'' = \{a_1, b_1, b_2\}$. This is because both projects will have the same quality which, in c_3's opinion, is determined by researchers a_1, b_1 and herself; but the relative importance of c_3 in getting the project's results is higher when she join S, and b_2 is excluded.

4. Tops Responsiveness and Stability

We mentioned in Section 3 that any research group formation problem whose agents' preferences satisfy tops responsiveness have stable research teams configurations. The aim of this section is to state formally this fact. We will introduce a procedure which selects a stable research teams configuration, for any problem whose agents' preferences exhibit tops responsiveness. This procedure can be viewed as a natural extension for the $\mathcal{RA}$-*algorithm* by Alcalde and Romero-Medina [3] (and henceforth to the Gale's *tops trading cycle* introduced in [13] by Shapley and Scarf).

Just to introduce our procedure, we present an auxiliary function applying the researchers set into itself. What this function does is to add to each research set the researchers belonging to the choice set of each of its components.

Definition 4.1. *We define the choice covering function $\mathcal{C}$ as the function applying to each research group formation problem $\{R, \succsim\}$ and set of researchers $S \subset R$ the set*

$$\mathcal{C}[S; \{R, \succsim\}] = S \bigcup_{i \in S} Ch_i(R)$$

where $Ch_i(R)$ is the choice set of r_i on R under $\succsim_i$, i.e., the (unique) $\succsim_i$ −maximal on R.

Let us observe that, for any given research group formation problem $\{R, \succsim\}$, the choice covering function, seen as a function from 2^R into itself, has fixed points. In particular, it can be straightforwardly seen that, for each $\{R, \succsim\}$, and any $r_i \in R$,

$$\mathcal{C}^{|R|}\left[\{r_i\};\{R,\succsim\}\right] = \mathcal{C}\left[\mathcal{C}^{|R|}\left[\{r_i\};\{R,\succsim\}\right];\{R,\succsim\}\right]$$

with $|R|$ being the cardinality of R, and for each positive integer k, $\mathcal{C}^k\left[S;\{R,\succsim\}\right]$ is the k-th composition of $\mathcal{C}$ applied to $[S;\{R,\succsim\}]$. For instance,

$$\mathcal{C}^2\left[S;\{R,\succsim\}\right] = \mathcal{C}\left[\mathcal{C}\left[S;\{R,\succsim\}\right];\{R,\succsim\}\right]$$

For simplification purposes, let us introduce the next notation

$$\mathcal{CC}\left[\{r_i\};\{R,\succsim\}\right] = \mathcal{C}^{|R|}\left[\{r_i\};\{R,\succsim\}\right]$$

where the r_i's covering choice under $\{R,\succsim\}$, $\mathcal{CC}\left[\{r_i\};\{R,\succsim\}\right]$, is the minimal (under inclusion) fix point for $\mathcal{C}$ containing r_i.

We now introduce the *tops covering algorithm*, a procedure which will help us to show the existence of a stable research teams configuration.

Definition 4.2. *Let $\{R,\succsim\}$ be a research group formation problem. The tops covering algorithm works as follows.*

Step 1.– *Let compute, for each $r_i \in R$, the set $\mathcal{CC}\left[\{r_i\};\{R,\succsim\}\right]$. Let check, for each $r_i \in R$ whether*

$$r_i \in \bigcap_{r_j \in \mathcal{CC}[\{r_i\};\{R,\succsim\}]} \left\{\mathcal{CC}\left[\{r_j\};\{R,\succsim\}\right]\right\} \tag{4.1}$$

or not. Let set, for each r_i such that (4.1) *is fulfilled,*

$$\tau^{tc}(r_i) = \mathcal{CC}\left[\{r_i\};\{R,\succsim\}\right],$$

and let denote by R^1 the set of researchers which have not been assigned to any group

$$R^1 = R \setminus \{r_i \in R : r_i \text{ satisfies Condition } (4.1)\}.$$

Finally, whenever R^1 is non-empty, let define the research group formation problem $\{R^1, \succsim|_{R^1}\}$, where $\succsim|_{R^1}$ is the restriction of $\succsim$ to the set of researchers R^1. Then if R^1 is empty, stop and produce as outcome the research teams configuration τ^{tc} above described. Otherwise, go to Step 2.

Step k.– *Let compute, for each* $r_i \in R^{k-1}$, *the set* $\mathcal{CC}\left[\{r_i\};\left\{R^{k-1}, \succsim|_{R^{k-1}}\right\}\right]$. *Let check, for each* $r_i \in R^{k-1}$ *whether*

$$r_i \in \bigcap_{r_j \in \mathcal{CC}\left[\{r_i\};\left\{R^{k-1}, \succsim|_{R^{k-1}}\right\}\right]} \left\{\mathcal{CC}\left[\{r_j\};\left\{R^{k-1}, \succsim|_{R^{k-1}}\right\}\right]\right\} \quad (4.2)$$

or not. Let set, for each $r_i \in R^{k-1}$ *such that* (4.2) *is fulfilled,*

$$\tau^{tc}(r_i) = \mathcal{CC}\left[\{r_j\};\left\{R^{k-1}, \succsim|_{R^{k-1}}\right\}\right],$$

and let denote by R^k *the set of researchers in* R^{k-1} *which have not been assigned to any group*

$$R^k = R^{k-1} \setminus \left\{r_i \in R^{k-1} : r_i \text{ satisfies Condition } (4.2)\right\}.$$

Finally, whenever R^k *is non-empty, let define the research group formation problem* $\left\{R^k, \succsim|_{R^k}\right\}$, *where* $\succsim|_{R^k}$ *is the restriction of* $\succsim$ *to the set of researchers* R^k. *Then if* R^k *is empty, stop and produce as outcome the research teams configuration* τ^{tc} *described throughout Steps* 1 *to* $k-1$. *Otherwise, go to Step* $k+1$.

Let us propose an illustrative example to show how to compute the choice covering sets and how the tops covering algorithm works.

Example 4.3. *Let* $R = \{r_1, r_2, r_3, r_4, r_5\}$, *with preferences*

$$\{r_2, r_3, r_4\} \succ_1 \{r_2, r_3, r_4, r_5\} \succ_1 \{r_2, r_4, r_5\} \succ_1 \{r_5\} \succ_1 \cdots$$

$$\{r_3\} \succ_2 \{r_1, r_3\} \sim_2 \{r_3, r_4\} \succ_2 \{r_1, r_3, r_4\} \sim_2 \{r_3, r_5\} \succ_2 \cdots$$

$$\{r_2, r_4\} \succ_3 \{r_1, r_2, r_4\} \sim_3 \{r_2, r_4, r_5\} \succ_3 \{r_1, r_2, r_4, r_5\} \succ_3 \{r_1, r_2, r_5\} \succ_3 \cdots$$

$$\{r_2, r_3\} \succ_4 \{r_1, r_2, r_3\} \sim_4 \{r_2, r_3, r_5\} \succ_4 \{r_1, r_2, r_3, r_5\} \succ_4 \{r_1, r_3, r_5\} \succ_4 \emptyset \succ_4 \cdots$$

$$\{r_1, r_3, r_4\} \succ_5 \{r_1, r_2, r_3, r_4\} \succ_5 \{r_1\} \succ_5 \cdots$$

Let us observe that

$$\begin{aligned}
\mathcal{C}\left[\{r_1\};\{R,\succsim\}\right] &= \{r_1,r_2,r_3,r_4\} \\
\mathcal{C}\left[\{r_2\};\{R,\succsim\}\right] &= \{r_2,r_3\} \\
\mathcal{C}\left[\{r_3\};\{R,\succsim\}\right] &= \{r_2,r_3,r_4\} \\
\mathcal{C}\left[\{r_4\};\{R,\succsim\}\right] &= \{r_2,r_3,r_4\} \\
\mathcal{C}\left[\{r_5\};\{R,\succsim\}\right] &= \{r_1,r_3,r_4,r_5\}
\end{aligned}$$

Therefore

$$\begin{aligned}
\mathcal{CC}\left[\{r_1\};\{R,\succsim\}\right] &= \{r_1,r_2,r_3,r_4\} \\
\mathcal{CC}\left[\{r_2\};\{R,\succsim\}\right] &= \{r_2,r_3,r_4\} \\
\mathcal{CC}\left[\{r_3\};\{R,\succsim\}\right] &= \{r_2,r_3,r_4\} \\
\mathcal{CC}\left[\{r_4\};\{R,\succsim\}\right] &= \{r_2,r_3,r_4\} \\
\mathcal{CC}\left[\{r_5\};\{R,\succsim\}\right] &= \{r_1,r_2,r_3,r_4,r_5\}
\end{aligned}$$

and hence

$$
\begin{array}{rclcl}
r_1 & \notin & \bigcap_{r_j \in \mathcal{CC}[\{r_1\};\{R,\succsim\}]} \{\mathcal{CC}[\{r_j\};\{R,\succsim\}]\} & = & \{r_2, r_3, r_4\} \\
r_2 & \in & \bigcap_{r_j \in \mathcal{CC}[\{r_2\};\{R,\succsim\}]} \{\mathcal{CC}[\{r_j\};\{R,\succsim\}]\} & = & \{r_2, r_3, r_4\} \\
r_3 & \in & \bigcap_{r_j \in \mathcal{CC}[\{r_3\};\{R,\succsim\}]} \{\mathcal{CC}[\{r_j\};\{R,\succsim\}]\} & = & \{r_2, r_3, r_4\} \\
r_4 & \in & \bigcap_{r_j \in \mathcal{CC}[\{r_4\};\{R,\succsim\}]} \{\mathcal{CC}[\{r_j\};\{R,\succsim\}]\} & = & \{r_2, r_3, r_4\} \\
r_5 & \notin & \bigcap_{r_j \in \mathcal{CC}[\{r_5\};\{R,\succsim\}]} \{\mathcal{CC}[\{r_j\};\{R,\succsim\}]\} & = & \{r_2, r_3, r_4\}
\end{array}
$$

Thus, the first step of the tops covering algorithm ends proposing the research team

$$\tau^{tc}(r_2) = \tau^{tc}(r_3) = \tau^{tc}(r_4) = \{r_2, r_3, r_4\}.$$

Then, let us proceed by analyzing the problem associated to $R^1 = \{r_1, r_5\}$. *Let us observe that for* R^1 *the restricted preferences profile is*

$$\{5\} \succ_1 \emptyset$$

$$\{1\} \succ_5 \emptyset$$

and, in such a case, $\mathcal{CC}[\{r_1\};\{R^1, \succsim|_{R^1}\}] = \mathcal{CC}[\{r_5\};\{R^1, \succsim|_{R^1}\}] = \{r_1, r_5\}$, *and thus, the second step of the tops covering algorithm proposes to form the researchers' team*

$$\tau^{tc}(r_1) = \tau^{tc}(r_5) = \{r_1, r_5\}.$$

Since $R^2 = R^1 \backslash \{r_1, r_5\}$ *is the empty set, the algorithm ends.*

It is easy to see that, for any research group formation problem, $\{R, \succsim\}$, and any stage k, at the end of which R^k is non-empty, we have that $R^{k-1} \subsetneq R^k$ (with $R^0 = R$). Therefore, provided that the number of researchers is finite, it is clear that the tops covering algorithm ends in finite steps. (See Lemma 6.1 for a formal proof of this fact.)

Next theorem, whose proof is relegated to the Appendix 1, is the main result of this section. It establishes that the general non-existence of stable research teams configurations is avoided when agents' preferences satisfy tops responsiveness.

Theorem 4.4. *Let $\{R, \succsim\}$ be a research group formation problem whose agents' preferences satisfy tops responsiveness, then the set of stable research teams configurations is non-empty.*

5. Strategic Behavior and the Tops Covering Algorithm

As we exposed in the Introduction, researchers' coordination to agree on forming stable research teams could be very hard. In such a case they could suffer an opportunity cost (in terms of time, for instance) that could be very high. A way to avoid such a cost is to employ some given procedure that (automatically) proposes a research teams configuration for each research group formation problem. A first property that such a procedure should verify is that of efficiency, i.e., for any given research group formation problem, $\{R, \succsim\}$, the recommendation that such a rule gives is a research teams configuration τ, stable for $\{R, \succsim\}$. Let us observe that the employ of unstable rules is not useful that to avoid the above-mentioned coordination cost, this is because agents will not commit on following the rules' recommendations.

The second question that we propose here is related to individual costs rather than collective costs. In fact, once agents commit on following the recommendations given by a stable rule a second question arises. Let suppose that each researcher only has information about her own characteristics, and also knows how the rule processes the information provided by each researcher. In such a case, it is natural to think that each researcher could explore whether she could benefit from misreporting her true characteristics or not. Note that the answer to this question eventually depends not only on the rule that is employed, but also on agents' characteristics. Clearly, when a researcher analyses if she could manipulate the mechanism in her own benefit, she is implicitly supporting an individual cost (at least in terms of time), which is expected to be increasing on the number of researchers involved in the problem. Therefore, when such a strategic behavior could report benefit to some researchers, we can think off that the employ of the rule is avoiding coordination costs but introducing individual "manipulation" costs.

In this section we explore the possibility of designing stable mechanisms, and hence to avoid the coordination costs, but without introducing "manipulation" costs. The way to do that is by designing stable strategy-proof mechanisms, i.e., stable rules whose employ does not induce any incentive to the researchers' strategic behavior.

Definition 5.1. *Let $\mathcal{P}$ be a family of group formation problems, and let Ψ be a research teams configuration rule on $\mathcal{P}$, i.e. for each $\{R, \succsim\} \in \mathcal{P}$, $\Psi[\{R, \succsim\}]$ is a research teams configuration for $\{R, \succsim\}$. We say that*

(*i*) *Ψ is stable in $\mathcal{P}$ if, for each $\{R, \succsim\} \in \mathcal{P}$, $\Psi[\{R, \succsim\}]$ is stable for $\{R, \succsim\}$, and*

(*ii*) *Ψ is strategy-proof in $\mathcal{P}$ if, for each $\{R, \succsim\} \in \mathcal{P}$, $r_i \in R$, and $\succsim_i'$ such that $\{R, [\succsim_i', \succsim_{-i}]\} \in \mathcal{P}$,*

$$\Psi_i[\{R, \succsim\}] \setminus \{r_i\} \succsim_i \Psi_i[\{R, [\succsim_i', \succsim_{-i}]\}] \setminus \{r_i\}$$

where $[\succsim_i', \succsim_{-i}]$ stands for the preference profile obtained by exchanging in $\succsim$ r_i's preferences $\succsim_i$ by $\succsim_i'$; and $\Psi_i[\{R, \succsim\}]$ is the research team to which r_i is allocated by Ψ for the problem $\{R, \succsim\}$.

Our first result in this section establishes the existence of strategy-proof stable rules for the family of research group formation problems whose agents preferences satisfy tops responsiveness. More than that, we also propose a rule fulfilling both properties. Such a rule is the one induced by the tops covering algorithm.

Theorem 5.2. *Let R be a fixed set of researchers, and let $\mathcal{TR}$ the family of research group formation problems $\{R, \succsim\}$ such that, for each $r_i \in R$, $\succsim_i$ satisfies tops responsiveness. Let us denote by Ψ^{tc} the rule that associates, to each $\{R, \succsim\}$ the researchers team configuration τ^{tc} given by the tops covering algorithm. Then Ψ^{tc} is strategy-proof in $\mathcal{TR}$.*

Proof. See Appendix 2 ∎

We next explore the possibility of designing other strategy-proof stable rules for research group formation problems. As the next example points out, the tops covering mechanism is not the unique stable rule in our framework, this is why our question reaches a particular relevance. Provided that it is possible to design different stable rules for the research group formation problem, whose of them avoid agents' strategic behavior? The answer to this question will be pointed out in the analysis given in Example 5.3, and formalized in Theorem 5.4.

Example 5.3. *Let* $R = \{r_1, r_2, r_3\}$ *a set of researchers with preferences over colleagues as follows*

$$\{r_2\} \succ_1 \{r_2, r_3\} \succ_1 \emptyset \succ_1 \{r_3\}$$

$$\{r_3\} \succ_2 \{r_1, r_3\} \succ_2 \emptyset \succ_2 \{r_1\}$$

$$\{r_1\} \succ_3 \{r_1, r_2\} \succ_3 \{r_2\} \succ_3 \emptyset$$

In this research group formation problem there are two stable solutions, namely

$$\tau^{tc}(r_1) = \tau^{tc}(r_2) = \tau^{tc}(r_3) = \{r_1, r_2, r_3\},$$

and

$$\tau'(r_1) = \{r_1\}; \tau'(r_2) = \tau'(r_3) = \{r_2, r_3\}$$

Nevertheless, it is easy to see that any research teams configuration rule, say Ψ*, whose recommendation were* τ' *for the above problem is manipulable by researcher* r_3 *via preferences* $\succsim_3'$ *as follows*

$$\{r_1\} \succ_3' \{r_1, r_2\} \succ_3' \emptyset \succ_3' \{r_2\}$$

The reason is quite simple. The unique research teams configuration, stable for $\{R, [\succsim_3', \succsim_{-3}]\}$ *is* τ^{tc}*, and*

$$\tau^{tc}(r_3) \setminus \{r_3\} = \{r_1, r_2\} \succ_3 \{r_2\} = \tau'(r_3) \setminus \{r_3\}.$$

Theorem 5.4. *Let* $\mathcal{PTC}$ *denote the set of research group formation problems* $\{R, \succsim\}$ *with a finite number of researchers, and agents' preferences satisfying tops responsiveness. Let* Ψ *a stable research teams configuration rule on* $\mathcal{PTC}$*. Then* Ψ *is strategy-proof if, and only if, for each* $\{R, \succsim\} \in \mathcal{PTC}$ *we have that*

$$\Psi[\{R, \succsim\}] \equiv \Psi^{tc}[\{R, \succsim\}]$$

Proof. See Appendix 3. ∎

6. Conclusions

This paper explores the problem that researchers face when they develop their tasks cooperatively. We propose a way to solve such a problem by designing rules proposing which agents should work collectively, depending on researchers' characteristics (or preferences). In particular, we concentrate on taking advantage of the possible researchers' complementation.

In particular, we are interested on designing rules to avoid the two main categories of costs (in terms of time) inherent to these problems. Firstly, the coordination (or collective) costs associated to the configuration of researchers' teams. And secondly the individual cost which appears when agents explore the possibility of manipulating the mechanisms employed to recommend the way in which they should be coordinated.

The process that we follow to solve the problem is the following. First, we explore the possibility of designing mechanisms selecting recommendations on how agents should form research teams being stables from their point of view. For, we identify a family of environments in which stable research teams configurations always exist. These environments are characterized by a property called *tops responsiveness*, which can be seen as a natural extension of what Alcalde and Romero-Medina [3] called Essentiality. The second step to avoid the coordination costs is to identify a rule that always produces stable recommendations throughout the family of problems satisfying tops responsiveness. To reach this objective we present a rule, whose description could resemble the top trading cycle used by Shapley and Scarf [13]. This resemblance is founded as follows. Let us generate a (directed) graph whose nodes were the researchers and arcs go from each agent to the researchers being in her choice set. Then, we look for a set of agents, minimal under inclusion, guaranteeing the existence of a cycle on this di-graph.

To avoid the individual costs, the second problem we propose, we proceed as follows. First we show that the mechanism induced by the *tops covering algorithm* is strategy-proof. Therefore, no researcher would have any incentive on behave strategically by misrepresenting her true characteristics. This is perhaps the best way to guarantee that agents will not incur any individual cost. Moreover, we study the possibility of designing strategy-proof mechanisms for the research group formation problems and we found that the only one selecting stable solutions is the mechanism induced by the tops covering algorithm.

Just to conclude, we would like to remark that our last result contrasts with the findings by Sönmez [14], who shows that in a more general setting, the existence of strategy-proof stable rules is conditioned to the case of frameworks whose core is (essentially) a singleton. Let us observe that, as Example 5.3 points out, under tops responsiveness the set of stable research teams configurations could be not a singleton. Nevertheless, there is no contradiction between Sönmez's results and ours. In fact, the findings by Sönmez are strongly inspired in a condition he imposes on the set of admissible agents' preferences. He establishes that for any researcher r_i and each two researchers' teams S, S' in $\mathcal{A}^i$, if $S \succ_i S'$, then there exists preferences $\succsim_i'$, admissible for r_i, such that $S \succ_i' \emptyset \succ_i' S'$. Let us observe that tops responsiveness does not allow this possibility. In fact, if we consider $R = \{r_1, r_2, r_3\}$, Condition 2 in Definition 3.1, establishes that if $\{r_2\} \succ_1 \{r_2, r_3\} \succ_1 \emptyset$ there are no preferences $\succsim_1'$, satisfying tops responsiveness, such that $\{r_2\} \succ_1' \emptyset \succ_1' \{r_2, r_3\}$.

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APPENDIX 1.

The aim of this appendix is to give a formal proof for Theorem 4.4. More precisely, we will study the tops covering algorithm (TCA), introduced in Definition 4.2, and as a conclusion of our analysis we will show that, when researchers' preferences satisfy tops responsiveness, the set of stable research teams configurations is non-empty.

We will proceed as follows. First, Lemma 6.1 establishes the convergence of the TCA in finite steps. Secondly, Lemma 6.2 informs us that the output of the TCA, when researchers' preferences satisfy tops responsiveness, is always a research teams configuration. Finally, Proposition 6.3 concludes a constructive proof for Theorem 4.4. In particular, the former result establishes that the research teams configuration proposed by the TCA will be stable relative to the underlying research group formation problem.

Lemma 6.1. *Let $\{R, \succsim\}$ be a research group formation problem whose agents' preferences satisfy tops responsiveness. Then the TCA applied to $\{R, \succsim\}$ ends in finite steps.*

Proof. To prove our result we just need to show that, in Step 1 of the TCA the set of researchers fulfilling Condition (4.1) is non-empty.

First, let us observe that, if for some $r_i \in R$, $\emptyset \succ_i S$ for each $S \in \mathcal{A}^i$, $S \neq \emptyset$, it can be straightforwardly seen that r_i fulfills Condition (4.1). Notice that, in such a case, $\{r_i\} = \mathcal{CC}\left[\{r_i\}\,;\{R, \succsim\}\right]$. Therefore, to proceed with our proof, let us consider that, for each $r_i \in R$ there is a non-empty set $S^i \in \mathcal{A}^i$ such that $S^i \succ_i \emptyset$. Let us observe that, by construction, we have that for any two researchers r_i and r_j if $r_j \in \mathcal{CC}\left[\{r_i\}\,;\{R, \succsim\}\right]$, then

$$\mathcal{CC}\left[\{r_j\}\,;\{R, \succsim\}\right] \subset \mathcal{CC}\left[\{r_i\}\,;\{R, \succsim\}\right] \tag{6.1}$$

Let us construct a directed graph G whose vertices are the researchers and, for each r_i the set of her successors is

$$\Gamma_G^+(r_i) = \{r_j \in \mathcal{CC}\left[\{r_i\}\,;\{R, \succsim\}\right] : r_i \notin \mathcal{CC}\left[\{r_j\}\,;\{R, \succsim\}\right]\}$$

Let us observe that G must have no cycle. In fact, if we had a cycle involving some researchers, say $r^0, \ldots, r^k$, with

$$r^h \in \Gamma_G^+\left(r^{h-1}\right),\ h = 1, \ldots, k, \text{ and } r^0 \in \Gamma_G^+\left(r^k\right)$$

it should be the case that, by (6.1)

$$\begin{aligned}\mathcal{CC}\left[\left\{r^0\right\};\{R,\succsim\}\right] &\subset \mathcal{CC}\left[\left\{r^k\right\};\{R,\succsim\}\right] \subset \mathcal{CC}\left[\left\{r^{k-1}\right\};\{R,\succsim\}\right] \subset \ldots \\ &\subset \mathcal{CC}\left[\left\{r^1\right\};\{R,\succsim\}\right] \subset \mathcal{CC}\left[\left\{r^0\right\};\{R,\succsim\}\right].\end{aligned}$$

Therefore, for each $h = 0, \ldots, k-1$,

$$\mathcal{CC}\left[\left\{r^h\right\};\{R,\succsim\}\right] = \mathcal{CC}\left[\left\{r^{h+1}\right\};\{R,\succsim\}\right]$$

which contradicts that $r^h \in \Gamma_G^+\left(r^{h-1}\right)$, for each $h = 1, \ldots, k$.

Since G contains no cycle and the number of vertices is finite, we have that, at least one of the vertices must have no successor, i.e. there is some researcher $r_{\hat{\imath}} \in R$ such that $\Gamma_G^+(r_{\hat{\imath}}) = \emptyset$. Since there is some non-empty set $S^{\hat{\imath}}$ such that $S^{\hat{\imath}} \succ_{\hat{\imath}} \emptyset$, we have that $\mathcal{CC}\left[\{r_{\hat{\imath}}\};\{R,\succsim\}\right] \neq \emptyset$, and since $\Gamma_G^+(r_{\hat{\imath}}) = \emptyset$, $r_{\hat{\imath}} \in \mathcal{CC}\left[\{r_j\};\{R,\succsim\}\right]$ for each $r_j \in \mathcal{CC}\left[\{r_{\hat{\imath}}\};\{R,\succsim\}\right]$. Therefore the non-empty set of agents $\mathcal{CC}\left[\{r_{\hat{\imath}}\};\{R,\succsim\}\right]$ fulfills Condition (4.1) in Definition 4.2.

To conclude this proof, let us observe that if researchers' preferences in $\{R,\succsim\}$ satisfy tops responsiveness, then $\{R^1,\succsim|_{R^1}\}$ is also a research group formation problem whose agents' preferences satisfy tops responsiveness. Therefore, if $R^1 \subsetneq R$, taking into account that R has a finite number of elements, a iterative argument yields the desired result. ∎

Lemma 6.2. *Let $\{R,\succsim\}$ be a research group formation problem, with $\succsim \in \mathcal{TR}$. Then $\Psi^{tc}\left[\{R,\succsim\}\right]$ is a research teams configuration for $\{R,\succsim\}$.*

Proof. Since, for each research group formation problem, say $\{R,\succsim\}$, we have that the application of the tops covering algorithm ends in finite steps (Lemma 6.1), we just need to show that for each two researchers r_i and r_j,

$$\Psi_i^{tc}\left[\{R,\succsim\}\right] \cap \Psi_j^{tc}\left[\{R,\succsim\}\right] \neq \emptyset \text{ implies that } \Psi_i^{tc}\left[\{R,\succsim\}\right] = \Psi_j^{tc}\left[\{R,\succsim\}\right].$$

Let us assume that there are two researchers, say r_i and r_j, such that

$$r_h \in \Psi_i^{tc}\left[\{R,\succsim\}\right] \cap \Psi_j^{tc}\left[\{R,\succsim\}\right] \neq \emptyset$$

for some $r_h \in R^{k-1}$. By Definition 4.2, we have that both agents are allocated at the same step of the tops covering algorithm. Let k be such a step. Therefore, we have that, since $r_h \in \Psi_i^{tc}\left[\{R,\succsim\}\right]$,

$$\begin{aligned}\mathcal{CC}\left[\{r_i\};\left\{R^{k-1},\succsim|_{R^{k-1}}\right\}\right] &= \mathcal{CC}\left[\{r_h\};\left\{R^{k-1},\succsim|_{R^{k-1}}\right\}\right] = \\ = \bigcap_{r_t \in \mathcal{CC}\left[\{r_h\};\left\{R^{k-1},\succsim|_{R^{k-1}}\right\}\right]} &\left\{\mathcal{CC}\left[\{r_t\};\left\{R^{k-1},\succsim|_{R^{k-1}}\right\}\right]\right\}.\end{aligned}$$

Similarly, since $r_h \in \Psi_j^{tc}[\{R, \succsim\}]$, we have that

$$\mathcal{CC}\left[\{r_j\};\left\{R^{k-1}, \succsim|_{R^{k-1}}\right\}\right] = \mathcal{CC}\left[\{r_h\};\left\{R^{k-1}, \succsim|_{R^{k-1}}\right\}\right] = \\ = \bigcap_{r_t \in \mathcal{CC}\left[\{r_h\};\left\{R^{k-1}, \succsim|_{R^{k-1}}\right\}\right]} \left\{\mathcal{CC}\left[\{r_t\};\left\{R^{k-1}, \succsim|_{R^{k-1}}\right\}\right]\right\}.$$

Then, we have that

$$\Psi_i^{tc}[\{R, \succsim\}] = \Psi_j^{tc}[\{R, \succsim\}]. \blacksquare$$

Proposition 6.3. *Let $\{R, \succsim\}$ be a research group formation problem whose agents' preferences satisfy tops responsiveness. Then $\Psi^{tc}[\{R, \succsim\}]$ is a stable research teams configuration for $\{R, \succsim\}$.*

Proof. Let $\{R, \succsim\}$ a research group formation problem whose agents' preferences satisfy tops responsiveness, and let us assume that the research teams configuration proposed by the tops covering algorithm, $\Psi^{tc}[\{R, \succsim\}]$, is unstable for $\{R, \succsim\}$. Then, there should be a set of researchers, say T, such that for each $r_i \in T$

$$T \setminus \{r_i\} \succ_i \Psi_i^{tc}[\{R, \succsim\}] \setminus \{r_i\} \tag{6.2}$$

Let $k(r_i)$ denote the step of the tops covering algorithm in which r_i is assigned to a research team. Let r_j be an agent in T such that $k(r_i) \geq k(r_j)$ for each $r_i \in T$. Then, we have that

$$Ch_j\left(R^{k(r_j)-1}\right) \subset \mathcal{CC}\left[\{r_j\};\left\{R^{k(r_j)-1}, \succsim|_{R^{k(r_j)-1}}\right\}\right] = \Psi_j^{tc}[\{R, \succsim\}] \tag{6.3}$$

Combining Equation (6.2) and Condition 1 in Definition 3.1, we have that

$$Ch_j\left(R^{k(r_j)-1}\right) \subset T$$

By the equation above we have that, for each $r_i \in Ch_j\left(R^{k(r_j)-1}\right)$,

$$Ch_i\left(R^{k(r_j)-1}\right) \subset T$$

This implies that

$$\mathcal{C}^2\left[\{r_j\};\left\{R^{k(r_j)-1}, \succsim|_{R^{k(r_j)-1}}\right\}\right] \subset T$$

Taking into account that, for each positive integer t

$$\mathcal{C}^t \left[\{r_j\} ; \left\{ R^{k(r_j)-1}, \succsim|_{R^{k\left(r_j\right)-1}} \right\} \right] \subset \mathcal{C}^{t+1} \left[\{r_j\} ; \left\{ R^{k(r_j)-1}, \succsim|_{R^{k\left(r_j\right)-1}} \right\} \right] \subset T$$

and applying an iterative argument, we get that

$$\mathcal{CC} \left[\{r_j\} ; \left\{ R^{k(r_j)-1}, \succsim|_{R^{k\left(r_j\right)-1}} \right\} \right] \subset T \tag{6.4}$$

Combining equations (6.3) and (6.4) we have that, by Condition 2 in Definition 3.1,

$$\Psi_j^{tc} \left[\{R, \succsim\} \right] \setminus \{r_j\} \succsim_j T \setminus \{r_j\} ,$$

which contradicts our hypothesis in Equation (6.2). ∎

APPENDIX 2.

The aim of this appendix is to provide a formal proof for Theorem 5.2, i.e., we want to show that the tops covering mechanism is strategy proof.

Proof of Theorem 5.2 To obtain a contradiction, let us assume that statement of Theorem 5.2 is not true. Then there should be a research group formation problem $\{R, \succsim\}$, an agent $r_i \in R$, and preferences $\succsim'_i$ satisfying tops responsiveness such that

$$\Psi_i^{tc}\left[\{R, [\succsim'_i, \succsim_{-i}]\}\right] \setminus \{r_i\} \succ_i \Psi_i^{tc}\left[\{R, \succsim\}\right] \setminus \{r_i\}.$$

For each agent $r_j \in R$, let $k(r_j)$ denote the step of the tops covering algorithm in which r_j is assigned to a researchers team when applied to $\{R, \succsim\}$.[2] Let us observe that, for each researcher r_j such that $k(r_j) < k(r_i)$ we have that

$$\Psi_j^{tc}\left[\{R, \succsim\}\right] = \Psi_j^{tc}\left[\{R, [\succsim'_i, \succsim_{-i}]\}\right].$$

Hence, it should be the case that

$$\Psi_i^{tc}\left[\{R, [\succsim'_i, \succsim_{-i}]\}\right] \subset R^{k(r_i)-1},$$

with $R^0 \equiv R$. Moreover, $r_i \in \mathcal{CC}\left[\{r_j\}; \left\{R^{k(r_i)-1}, \succsim|_{R^{k(r_i)-1}}\right\}\right]$ for each $r_j \in \Psi_i^{tc}\left[\{R, \succsim\}\right]$. This implies that, for the problem $\{R, [\succsim'_i, \succsim_{-i}]\} \equiv \{R, \succsim'\}$ it should be the case that

$$\Psi_i^{tc}\left[\{R, [\succsim'_i, \succsim_{-i}]\}\right] \supset \bigcup_{r_j \in Ch_i\left(R^{k(r_i)-1}; \succsim'_i\right)} \mathcal{CC}\left[\{r_j\}; \left\{R^{k(r_i)-1}, \succsim'|_{R^{k(r_i)-1}}\right\}\right].$$

Let consider the following two possible cases:

(*a*) $\Psi_i^{tc}\left[\{R, \succsim\}\right] \subset \Psi_i^{tc}\left[\{R, [\succsim'_i, \succsim_{-i}]\}\right]$. Then it should be the case that

$$Ch_i\left(R^{k(r_i)-1}; \succsim_i\right) \subset \Psi_i^{tc}\left[\{R, \succsim\}\right] \cap \Psi_i^{tc}\left[\{R, [\succsim'_i, \succsim_{-i}]\}\right],$$

and thus, by Condition 2 in Definition 3.1, we have that

$$\Psi_i^{tc}\left[\{R, \succsim\}\right] \setminus \{r_i\} \succsim_i \Psi_i^{tc}\left[\{R, [\succsim'_i, \succsim_{-i}]\}\right] \setminus \{r_i\},$$

which contradicts that r_i manipulates Ψ^{tc} at $\{R, \succsim\}$ via $\succsim'_i$;

[2] Following the notation employed in Definition 4.2, $k(r_j)$ is the integrer such that $r_i \in R^{k(r_j)-1} \setminus R^{k(r_j)}$, with $R = R^0$.

(*b*) $\Psi_i^{tc}[\{R, \succsim\}] \not\subset \Psi_i^{tc}[\{R, [\succsim_i', \succsim_{-i}]\}]$. Then it should be a researcher r_h such that $r_h \in \Psi_i^{tc}[\{R, \succsim\}]$ and $r_h \notin \Psi_i^{tc}[\{R, [\succsim_i', \succsim_{-i}]\}]$. Let us observe that, it must be the case that

$$Ch_i\left(R^{k(r_i)-1}; \succsim_i\right) \not\subset \Psi_i^{tc}[\{R, [\succsim_i', \succsim_{-i}]\}],$$

and hence

$$\begin{aligned} Ch_i\left(R^{k(r_i)-1}; \succsim_i\right) &= Ch_i\left(\Psi_i^{tc}[\{R, \succsim\}] \setminus \{r_i\}; \succsim_i\right) \neq \\ &\neq Ch_i\left(\Psi_i^{tc}[\{R, [\succsim_i', \succsim_{-i}]\}] \setminus \{r_i\}; \succsim_i\right), \end{aligned}$$

then, applying Condition 1 in Definition 3.1, we have that

$$\Psi_i^{tc}[\{R, \succsim\}] \setminus \{r_i\} \succ_i \Psi_i^{tc}[\{R, [\succsim_i', \succsim_{-i}]\}] \setminus \{r_i\},$$

which contradicts that r_i manipulates Ψ^{tc} at $\{R, \succsim\}$ via $\succsim_i'$. ∎

APPENDIX 3.

The aim of this appendix is to give a formal proof for Theorem 5.4.

Let us introduce a way of describing how agents could manipulate. The idea beyond this possibility of manipulation is somehow similar to the one used in an impossibility result due to Alcalde and Barberà [2]. Let $\{R, \succsim\}$ be a research group formation problem whose agents' preferences satisfy tops responsiveness. For each researcher r_i let construct preferences $\succsim_i^{tc}$ as follows. First, for each two non-empty researchers' teams S, S' in $\mathcal{A}^i$

$$S \succsim_i S' \Rightarrow S \succsim_i^{tc} S',$$

and, secondly, for each $S \in \mathcal{A}^i$, $S \succ_i^{tc} \emptyset$ only if $Ch_i(S) \succsim_i \Psi_i^{tc}[\{R, \succsim\}] \setminus \{r_i\}$. In words, $\succsim_i^{tc}$ is obtained by $\succsim_i$ by establishing that r_i considers unacceptable under $\succsim_i^{tc}$ any research team worse than her set of partners in $\Psi_i^{tc}[\{R, \succsim\}]$, except the researchers teams being supersets of $\Psi_i^{tc}[\{R, \succsim\}]$, just for $\succsim_i^{tc}$ to satisfy tops responsiveness. Let us observe that, since $\Psi^{tc}[\{R, \succsim\}]$ is stable for $\{R, \succsim\}$, it is easy to see that $\Psi^{tc}[\{R, \succsim\}]$ is stable for any research group formation problem $\{R, \succsim'\}$ where

$$\succsim_i' \in \left\{\succsim_i, \succsim_i^{tc}\right\} \text{ for each researcher } r_i.$$

Moreover, it also holds that, for each such a problem $\{R, \succsim'\}$

$$\Psi^{tc}[\{R, \succsim\}] = \Psi^{tc}[\{R, \succsim'\}].$$

Thus, a way in which a researcher r_i might manipulate some stable rule Ψ is by declaring preferences $\succsim_i^{tc}$. In such a case, r_i must be in a research team $\Psi_i[\{R, \succsim\}]$ satisfying that

$$\Psi_i^{tc}[\{R, \succsim\}] \setminus \{r_i\} \succ_i \Psi_i[\{R, \succsim\}] \setminus \{r_i\}.$$

An iterative argument on the researcher belonging to $\Psi_i^{tc}[\{R, \succsim\}]$ will provide the desired result. We now proceed to formally prove our Theorem 5.4.

Proof of Theorem 5.4 By Theorem 5.2 we know that the stable mechanism induced by the tops covering algorithm is strategy-proof. To show that the statement of Theorem 5.4 is true we will proceed by contradiction. Then, let us assume that there is some strategy-proof stable mechanism Ψ whose domain is $\mathcal{PTC}$, different from Ψ^{tc}. Then, there should be a research group formation problem $\{R, \succsim\} \in \mathcal{PTC}$ such that

$$\Psi\left[\{R, \succsim\}\right] \neq \Psi^{tc}\left[\{R, \succsim\}\right].$$

Since both $\Psi\left[\{R, \succsim\}\right]$ and $\Psi^{tc}\left[\{R, \succsim\}\right]$ are stable for $\{R, \succsim\}$ there should be a researcher r_i preferring her colleagues under $\Psi\left[\{R, \succsim\}\right]$ rather than the ones that $\Psi^{tc}\left[\{R, \succsim\}\right]$ assigned to her. Moreover, by the stability of the above research teams configurations we have that for each r_i such that

$$\Psi_i\left[\{R, \succsim\}\right] \setminus \{r_i\} \succ_i \Psi_i^{tc}\left[\{R, \succsim\}\right] \setminus \{r_i\} \tag{6.5}$$

there should be a researcher $r_j \in \Psi_i\left[\{R, \succsim\}\right]$ such that

$$\Psi_j^{tc}\left[\{R, \succsim\}\right] \setminus \{r_j\} \succ_j \Psi_i\left[\{R, \succsim\}\right] \setminus \{r_j\}.$$

Let $k(r_i)$ denote the step of the tops covering algorithm in which r_i is assigned to a research team, when applied to $\{R, \succsim\}$.

Let us assume that there is some researcher r_i such that $k(r_i) = 1$ for which Equation (6.5) is fulfilled. Then, by Condition 2 in Definition 3.1, it should be the case that

$$Ch_i(R; \succsim_i) \subset \Psi_i\left[\{R, \succsim\}\right] \setminus \{r_i\}$$

By construction of $\Psi_i^{tc}\left[\{R, \succsim\}\right]$, and given that $\Psi^{tc}\left[\{R, \succsim\}\right]$ satisfies individual rationality, we have that $\Psi_i^{tc}\left[\{R, \succsim\}\right] \neq \{r_i\}$. Then, there should be an agent, say $r_{i(1)}$, such that

$$r_{i(1)} \in \Psi_i\left[\{R, \succsim\}\right] \cap \Psi_i^{tc}\left[\{R, \succsim\}\right] \tag{6.6}$$

and, by stability of Ψ^{tc}, for some researcher satisfying (6.6) above,

$$Ch_{i(1)}\left(R; \succsim_{i(1)}\right) \nsubseteq \Psi_{i(1)}\left[\{R, \succsim\}\right] \setminus \left\{r_{i(1)}\right\}. \tag{6.7}$$

Now, let us consider that $r_{i(1)}$ declares preferences $\succsim_{i(1)}^{tc}$. Let us observe that, in such a case, we have that any research teams configuration τ, stable for $\left\{R, \left[\succsim_{i(1)}^{tc}, \succsim_{-i}\right]\right\}$ must satisfy that

1. $\tau\left(r_{i(1)}\right) = \left\{r_{i(1)}\right\}$, or

2. $\tau\left(r_{i(1)}\right)\setminus\left\{r_{i(1)}\right\}\supset Ch_{i(1)}\left(R;\succsim_{i(1)}^{tc}\right)=Ch_{i(1)}\left(R;\succsim_{i(1)}\right)$.

In particular, we have that mechanism Ψ should produce a research teams configuration $\Psi\left\{R,\left[\succsim_{i(1)}^{tc},\succsim_{-i}\right]\right\}$ such that, if $\Psi_{i(1)}\left\{R,\left[\succsim_{i(1)}^{tc},\succsim_{-i}\right]\right\}\neq\left\{r_{i(1)}\right\}$ then

$$Ch_{i(1)}\left(R;\succsim_{i(1)}^{tc}\right)\subset\Psi_{i(1)}\left\{R,\left[\succsim_{i(1)}^{tc},\succsim_{-i}\right]\right\}\setminus\left\{r_{i(1)}\right\} \tag{6.8}$$

Notice that, since Ψ is strategy-proof, (6.8) must not be satisfied because in such a case, taking into account Condition 1 in Definition 3.1 and Equation (6.7), we have that

$$\Psi_{i(1)}\left\{R,\left[\succsim_{i(1)}^{tc},\succsim_{-i(1)}\right]\right\}\setminus\left\{r_{i(1)}\right\}\succ_{i(1)}\Psi_{i(1)}\left\{R,\succsim\right\}\setminus\left\{r_{i(1)}\right\},$$

which is equivalent to say that $r_{i(1)}$ could manipulate Ψ at $\{R,\succsim\}$ via $\succsim_{i(1)}^{tc}$. Thus Ψ's strategy-proofness must imply that

$$\Psi_{i(1)}\left\{R,\left[\succsim_{i(1)}^{tc},\succsim_{-i}\right]\right\}=\left\{r_{i(1)}\right\}. \tag{6.9}$$

Moreover, given that $\Psi_{i(1)}^{tc}\left[\{R,\succsim\}\right]\setminus\left\{r_{i(1)}\right\}$ is non-empty, there should be a researcher, say $r_{i(2)}$ such that

$$r_{i(1)}\in Ch_{i(2)}\left(R;\succsim_{i(2)}\right). \tag{6.10}$$

This is due to the fact that $k\left(r_{i(1)}\right)=1$ and that $\Psi^{tc}\left[\{R,\succsim\}\right]$ is individually rational for $\{R,\succsim\}$.

Given that

$$\Psi^{tc}\left[\{R,\succsim\}\right]\equiv\Psi^{tc}\left[\left\{R,\left[\succsim_{i(1)}^{tc},\succsim_{-i(1)}\right]\right\}\right],\text{ and}$$

$$r_{i(1)}\notin\Psi_{i(2)}\left[\left\{R,\left[\succsim_{i(1)}^{tc},\succsim_{-i(1)}\right]\right\}\right],$$

we have that

$$\Psi_{i(2)}^{tc}\left[\left\{R,\left[\succsim_{i(1)}^{tc},\succsim_{-i(1)}\right]\right\}\right]\setminus\left\{r_{i(2)}\right\}\succ_{i(2)}\Psi_{i(2)}\left[\left\{R,\left[\succsim_{i(1)}^{tc},\succsim_{-i(1)}\right]\right\}\setminus\left\{r_{i(2)}\right\}\right].$$

Now, let us consider the research group formation problem $\{R, \succsim^2\}$, with $\succsim_h^2 = \succsim_h^{tc}$ if $h \in \{i(1), i(2)\}$ and $\succsim_h^2 = \succsim_h$ otherwise. By applying the above arguments (on Ψ's manipulability by $r_{i(1)}$), we have that it must be the case that

$$\Psi_{i(2)}\left[\left\{R, \succsim^2\right\}\right] = \left\{r_{i(2)}\right\}.$$

Let us observe that, otherwise, it should be the case that $Ch_{i(2)}\left(R; \succsim_{i(2)}\right) \subset \Psi_{i(2)}\left[\{R, \succsim^2\}\right] \backslash \left\{r_{i(2)}\right\}$, and hence $r_{i(2)}$ would manipulate Ψ at $\left\{R, \left[\succsim_{i(1)}^{tc}, \succsim_{-i(1)}\right]\right\}$ via $\succsim_{i(2)}^{tc}$. At this point we have the following two possibilities:

(a_2) $\Psi_i^{tc}\left[\{R, \succsim\}\right] = \left\{r_{i(1)}, r_{i(2)}\right\}$. Then, it should be the case that $Ch_{i(h)}\left(R; \succsim_{i(h)}\right) \cup r_{i(h)} = \left\{r_{i(1)}, r_{i(2)}\right\}$ for $h = 1, 2$, which contradicts that $\Psi\left[\left\{R, \left[\succsim_{i(1)}^{tc}, \succsim_{-i(1)}\right]\right\}\right]$ were stable for $\left[\left\{R, \left[\succsim_{i(1)}^{tc}, \succsim_{-i(1)}\right]\right\}\right]$ because $\left\{r_{i(1)}, r_{i(2)}\right\}$ could block such a research teams configuration; or

(b_2) $\Psi_i^{tc}\left[\{R, \succsim\}\right] \neq \left\{r_{i(1)}, r_{i(2)}\right\}$. Then, there should be a researcher, say $r_{i(3)} \in \Psi_i^{tc}\left[\{R, \succsim\}\right] \setminus \left\{r_{i(1)}, r_{i(2)}\right\}$ such that

$$\left\{r_{i(1)}, r_{i(2)}\right\} \cap Ch_{i(3)}\left(R; \succsim_{i(3)}\right) \neq \emptyset$$

Let us select $r_{i(3)}$ such that

$$r_{i(2)} \in Ch_{i(3)}\left(R; \succsim_{i(3)}\right). \tag{6.11}$$

Since $\Psi_{i(2)}\left[\{R, \succsim^2\}\right] = \left\{r_{i(2)}\right\}$, it must be the case that

$$Ch_{i(3)}\left(R; \succsim_{i(3)}\right) \nsubseteq \Psi_{i(3)}\left[\left\{R, \succsim^2\right\}\right]$$

Now, let us consider the research group formation problem

$$\left\{R, \succsim^3\right\} \equiv \left\{R, \left[\succsim_{i(3)}^{tc}, \succsim_{-i(3)}^2\right]\right\}.$$

Let us observe that

$$\Psi_{i(3)}\left[\left\{R, \succsim^3\right\}\right] = \left\{r_{i(3)}\right\}$$

because otherwise $Ch_{i(3)}\left(R; \succsim_{i(3)}\right) \subset \Psi_{i(3)}\left[\{R, \succsim^3\}\right] \backslash \{r_3\}$, which implies that $r_{i(3)}$ might manipulate Ψ at $\{R, \succsim^2\}$ via $\succsim_{i(3)}^{tc}$.

If there is no $r_{i(3)}$ satisfying (6.11), then choose any $r_{i(3)}$ such that $r_{i(1)} \in Ch_{i(3)}\left(R; \succsim_{i(3)}\right)$. Given that $r_{i(2)} \in \Psi_i^{tc}\left[\{R, \succsim\}\right]$ and $r_{i(2)} \notin Ch_{i(3)}\left(R; \succsim_{i(3)}\right)$ for all $r_{i(3)} \in \Psi_i^{tc}\left[\{R, \succsim\}\right] \setminus \left\{r_{i(1)}, r_{i(2)}\right\}$, then $r_{i(2)} \in Ch_{i(1)}\left(R; \succsim_{i(1)}\right)$. Let us note that, in such a case, we have that

$$\Psi_{i(2)}\left[\left\{R, \succsim^2\right\}\right] = \left\{r_{i(2)}\right\},$$

and, for each (non-empty) $S \in \mathcal{A}^{i(1)}$ such that $r_{i(2)} \notin S$

$$\emptyset \succ_{i(1)}^{tc} S.$$

Then, it must be the case that

$$\Psi_{i(3)}\left[\left\{R, \succsim^3\right\}\right] = \left\{r_{i(3)}\right\}.$$

Let us consider the following possibilities:

(a_3) $\Psi_i^{tc}\left[\{R, \succsim\}\right] = \left\{r_{i(1)}, r_{i(2)}, r_{i(3)}\right\}$. Then, it should be the case that

$$Ch_{i(h)}\left(R; \succsim_{i(h)}\right) \cup r_{i(h)} = \left\{r_{i(1)}, r_{i(2)}, r_{i(3)}\right\}$$

for $h = 1, 2, 3$, which contradicts that $\Psi\left[\{R, \succsim^2\}\right]$ were stable for $\left[\{R, \succsim^2\}\right]$ because $\left\{r_{i(1)}, r_{i(2)}, r_{i(3)}\right\}$ could block such a research teams configuration; or

(b_3) $\Psi_i^{tc}\left[\{R, \succsim\}\right] \neq \left\{r_{i(1)}, r_{i(2)}, r_{i(3)}\right\}$. Then there should be a researcher, say $r_{i(4)} \in \Psi_i^{tc}\left[\{R, \succsim\}\right] \setminus \left\{r_{i(1)}, r_{i(2)}, r_{i(3)}\right\}$ such that

$$\left\{r_{i(1)}, r_{i(2)}, r_{i(3)}\right\} \cap Ch_{i(4)}\left(R; \succsim_{i(4)}\right) \neq \emptyset.$$

Let us select $r_{i(4)}$ according the following priorities rule:

[1] $r_{i(3)} \in Ch_{i(4)}\left(R; \succsim_{i(4)}\right)$. In this case, since

$$\Psi_{i(3)}\left[\left\{R, \succsim^3\right\}\right] = \left\{r_{i(3)}\right\},$$

we have that

$$Ch_{i(4)}\left(R; \succsim_{i(4)}\right) \nsubseteq \Psi_{i(4)}\left[\left\{R, \succsim^3\right\}\right]$$

If there is no researcher $r_{i(4)}$ satisfying [1], let select $r_{i(4)}$ to satisfy [2] following.

[2] If there is no agent satisfying [1], we have that

$$r_{i(3)} \in Ch_{i(1)}\left(R; \succsim_{i(1)}\right) \cup Ch_{i(2)}\left(R; \succsim_{i(2)}\right)$$

Let us select $r_{i(4)}$ such that

[α] $r_{i(2)} \in Ch_{i(4)}\left(R; \succsim_{i(4)}\right)$ if $r_{i(3)} \in Ch_{i(2)}\left(R; \succsim_{i(2)}\right)$. Then, it must be the case that $Ch_{i(4)} \nsubseteq \Psi_{i(4)}\left[\{R, \succsim^3\}\right]$ because

$$\Psi_{i(3)}\left[\left\{R, \succsim^3\right\}\right] = r_{i(3)}$$

and, for each (non-empty) $S \in \mathcal{A}^{i(2)}$ such that $r_{i(3)} \notin S$

$$\emptyset \succ^{tc}_{i(2)} S$$

If no $r_{i(4)}$ satisfies [α], let us select $r_{i(4)}$ to satisfy [β].

[β] $r_{i(1)} \in Ch_{i(4)}\left(R; \succsim_{i(4)}\right)$ if $r_{i(3)} \in Ch_{i(1)}\left(R; \succsim_{i(1)}\right)$. Let us observe that using the arguments employed in [α] above, just exchanging the roles of $r_{i(2)}$ and $r_{i(1)}$, we have that it must be satisfied that $Ch_{i(4)} \nsubseteq \Psi_{i(4)}\left[\{R, \succsim^3\}\right]$.

[3] Finally, if there is no $r_{i(4)}$ satisfying either [1] nor [2], let choose $r_{i(4)}$ as follows.

[α] $r_{i(1)} \in Ch_{i(4)}\left(R; \succsim_{i(4)}\right)$ if $r_{i(3)} \notin Ch_{i(1)}\left(R; \succsim_{i(1)}\right)$. Let us observe that, in such a case, by Condition (b_2) above, we have that $r_{i(3)} \in Ch_{i(2)}\left(R; \succsim_{i(2)}\right)$; since condition [2.$\alpha$] above does not apply, there is no $r_{i(4)}$ such that $r_{i(2)} \in Ch_{i(4)}\left(R; \succsim_{i(4)}\right)$. Because $r_{i(4)} \in \Psi^{tc}_i\left[\{R, \succsim\}\right] \setminus \left\{r_{i(1)}, r_{i(2)}, r_{i(3)}\right\}$, it must be the case that $r_{i(2)} \in Ch_{i(1)}\left(R; \succsim_{i(1)}\right)$. Then, $Ch_{i(4)}\left(R; \succsim_{i(4)}\right) \nsubseteq \Psi_{i(4)}\left[\{R, \succsim^3\}\right]$. This is because

$$\Psi_{i(3)}\left[\left\{R, \succsim^3\right\}\right] = \left\{r_{i(3)}\right\},$$

for each (non-empty) $S \in \mathcal{A}^{i(2)}$ such that $r_{i(3)} \notin S$

$$\emptyset \succ^{tc}_{i(2)} S,$$

and for each $S' \in \mathcal{A}^{i(1)}$, $S' \neq \emptyset$, such that $r_{i(2)} \notin S'$

$$\emptyset \succ^{tc}_{i(1)} S'.$$

Otherwise, if there is not such a $r_{i(4)}$, let consider the next only possibility,

[β] $r_{i(2)} \in Ch_{i(4)}\left(R; \succsim_{i(4)}\right)$ if $r_{i(3)} \notin Ch_{i(2)}\left(R; \succsim_{i(2)}\right)$. Let us observe that the above arguments, by exchanging the roles by $r_{i(1)}$ and $r_{i(2)}$ will give us that

$$Ch_{i(4)}\left(R; \succsim_{i(4)}\right) \nsubseteq \Psi_{i(4)}\left[\left\{R, \succsim^{3}\right\}\right]$$

Note that, since $\Psi_i^{tc}[\{R, \succsim\}] \neq \left\{r_{i(1)}, r_{i(2)}, r_{i(3)}\right\}$ there should be $r_{i(4)}$ satisfying some of the above three cases. Now, let us note that Ψ's stability will imply that, when considering the problem $\{R, \succsim^4\} = \left\{R, \left[\succsim_{i(4)}^{tc}, \succsim_{-i(4)}^{3}\right]\right\}$

$$\Psi_{i(4)}\left[\left\{R, \left[\succsim_{i(4)}^{tc}, \succsim_{-i(4)}^{3}\right]\right\}\right] = \left\{r_{i(4)}\right\}$$

whenever Ψ is strategy-proof.

Let us observe that, when analyzing $\Psi_i^{tc}[\{R, \succsim^4\}]$ there are also two possibilities:

(a_4) $\Psi_i^{tc}[\{R, \succsim\}] = \left\{r_{i(1)}, r_{i(2)}, r_{i(3)}, r_{i(4)}\right\}$. This case conflicts with the fact that Ψ is a stable mechanism, since

$$\Psi_{i(h)}\left[\left\{R, \succsim^{4}\right\}\right] = \left\{r_{i(h)}\right\}, \text{ for all } h = 1, \ldots, 4$$

and thus the researchers team $\left\{r_{i(1)}, r_{i(2)}, r_{i(3)}, r_{i(4)}\right\}$ blocks $\Psi[\{R, \succsim^4\}]$, or

(b_4) $\Psi_i^{tc}[\{R, \succsim\}] \neq \left\{r_{i(1)}, r_{i(2)}, r_{i(3)}, r_{i(4)}\right\}$. Then, there should be a researcher, say $r_{i(5)}$ in $\Psi_i^{tc}[\{R, \succsim\}] \setminus \left\{r_{i(1)}, r_{i(2)}, r_{i(3)}, r_{i(4)}\right\}$ such that

$$\left\{r_{i(1)}, r_{i(2)}, r_{i(3)}, r_{i(4)}\right\} \cap Ch_{i(5)}\left(R; \succsim_{i(5)}\right) \neq \emptyset.$$

Just to conclude, let us note that we can now select $r_{i(5)}$ in a similar way to the one explained for $r_{i(4)}$, i.e., first select $r_{i(5)}$ such that $r_{i(4)} \in Ch_{i(5)}\left(R; \succsim_{i(5)}\right)$. If such an $r_{i(5)}$ does not exist, since $r_{i(5)} \in \Psi_i^{tc}[\{R, \succsim\}]$ then there should be a researcher $r_{i(5)}$ such that

$$r_{i(5)} \in \bigcup_{h=1}^{3} Ch_{i(h)}\left(R; \succsim_{i(h)}\right).$$

Let us replicate the steps [2] and [3] above (done for $r_{i(4)}$) exhausting all the possibilities to show that

$$\Psi_{i(5)}\left[\left\{R, \succsim^{4}\right\}\right] = \left\{r_{i(5)}\right\}.$$

Since $\Psi_i^{tc}[\{R, \succsim\}]$ has a finite number of researchers, an iterative argument will recover this set, i.e., we will find $r_{i(1)}, \ldots, r_{i(t)}$ such that

$$\Psi_i^{tc}[\{R, \succsim\}] = \bigcup_{h=1}^{t} \{r_{i(h)}\},$$

in such a way that, since Ψ proposes stable research teams configurations, agent $r_{i(t)}$ will manipulate Ψ at $\{R, \succsim^{t-1}\}$ via $\succsim_{i(t)}^{tc}$, with $\succsim_h^{t-1} := \succsim_h^{tc}$ if $h \in \bigcup_{h=1}^{t-1} \{r_{i(h)}\}$ or $\succsim_h^{t-1} := \succsim_h$ otherwise.

To conclude this proof, let us observe that if there is no r_i such that $k(r_i) = 1$ and $\Psi_i[\{R, \succsim\}] \setminus \{r_i\} \succ_i \Psi_i^{tc}[\{R, \succsim\}] \setminus \{r_i\}$, we can identify a researcher, say r_j such that

$$\Psi_j[\{R, \succsim\}] \setminus \{r_j\} \succ_j \Psi_j^{tc}[\{R, \succsim\}] \setminus \{r_j\} \tag{6.12}$$

minimizing $k(r_j)$ among all the researchers satisfying (6.12). Then the above arguments could be replicated for the problem

$$\left\{R^{k(r_j)-1}, \succsim|_{R^{k(r_j)-1}}\right\},$$

and obtaining the desired results. ∎