Going Multinational under Exchange Rate Uncertainty

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Abstract

A domestic exporting firm faces exchange rate uncertainty and has the option to install capacity abroad becoming multinational. We analyze when should the firm exercise such option optimally. We extend the option theory of investment by considering a Cournot market equilibrium. There are four main findings. First, the degree of hysteresis in foreign direct investment (FDI) grows as the number of firms increases. Second, a maintenance cost may induce the exporting firm to sustain losses, i.e. dumping. Third, the FDI-inducing effect of tariffs is decreasing in the number of firms. Fourth, FDI reduces exchange rate pass-through, specially for the range of exchange rate values that it would have been maximal otherwise.

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1 Introduction

When a domestic firm decides to install capacity in a foreign country it creates a flow of foreign direct investment (FDI). This decision might be triggered by several real factors as studied in the international trade literature, e.g. Wong (1995). Nominal exchange rate movements may also affect FDI. It can be argued that going multinational will be more profitable for domestic firms as the domestic currency appreciates. There are two reasons why this can be the case. First, with an appreciating domestic currency, exporting firms become less competitive in foreign markets and, hence, more likely to go multinational. Second, the sunk cost of setting a plant in a foreign country will be cheaper the more appreciated the domestic currency. Symmetrically, setting a plant in the domestic economy will be more attractive for foreign firms as the domestic currency depreciates. Therefore, intuitively at least, exchange rate fluctuations may trigger FDI flows.

Most flows of FDI take place among industrial economies whose exchange rates float freely. This observation has given rise to an interesting field in international economics which explores the relation between exchange rates and FDI. Theoretical models such as Golbergerg and Kolstad (1995) and Sung and Lapan (2000) show that exchange rate movements influence the location decision of firms. The empirical evidence available on the relationship between exchange rate fluctuations and FDI includes Blonigen (1997), who argues that the exchange rate may affect FDI because acquisitions involve firm-specific assets which can generate returns in domestic currency, and Campa (1993), who finds a negative effect of exchange rate volatility on the number of foreign firms entering the U.S. market.

A firm who decides to go multinational faces investment costs and expects a stream of future earnings. The orthodox investment theory suggests that a firm should incur in FDI expense when the net present value of the investment in a foreign country is positive. The new theory of investment recognizes that exchange rate movements may induce firms to wait for more favorable conditions. The arrival of new information might affect the timing of the investment. On the other hand, FDI is costly and at least partly irreversible.

Hence, the possibility of delay and irreversibility are two very important features of the investment that the firm takes into account before undertaking a FDI project. A firm facing this problem can be understood as having a financial option by which the firm has the right to buy an asset (the plant in a foreign country) at any future time. The price that the firm has to pay in order to exercise the option, the strike price, is the sunk cost of the investment. Once the firm has decided to undertake a FDI expense, the firm has the option to revert to the initial situation by withdrawing from the foreign country, incurring in another sunk cost. The theoretical framework to deal with this investment problem has been developed by Dixit (1989a, 1989b) and Dixit and Pindyck (1994), building on previous work by McDonald and Siegel (1986). As suggested by Dixit (1989b), we extend the option theory of investment by considering a Cournot market equilibrium.

A stylized characteristic of the relation between exchange rates and FDI is that the FDI response to exchange rate movements may exhibit a hysteric pattern. Informally, it can be argued that a weak dollar encourages foreign firms to purchase U.S. assets, however, if the dollar strengthens investors need not reverse their investments. Formally, Darby, Hallett, Ireland and Piscitelly (1999) use Dixit's approach to study the effect of exchange rate variability on the degree of hysteresis in FDI flows. Their analysis is based on the assumption that the domestic firm is a price taker in the foreign country. In this paper we extend the analysis of FDI decisions under exchange rate uncertainty to other scenarios with different degrees of market power. Our work extends to an uncertain and dynamic setting the work by Campa, Donnenfeld and Weber (1998) who study the effect of strategic interaction among domestic and foreign firms on FDI. We analyze the case of an exporting firm willing to set a plant in a host country where the exporting and foreign firms compete *à la* Cournot. Our model embodies the monopoly as a special case when there is one exporting firm and the number of foreign firms is equal to zero, the duopoly with one foreign firm, the triopoly with two foreign firms, and so on. Perfect competition is reached when the number of firms goes to infinity. We study how the degree of hysteresis varies with the number of firms, exchange rate volatility, market size and demand elasticity. We also analyze the effect of maintenance costs, tariffs and export subsidies on the market equilibrium. Finally, we address the relationship between exchange rate pass-through and FDI. Interesting as it may be, we do not consider the possibility of tacit nor explicit collusion among firms.

The article is organized as follows. The next section describes the market equilibrium when the domestic firm is exporting and when it goes multinational and the optimal timing rule to undertake a foreign project. Section 3 shows the numerical solution of the model and the effects of changes in the parameters of the model. Finally section 4 summarizes our main findings.

2 The Model

We consider a market located in a foreign country where one domestic firm and $N-1$ foreign firms sell their entire production of a homogeneous good competing *à la* Cournot.[1] The analysis is dynamic and time is continuous. At the beginning the domestic firm exports to the foreign country and we say the firm is in state $j=0$. The firm may set a plant in the foreign country becoming multinational, and we say the firm is in state $j=1$.

The domestic and foreign countries have different currencies. Let S be the exchange rate, defined as the number of domestic currency units necessary to buy one unit of foreign currency. In this paper we study the role of exchange rate risk in the decision to go multinational. For simplicity we will assume that the only source of uncertainty comes from exchange rate movements. Assume that the exchange rate evolves over time exogenously as a geometric Brownian motion

$$\frac{dS}{S} = \mu dt + \sigma dz, \tag{1}$$

where dz is the increment of a standard Wiener process. The parameter μ is the expected depreciation rate and σ is a measure of exchange rate volatility. Under this assumption the expected discounted value of the domestic firm follows a stochastic process and the timing of FDI flows can be thought as

[1]This model is very easily extended to the case of M domestic firms and N foreign firms. However, if all domestic firms are identical, the choice between exporting and going multinational is the same for all domestic firms.

an option pricing problem.

Let the foreign currency market price be given by the inverse demand function

$$p_j = \alpha - \beta Q_j$$

where α and β are positive parameters and $Q_j = q_j + \sum_{i=1}^{N-1} q_{ij}^*$ is the total quantity of the good supplied in state $j = 0, 1$.

Production costs in the domestic country are denominated in domestic currency while production costs and sales in the foreign country are denominated in foreign currency. In state $j = 0$, when the domestic firm exports, the domestic currency cost of producing q_0 units is assumed to be linear

$$C(q_0) = f + \gamma q_0,$$

where f is a sunk cost and γ is a constant marginal cost. The sunk cost f refers to a maintenance expenditure and or advertising expenditure necessary to retain some brand awareness and is independent of the level of output. In state $j = 1$, when the domestic firm has gone multinational, the foreign currency cost of producing q_1 is

$$C^*(q_1) = f^* + \gamma^* q_1.$$

For foreign firm i the foreign currency cost of producing q_{ij}^* units is

$$C^*(q_{ij}^*) = f^* + \gamma^* q_{ij}^*.$$

From the point of view of the domestic firm, both exporting and going multinational imply some foreign exchange risk. When the domestic firm is exporting, its sales are denominated in foreign currency while it incurs in production costs denominated in domestic currency. In the absence of tariffs or transportation costs, for each unit exported to the foreign country the domestic firm receives Sp_0 units of domestic currency and pays $C(q_0)/q_0$ units of domestic currency. When the domestic firm goes multinational sales and production costs are denominated in foreign currency. For each unit sold in the foreign market as a multinational, the firm collects $S(p_1 - C^*(q_1))$ units of domestic currency as markup.

In the next two subsections we compute the profits under two different scenarios. First we compute profits when the domestic firm exports to the foreign market and remains in that state forever. Second we compute profits when the domestic firm sets a plant in the foreign country, that is, it goes multinational and remains in that state forever.

2.1 Exporting State

Suppose that the domestic firm stays in the exporting state forever and chooses quantities such that they maximize the expected present discounted value of profits subject to (1). Since the control variable of the firm, the quantity produced, does not affect the time pattern of the state variable, the exchange rate, intertemporal optimization is equivalent to period by period optimization. Therefore, we are neglecting any general equilibrium effect of the aggregate outcome of this industry on the level of the exchange rate. In other words, the exchange rate is assumed to be exogenously given.

At each point in time, the domestic firm solves

$$\max_{q_0 \geq 0} S\left(1-\tau\right)\left(\alpha - \beta Q_0\right) q_0 - f - \gamma q_0,$$

where $\tau \in (-\infty, 1)$. Positive values of τ can be interpreted as iceberg-type transport costs or ad-valorem tariffs and negative values as export subsidies. The nonnegativity constraint is necessary to avoid negative production for very low values of the exchange rate.

A typical foreign firm solves

$$\max_{q_0^* \geq 0} \left(\alpha - \beta Q_0\right) q_0^* - f^* - \gamma^* q_0^*.$$

Since all foreign firms have identical cost functions, all of them will produce exactly the same quantity, say q_0^*. The total quantity sold in the foreign market is $Q_0 = q_0 + (N-1)q_0^*$. The appendix shows that the total production, the production of the domestic firm and the production of the typical foreign firm in the Cournot equilibrium are

$$Q_0 = \frac{S(1-\tau)(N\alpha - (N-1)\gamma^*) - \gamma}{\beta(N+1)S(1-\tau)}, \tag{2}$$

$$q_0 = \frac{S(1-\tau)(\alpha + (N-1)\gamma^*) - N\gamma}{\beta(N+1)S(1-\tau)}, \tag{3}$$

$$q_0^* = \frac{S(1-\tau)(\alpha - 2\gamma^*) + \gamma}{\beta(N+1)S(1-\tau)}. \tag{4}$$

We will assume that $\alpha > 2\gamma^*$, so that foreign firms produce positive quantities for all exchange rate values. A domestic currency depreciation increases the market share of the exporting firm and reduces that of foreign competitors.

For exchange rate values above

$$\widetilde{S} = \frac{N\gamma}{(1-\tau)(\alpha + (N-1)\gamma^*)}$$

it is optimal for the domestic firm to produce and export positive quantities while for exchange rate values below $\widetilde{S}$ its optimal production is zero.

Substituting (2)-(4) in the objective of the exporting firm yields operating profits as a function of the exchange rate

$$\pi_0(S) = \begin{cases} \frac{[S(1-\tau)(\alpha + (N-1)\gamma^*) - N\gamma]^2}{\beta(N+1)^2 S(1-\tau)} - f & if \quad S > \widetilde{S} \\ -f & if \quad S \leq \widetilde{S} \end{cases} \tag{5}$$

Figure 1 shows the profit function when exporting as a function of the exchange rate for given parameter values.[2] The profit function exhibits a kink at $\widetilde{S}$. Therefore, the operating profits function in state 0 is continuous but not differentiable at $\widetilde{S}$.

[2]The parameter values used to draw figure 1 were $\alpha = 100$, $\beta = 1$, $\gamma = \gamma^* = 1$, $N = 35$, $\tau = 0$ $f = f^* = 5$.

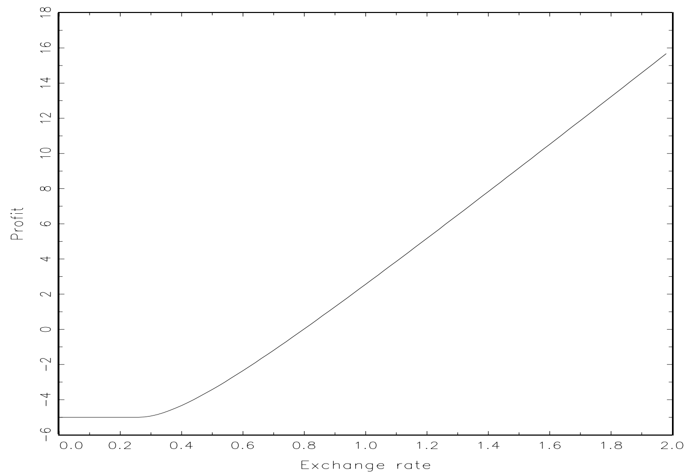

Figure 1: Profits when exporting as a function of the exchange rate.

2.2 Multinational State

Suppose that the domestic firm decides to set a plant in the foreign country and, therefore, faces the same cost function as local firms. The firm will compete as a multinational with the $N-1$ foreign firms. Even though the domestic firm has gone multinational, its headquarters in the source country maximize profits in domestic currency, that is

$$\max_{q_1 \geq 0} S\left[(\alpha - \beta Q_1)\, q_1 - f^* - \gamma^* q_1\right].$$

Foreign firms maximize profits in foreign currency, that is

$$\max_{q_1^* \geq 0} \left[(\alpha - \beta Q_1)\, q_1^* - f^* - \gamma^* q_1^*\right].$$

The appendix shows that the Cournot equilibrium production values are

$$Q_1 = \frac{N(\alpha - \gamma^*)}{\beta(N+1)},$$

$$q_1 = q_{11}^* = q_{21}^* = ... = \frac{\alpha - \gamma^*}{\beta(N+1)}.$$

Notice that, our previous assumption that $\alpha > 2\gamma^*$ guarantees positive production values regardless of the value of the exchange rate.

Operating profits for the multinational firm are now a linear function of the exchange rate

$$\pi_1(S) = S\left[\frac{1}{\beta}\left(\frac{\alpha-\gamma^*}{N+1}\right)^2 - f^*\right].$$

Notice that for positive fixed costs a sufficiently large number of firms in the industry will yield negative profits. In what follows we will restrict the attention to market structures where the number of firms is low enough to generate non negative profits of the multinational firm. This is a reasonable thing to do, since no firm will go multinational if that move generates losses.

2.3 Option pricing and optimal exercising

The firm decides how much to produce and sell in the foreign market and when to switch from exporting to multinational and viceversa. In state $j = 0$ the firm decides whether to produce only in the home country or to stop producing domestically and setting a plant in the foreign country, which means exercising the option to go multinational. In state $j = 1$ the firm decides whether to continue producing abroad or to go domestic again, which means exercising the option to reverse. A firm facing such a problem and able to change flexibly from one state to another has to price both options simultaneously.

Let $V(S, j)$ be the value of the firm given an initial exchange rate value and state and following optimal policies thereafter. Define $V_0(S) = V(S, 0)$ and $V_1(S) = V(S, 1)$. When the domestic firm exports to the foreign country, it earns a profit given by $\pi_0(S)$ each instant of time. In addition, the value of the firm is expected to change yielding a capital gain of $E(dV_0(S))/dt$. Under no-arbitrage opportunities it must be the case that

$$\frac{E(dV_0(S))}{dt} + \pi_0(S) = rV_0(S) \tag{6}$$

where r is the risk free rate of return. We will assume that $r > \mu$, otherwise the present discounted value of the firm is unbounded. Similarly, when the firm is multinational we have

$$\frac{E\left(dV_1\left(S\right)\right)}{dt} + \pi_1\left(S\right) = rV_1\left(S\right). \tag{7}$$

Using Itô's Lemma in each case, we have

$$\frac{1}{2}\sigma^2 S^2 V_0''\left(S\right) + \mu S V_0'\left(S\right) - rV_0\left(S\right) = -\pi_0\left(S\right), \tag{8}$$

$$\frac{1}{2}\sigma^2 S^2 V_1''\left(S\right) + \mu S V_1'\left(S\right) - rV_1\left(S\right) = -\pi_1\left(S\right). \tag{9}$$

Equations (8) and (9) have the same homogeneous part, so the solution to the homogeneous part must be the same. Trying a complementary function $g\left(S\right) = S^\eta$ yields

$$\frac{1}{2}\sigma^2 \eta\left(\eta - 1\right) S^\eta + \mu\eta S^\eta - rS^\eta = 0. \tag{10}$$

Define the polynomial

$$\varphi\left(\eta\right) = \frac{1}{2}\sigma^2\eta^2 + \left(\mu - \frac{\sigma^2}{2}\right)\eta - r = 0$$

whose roots are

$$\eta_0, \eta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}},$$

where $\eta_0 < 0$ and $\eta_1 > 1$.

The particular solution to the differential equation (8) requires special attention. Notice that the profit function (5) is not differentiable at $\widetilde{S}$. We will derive a solution to (8) not for all values of the exchange rate but only those values of the exchange rate above $\widetilde{S}$. Later on, in the numerical solution to the model, we will verify that the value of the exchange rate that makes the exporting firm go multinational is in fact above $\widetilde{S}$. Therefore, while the domestic firm exports, the exchange rate will be greater than $\widetilde{S}$ and the relevant part of the profit function does not include the kink.

The appendix shows that the functions

$$Y_0(S) = aS + bS^{-1} + c, \tag{11}$$

and

$$Y_1(S) = eS, \tag{12}$$

where

$$\begin{aligned}
a &= -\frac{(1-\tau)\left(\alpha + (N-1)\gamma^*\right)^2}{\beta (N+1)^2 (\mu - r)} \\
b &= -\frac{(N\gamma)^2}{\beta (N+1)^2 (1-\tau)\left(\sigma^2 - \mu - r\right)} \\
c &= -\frac{1}{r}\left(\frac{2\left(\alpha + (N-1)\gamma^*\right) N\gamma}{\beta (N+1)^2} + f\right) \\
e &= \frac{1}{\mu - r}\left[f^* - \frac{1}{\beta}\left(\frac{\alpha - \gamma^*}{N+1}\right)^2\right]
\end{aligned}$$

are particular solutions to the functional equations (8) and (9) respectively.

Economically speaking, these particular solutions are the expected present value of the operating profits at each state. In other words, equation (11) is the expected present value of the domestic firm when she is in state $j = 0$, i.e. exporting, and remains in that state forever. The appendix shows that

$$E\left[\int_0^\infty \pi_0 (S(t))\, e^{-rt} dt\right] = aS + bS^{-1} + c.$$

Similarly, equation (12) is the expected present value of the domestic firm when she is in state $j = 1$, i.e. multinational, and remains in that state forever. The appendix shows that

$$E\left[\int_0^\infty \pi_1 (S(t))\, e^{-rt} dt\right] = eS.$$

The general solution can be written as

$$V_0(S) = A_0 S^{\eta_0} + B_0 S^{\eta_1} + aS + bS^{-1} + c, \tag{13}$$

$$V_1(S) = A_1 S^{\eta_0} + B_1 S^{\eta_1} + eS, \tag{14}$$

where A_0, A_1, B_0 and $B_1$are constants to be determined.

Conventional wisdom says that exporting is profitable when the domestic currency is depreciated. Similarly, buying a foreign firm or setting a plant abroad is profitable when the domestic currency is appreciated. This simple reasoning places some restrictions on the general solution given above.

As the exchange rate tends to infinity, the option value of investing abroad becomes worthless, therefore, we should impose the restriction that the coefficient B_0 corresponding to the positive root must be zero, otherwise the value of the firm in the state 0 explodes. Should the exchange rate tend to zero the option value of exporting tends to zero. Hence, the coefficient A_1 corresponding to the negative root must be zero, otherwise the value of the firm in state 1 explodes. Rewriting (13) and (14) and sweeping off subscripts we have

$$V_0(S) = AS^{\eta_0} + aS + bS^{-1} + c,$$

$$V_1(S) = BS^{\eta_1} + eS.$$

The economic interpretation of these equations is simple. The value of the firm when exporting, $V_0(S)$, is the sum of two components: the expected present value of exporting, $aS + bS^{-1} + c$, plus the value of the option to go multinational, AS^{η_0}. Similarly, the value of the firm when multinational is the sum of two terms: the expected present value of selling in the foreign market as a multinational, eS, plus the value of the option to abandon the foreign country, BS^{η_1}. Logically, the value of the options can never become negative, thus we restrict A and B to be non-negative. Mathematically speaking, the value of the firm in either state is the sum of the expected present value of the firm in that state plus an intrinsic bubble (a bubble that depends on fundamentals).

Let $\underline{S}$ be the level of exchange rate low enough to induce the firm to invest in the host country, that is, the exchange rate at which the firm exercises the option to go multinational. Similarly, $\overline{S}$ is the level of exchange rate high enough to induce the firm to reverse, that is, the exchange rate at which the multinational firm exercises the option of abandoning the host country. Thus, the firm retains its option of going multinational over the interval $(\underline{S}, \infty)$. However, a multinational firm will follow operating in the host country over the interval $\left(0, \overline{S}\right)$. The ratio $(\overline{S} - \underline{S})/\underline{S}$ can be interpreted as a measure of the degree of hysteresis in FDI flows.

When the domestic firm goes multinational it has to pay I units of foreign currency to exercise that option. This is the sunk costs of closing down the plant in the domestic country and setting up a plant in the foreign country. The exchange rate value that makes the domestic firm indifferent between exporting and going multinational, $\underline{S}$, must satisfy the value matching condition

$$V_0\left(\underline{S}\right) = V_1\left(\underline{S}\right) - \underline{S}I. \tag{15}$$

Optimal exercising also requires that the smooth pasting condition

$$V_0'\left(\underline{S}\right) = V_1'\left(\underline{S}\right) - I, \tag{16}$$

be satisfied. Similarly, when a multinational firm suspends operations in the host country it must pay a lump-sum exit cost L in foreign currency to exercise that option. Thus, L represents the sunk cost of closing down the plant in the foreign country and setting up a new plant in the domestic country. Let $\overline{S}$ be the exchange rate value that makes the multinational firm indifferent between producing abroad and going domestic again. The value matching condition is

$$V_1\left(\overline{S}\right) = V_0\left(\overline{S}\right) - \overline{S}L, \tag{17}$$

and the smooth pasting condition is

$$V_1'\left(\overline{S}\right) = V_0'\left(\overline{S}\right) - L. \tag{18}$$

Equations (15) to (18) define a nonlinear system of equations

$$\begin{aligned}
B\underline{S}^{\eta_1} - A\underline{S}^{\eta_0} + (e-a)\,\underline{S} - b\underline{S}^{-1} - c - \underline{S}I &= 0 \\
\eta_1 B\underline{S}^{\eta_1 - 1} - \eta_0 A\underline{S}^{\eta_0 - 1} + (e-a) + b\underline{S}^{-2} - I &= 0 \\
B\overline{S}^{\eta_1} - A\overline{S}^{\eta_0} + (e-a)\,\overline{S} - b\overline{S}^{-1} - c + \overline{S}L &= 0 \\
\eta_1 B\overline{S}^{\eta_1 - 1} - \eta_0 A\overline{S}^{\eta_0 - 1} + (e-a) + b\overline{S}^{-2} + L &= 0
\end{aligned} \tag{19}$$

where $A, B, \overline{S}$ and $\underline{S}$ are to be determined. We are unable to find an analytical solution to this system of equations. The following section provides a numerical solution.

3 Numerical results

In this section we obtain numerical solutions to the system of equations (19) for different values of the parameters. First we find the solution for a baseline parameter configuration and then analyze the effects of changes in the parameter values one at a time.

3.1 The baseline case

Suppose that $\tau = 0$ and the demand function is

$$p = 100 - Q.$$

Let the parameters of the cost function be the same wherever the production is located with the following values $\gamma = \gamma^* = 1$ and $f = f^* = 0$, that is

$$\begin{aligned}
C(q) &= q, \\
C^*(q^*) &= q^*.
\end{aligned}$$

As in Dixit(1989a, 1989b), the interest rate is chosen to be $r = 0.025$, $\sigma = 0.1$ and $\mu = 0$. The entry sunk cost is $I = 30$ and the exit cost is $L = 10$. This parametrization ensures that $V_0(S)$ and $V_1(S)$ are positive.

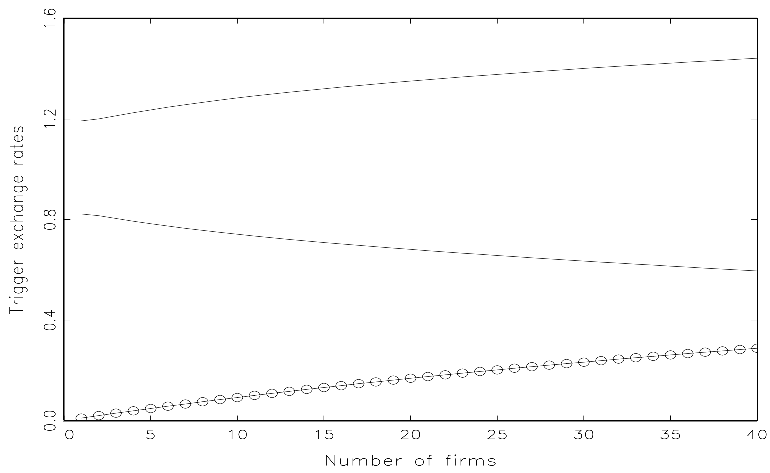

Figure 2: Trigger exchange rates as a function of the number of firms. $\overline{S}$ and $\underline{S}$ (solid lines), $\widetilde{S}$ (circle line).

Let us begin with a monopolistic industry, $N = 1$. This is the case when there are no foreign firms and the only supplier is the exporting firm. With this parameter configuration, the trigger exchange rate values are $\underline{S} = 0.8222$ and $\overline{S} = 1.1925$. The interpretation of these values is as follows. Suppose that initially the exchange rate is above $\underline{S}$ and the monopolist is exporting to the foreign economy. If S reaches the value $\underline{S} = 0.8222$, the domestic monopolist will set a plant in the foreign country becoming a multinational firm. However, the firm will only reverse to exporting if the exchange rate reaches the value $\overline{S} = 1.1925$. This numerical example shows that for reasonable parameter values, the degree of hysteresis can be quite high. In particular, the measure of hysteresis $(\overline{S} - \underline{S})/\underline{S} = 0.4504$. Thus, once the domestic firm goes multinational, the exchange rate has to depreciate 45% to revert to exporting.

At this point it is necessary to verify that, for this parameter configuration, the exchange rate value at which it is optimal for the domestic exporting firm to produce zero units, $\widetilde{S} = 0.01$, is below the level, $\underline{S} = 0.8222$, at which the domestic firm becomes multinational.

Now let us consider other market sizes by increasing the number of firms. When $N = 2$ the foreign market is served by the domestic exporter and a local firm, when $N = 3$ by the domestic exporter and two local firms, and so on. Figure 2 represents the trigger exchange rate values as functions of the number of firms in the industry. As we increase N, $\overline{S}$ rises and $\underline{S}$ diminishes. The degree of hysteresis, $(\overline{S} - \underline{S})/\underline{S}$, is increasing in the number firms. For $N = 40$ the measure of hysteresis, $(\overline{S} - \underline{S})/\underline{S} = 1.4225$, indicates that once the domestic firm goes multinational it would require a 142% depreciation to revert to exporting.

We have bounded the market size to forty firms in order to ensure positive profits when the firm goes multinational, since it is not sensible for the domestic firm to undertake a project abroad with negative operating profits. On the other hand, all exchange rate trigger values shown in the graph are greater than the minimum value of the exchange rate for which exporting makes sense, $\widetilde{S}$.

3.2 Parameter changes

Figure **??** shows the effect of changing the parameters of the Brownian Motion. The results are similar to those of Dixit (1989b). The left side panel shows the trigger exchange rates for $\sigma = 0.1$ (solid lines) and $\sigma = 0.2$ (broken lines). The entry trigger exchange rates shift down and the exit trigger exchange rates shift up, the hysteresis widens for all N. So higher exchange rate volatility deters FDI into the foreign economy, but if it takes place it is less likely to abandon the foreign economy.

The right side panel of Figure **??** shows the effect of changing the exchange rate depreciation drift from $\mu = 0$ to $\mu = 0.01$. It shows that a tendency towards depreciation reduces the entry trigger exchange rate $\underline{S}$ a bit while it has a larger effect on the exit trigger exchange rate. Hence, once the firm has gone multinational, under a depreciating exchange rate environment, it will demand a lower exit trigger exchange rate for any N.

Figure 4 shows the effect of changes in the sunk entry cost from $I = 30$ to $I = 60$ and exit cost from $L = 10$ to $L = 20$ respectively. As we can see in left side panel of Figure 4, when the entry cost is twice the initial value, the curve $\overline{S}$ shifts up and the curve $\underline{S}$ shifts down. As in Dixit (1989b) the

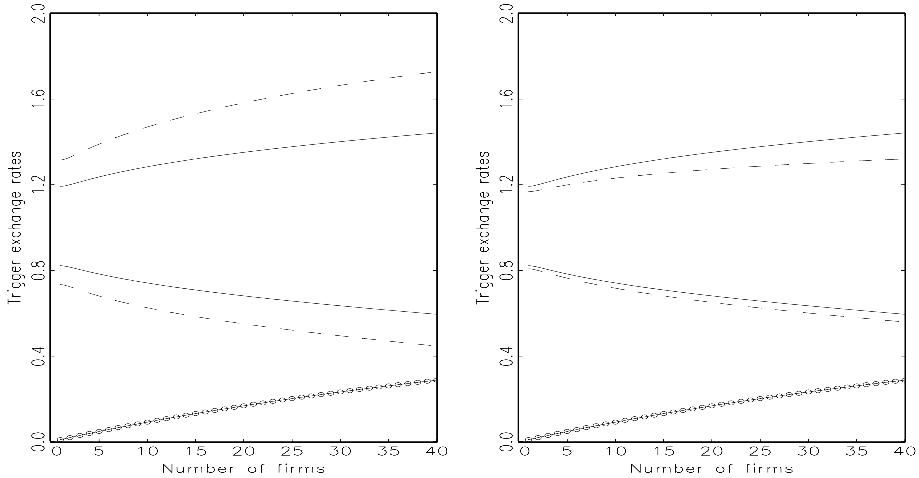

Figure 3: Effects of higher σ and μ.

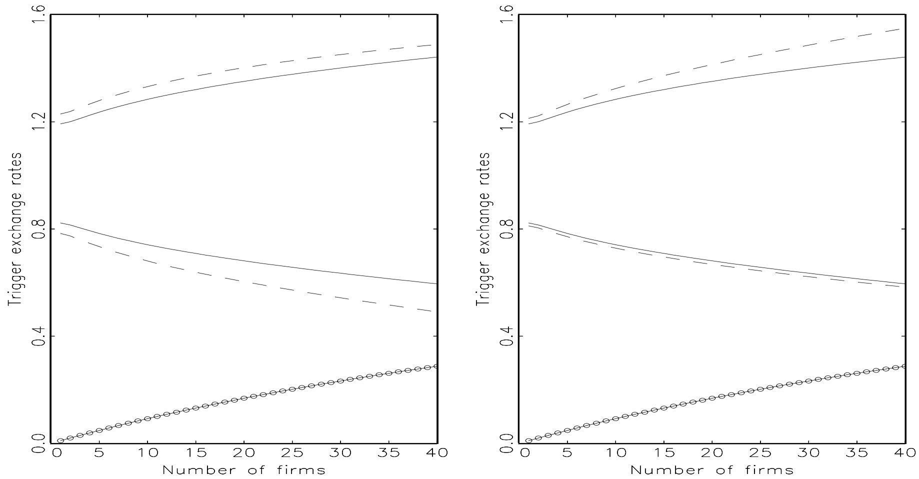

Figure 4: Effects of higher entry and exit costs.

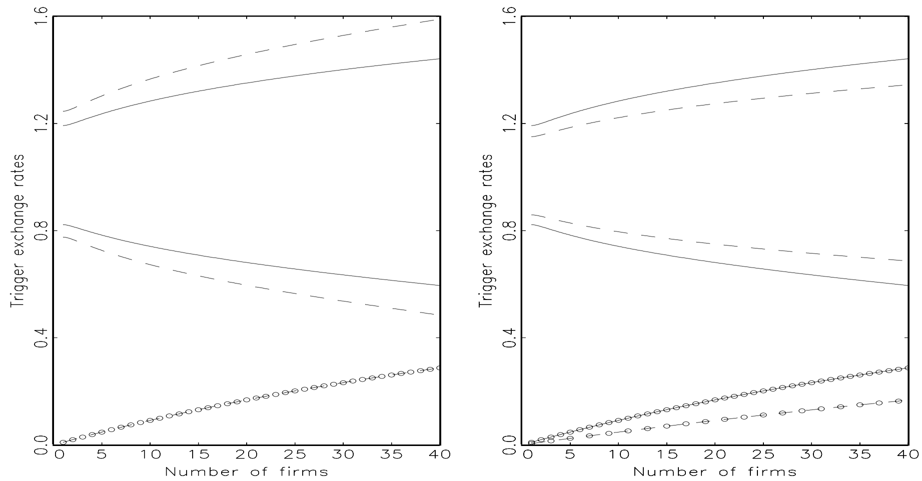

Figure 5: Effects of higher β and α.

degree of hysteresis increases with the entry cost. With a higher entry cost the exporting firm will demand a lower exchange rate to go multinational, since the foreign investment has become more expensive in domestic currency. The multinational firm, however, will turn back to exporting at a higher exit trigger exchange rate, because reentry has a higher cost.

The effect of a higher exit cost are shown in the right side panel of Figure 4. It seems to be the case that the exit cost does not affect the entry trigger exchange rate as much as the exit trigger exchange rate. Thus, hysteresis rises only as a consequence of the increases in $\overline{S}$. When the firm is multinational and stops producing in the host country a higher exit cost implies that it is more expensive to abandon the foreign country, and it will demand a higher exchange rate to go back to exporting.

Figure 5 shows the effect of changes in the parameters of the demand function. The left side panel shows the effects of a rise in the (absolute value of the) slope of the inverse demand function from $\beta = 1$ to $\beta = 2$. A lower price elasticity of the demand function, makes the hysteresis rise. The entry exchange rate $\underline{S}$ is now lower and the exit exchange rate $\overline{S}$ is higher. A higher slope of the demand function makes the equilibrium price and quantity to fall in both states. Therefore the exporting firm will get lower operating profits in foreign currency and it will undertake the project in the host country if the cost of the investment is lower, thereby the entry trigger exchange rate has to be lower. On the other hand, when the firm is multinational it will demand a higher exchange rate to stop producing in the host country.

The right side panel of Figure 5 shows the effect of a rise in the intercept of the inverse demand function from $\alpha = 100$ to $\alpha = 200$. This is the case of an exogenous shift in demand that causes the equilibrium prices and quantities to rise. The entry trigger exchange rate curve shifts up and the exit trigger exchange rate shifts down and the degree of hysteresis falls. The higher price and quantity make the operating profits rise in foreign currency, so going multinational would be profitable at a higher $\underline{S}$. However, to abandon the host country, the multinational firm will demand a lower $\overline{S}$ because the higher price at the new exit exchange rate makes exporting more profitable. A shift in demand also has an effect on the range of exchange rate values for which it is optimal to produce nothing for an exporting firm, the $\widetilde{S}$ line shifts down.

The left side panel of Figure 6 exhibits trigger exchange rate values for a sunk cost of production $f = f^* = 5$ (broken lines) and different number of firms. With respect to the no sunk cost case (solid lines), the entry and exit trigger exchange rates do not change for the monopoly and hardly move when the number of firms is below $N = 5$. On the other hand, the effect of an increase in the sunk costs on the trigger exchange rates is increasing in the number of firms. $\overline{S}$ moves down and $\underline{S}$ moves up and, therefore, the degree of hysteresis decreases.

The right side panel of Figure 6 shows the effect of an increase in the marginal cost from $\gamma = 1$ to $\gamma = 1.25$. This change increases the entry trigger exchange rate and reduces the exit trigger exchange rate.

3.3 Dumping

When production requires a sunk fixed cost, the exporting firm may incur in dumping. Notice that since the exporting firm does not sell in the home country, international price discrimination can not occur. However, when there is a fixed cost of production, dumping may arise because the domestic firm may optimally decide to export at a loss.

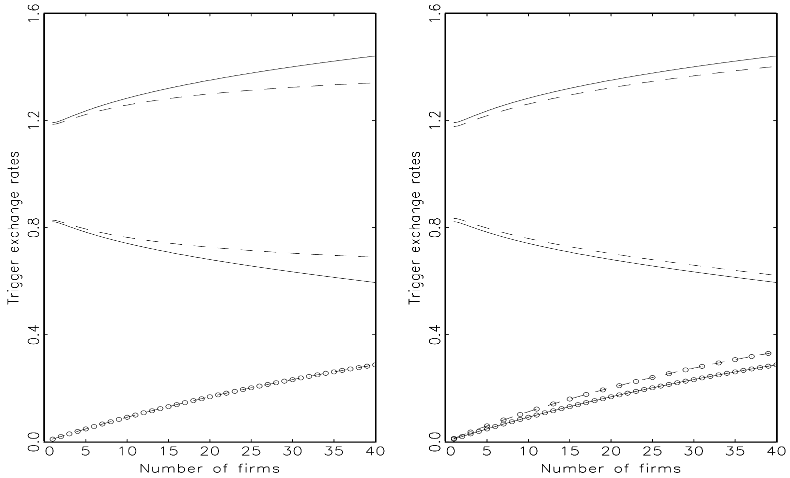

Figure 6: Effect of sunk and marginal costs.

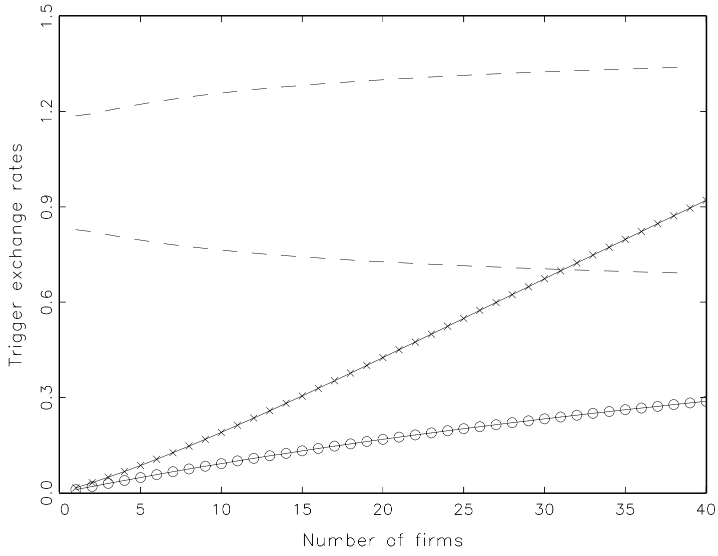

Figure 7: With a fixed cost $f = f^* = 5$ and $N > 31$, exchange rate values in the interval $(\underline{S}, \widehat{S})$ generate dumping.

Figure (7) represents entry and exit trigger exchange rates with a sunk cost $f = f^* = 5$. Let $\widehat{S}$ be the largest root of $\pi_0(S) = 0$. For exchange rate values below $\widehat{S}$ (crossed line) and above $\underline{S}$, the domestic firm is dumping in the foreign market. For industry sizes below $N = 31$, dumping never appears because the domestic firm goes multinational before the exchange rate hits the dumping trigger exchange rate $\widehat{S}$. Dumping appears for more competitive industries where the exporting firm may find it optimal to export at a loss for exchange rate values in the interval $(\underline{S}, \widehat{S})$. The parametric configuration used in drawing figure 7 sets export subsidies equal to zero. Of course, we could have generated dumping by setting τ sufficiently below zero.

When the domestic firm is dumping in the foreign market, it does not do so to drive competitors out of the market since local firms enjoy positive profits, hence, dumping is not *predatory* (e.g. Davies and McGuiness, 1982). Since we are studying a single market, there can not be international price discrimination, so dumping is not *persistent* (e.g. Brander and Krugman, 1983). Neither is this type of dumping *sporadic*, as it would be the case if the exporting firm was getting rid of unsold stocks. This type of dumping is of the same type as that found by Sercu and Vanhulle (1992), who show how an exporting firm "will dump when re-entry entails a cost".[3] The type of dumping that may arise in this model has the following characteristics. First, the number of firms in the market has to be large enough, in the numerical example $N \geq 31$. Second, the exporting firm has to face a large enough sunk fixed cost of production. Third the exchange rate should be in the interval $(\underline{S}, \widehat{S})$. Notice that, in this scenario, operating profits of the exporting firm are negative and, therefore, the expected present value of exporting (and remaining in that state forever) is negative. Why should the domestic firm be interested in exporting if the present value of that activity is negative? The reason is that the value of the exporting firm is the sum of two terms, the present value of exporting plus the value of the option to become multinational. When dumping appears, the present value of exporting is negative, but the value of the option to become multinational

[3]Sercu and Vanhulle (1992) cite the working paper versions of Dixit (1989a, 1989b) and Delgado (1991) where apparently this type of dumping was first mentioned.

is positive and compensates the negative present value of exporting. Furthermore, while the exporting firm dumps the value of being multinational is greater than the initiation cost, i.e. $V_1(S) > IS$. Therefore, the orthodox theory or investment would suggest to go multinational instead of losing money exporting. However, waiting is a better strategy since $V_0(S) > V_1(S) - IS$. The domestic firm keeps on exporting and waits: if the domestic currency depreciates, it will continue exporting, but in case of an appreciation that makes the exchange rate to go below $\underline{S}$, it will go multinational. This type of dumping is temporary and appears because the relationship between FDI and the exchange rate exhibits a hysteric pattern.

The empirical evidence provided by Knetter and Prusa (2003) suggests that a real appreciation of the importing country's currency increases antidumping filings. In this model, however, dumping appears when the currency of the importing country is sufficiently depreciated. This discrepancy between theory and empirical evidence could be due at least to two reasons. First, the type of dumping suggested by this model could just be a theoretically feasible but empirically irrelevant outcome. Second, it could be the case that, as Knetter and Prusa (2003) are inclined to think, foreign firms are held responsible for factors beyond their control. In the latter case, antidumping laws allow abuse of the statute.

3.4 Tariffs and Export Subsidies

Figure 8 shows the effect of a tariff and an export subsidy. The left side panel plots the trigger exchange rates when $\tau = 0$ and $\tau = 0.01$. The effect of such a small change in the tariff rate is very large in less competitive industries. In the case of a monopoly, the entry trigger exchange rate, $\underline{S}$, moves from 0.8222 to 1.5633 and the exit trigger exchange rate, $\overline{S}$, moves from 1.1926 to 2.5111 (not shown in Figure 6). A small tariff enlarges the range of exchange rate values for which going multinational makes sense. A small tariff has a similar, but less strong, effect on a duopolistic industry, the exit trigger exchange rate rises from 1.2005 to 1.6448 and the entry trigger exchange rate rises from 0.8151 to 1.0714. The effect of the tariff is smaller the larger

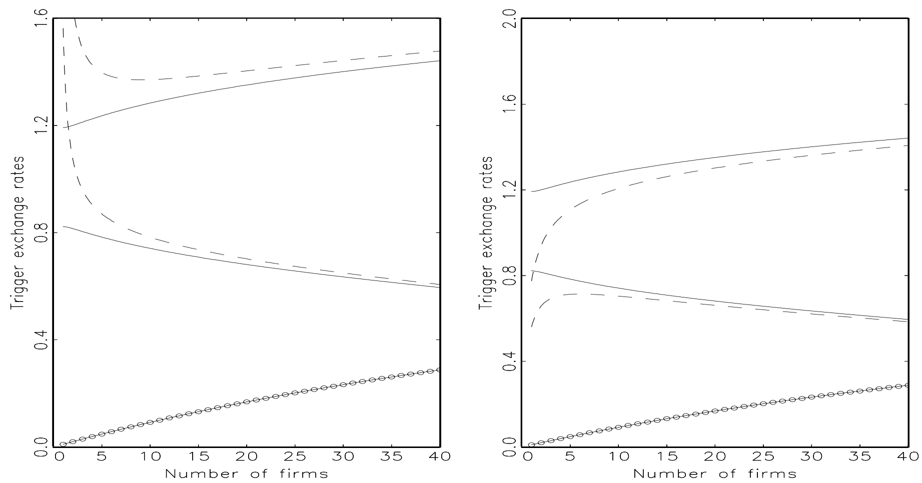

Figure 8: Effect of $\tau = \pm 0.01$

the number of firms in the industry. In fact, for large values of N the effect is not visually perceptible. Thus, a small tariff has a FDI-inducing effect and this effect is smaller the larger the number of firms in the industry. The fact that a tariff makes FDI more likely is in line with the literature on tariff-jumping FDI. The contribution here lays on the fact that the FDI-inducing effect of a tariff is lower the more competitive the industry is.

On the other hand, although perhaps it is not clear from a visual inspection of the left side panel of Figure 8, a small tariff increases the degree of hysteresis since $\underline{S}$ rises less than $\overline{S}$. This suggests that a tariff may be used not only to induce the exporting firm to go multinational, but also, and more effectively, to deter withdrawing.

The effect of a small export subsidy is shown in the right side panel of Figure 8. In this case we do find a solution to the system of equations for all industry sizes. In terms of the $\overline{S}$ and $\underline{S}$ curves, the effect is just the opposite to the introduction of a tariff. However, the degree of hysteresis decreases for less competitive industries.

So far we have analyzed the effects of a very small tariff (subsidy). Tariffs are typically a lot larger than 1%. When we set the tariff rate at higher levels a solution to the system of equations (19) does not exist for small industry sizes. For instance, the left side panel of Figure 9 shows the case of a 5%

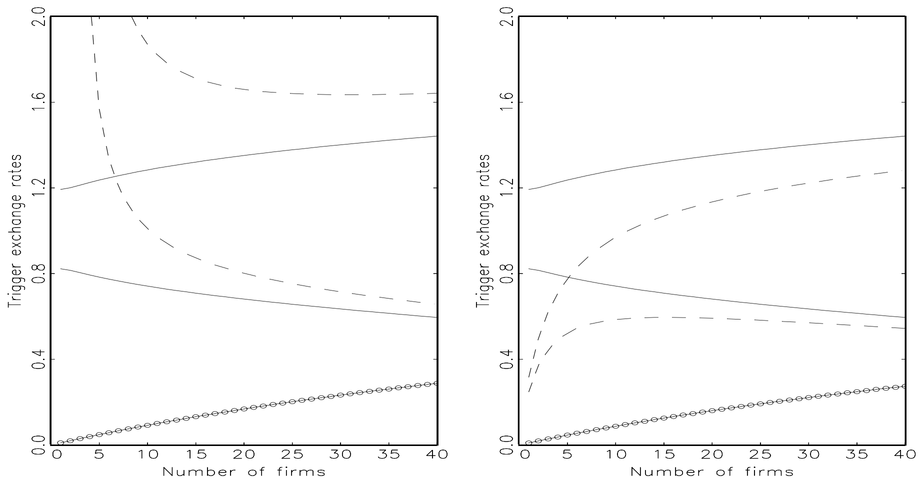

Figure 9: Effect of $\tau = \pm 0.05$

tariff rate. There is no solution to the nonlinear system of equations (19) for $N \leq 2$.[4] The interpretation of this result is that for $N \leq 2$ and a 5% tariff, there are no exchange rate values for which exporting is a sensible strategy. Hence, small tariffs can be used as a tool to attract FDI in markets with few competitors.

The case for an export subsidy of 5% is represented in the right side panel of figure 9. The results are similar to those of Figure 8 only that now the effects on the trigger exchange rate values are larger.

3.5 Exchange rate pass-through

The model developed so far has interesting implications for exchange rate pass-through. When the domestic firm exports to the foreign country, the

[4] The left side panel of Figure 9 plots entry and exit trigger exchange rates within the range $[0, 2]$. For $N \leq 2$ there is no solution to the system. For $N \in \{3, 4, 5, 6, 7\}$ a solution exists with the exit trigger exchange rate being greater than 2 and hence it is not shown in the figure.

market price is a function of the exchange rate given by

$$\begin{aligned} p &= \frac{S\,(1-\tau)\,(\alpha+(N-1)\,\gamma^{*})+\gamma}{(N+1)\,S\,(1-\tau)} \\ &= \frac{\alpha+(N-1)\,\gamma^{*}}{(N+1)}+\frac{R\gamma}{(N+1)\,(1-\tau)} \end{aligned}$$

where $R = 1/S$ is the number of foreign currency units necessary to buy a unit of domestic currency, that is, the exchange rate from the point of view of the foreign country. The foreign currency price is a linear function of the (foreign) exchange rate. A foreign currency depreciation (a rise in R) increases the foreign currency price of foreign imports. This effect is lower the larger the industry size.

The exchange rate pass-through is

$$\frac{\partial p}{\partial R}\frac{R}{p} = \frac{1}{1+\frac{(1-\tau)(\alpha+(N-1)\gamma^{*})}{\gamma R}}. \tag{20}$$

Therefore, the exchange rate pass-through is increasing in R, reaching a maximum value of 1. Accordingly, a foreign currency depreciation will increase the exchange rate pass-through. However, the domestic firm will not export for exchange rate values below $\underline{S}$, or values of R above $1/\underline{S}$. When the domestic firm goes multinational the foreign currency price is

$$p = \frac{\alpha+N\gamma^{*}}{N+1}$$

and the exchange rate no longer affects the foreign currency price of foreign imports. The exchange rate pass-through is suddenly reduced to zero precisely for the range of exchange rate values for which it would have been maximal in the absence of the option to go multinational. This result is exacerbated by the fact that there is only one domestic firm in our model, but the introduction of more domestic firms would only reduce the strength of the result. This result has an empirical implication: exchange rate pass-through should be lower in industries open to foreign direct investment. Put it another way, opening industries to FDI reduces the exchange rate pass-through.

4 Conclusions

In this paper we have presented a model of entry and exit decisions of an exporting firm who has the option to undertake a FDI project under exchange rate uncertainty. Real option pricing techniques are used to determine the optimal timing rule of the investment. We consider the case of a domestic firm competing in a foreign oligopolist market. The flexibility of the market structure allows us to show how the number of firms and hysteresis are related.

We find that the degree of hysteresis grows with the number of firms in the industry, entry costs, exchange rate volatility and the (absolute value of the) slope of the inverse demand function. Thus higher values of these variables deter FDI. When an exporting firm has the option to go multinational and there are fixed costs, dumping can occur for large enough industry sizes. We also find that very small tariffs encourage FDI and have a greater impact the less competitive the market is. Finally, in our model, since a domestic firm will go multinational in an appreciating exchange rate environment, FDI reduces the degree of exchange rate pass-through.

Appendix: Math worksheet

Market equilibrium when exporting

The exporting firm maximizes

$$\pi\left(q_0\right)=S\left(1-\tau\right)\left(\alpha-\beta Q_0\right)q_0-f-\gamma q_0.$$

The first order condition is

$$\frac{\partial\pi\left(q_0\right)}{\partial q_0}=S\left(1-\tau\right)\left(\alpha-\beta Q_0-\beta q_0\right)-\gamma=0$$

which gives the reaction function of the exporting firm

$$q_0=\frac{S\left(1-\tau\right)\left(\alpha-\beta\sum_{i=1}^{N-1}q_{i0}^*\right)-\gamma}{2\beta S\left(1-\tau\right)}.$$

Foreign firm i maximizes

$$\pi\left(q_{i0}^*\right)=\left(\alpha-\beta Q_0\right)q_{i0}^*-f^*-\gamma^* q_{i0}^*$$

$$\frac{\partial\pi\left(q_{i0}^*\right)}{\partial q_{i0}^*}=\alpha-\beta Q_0-\beta q_{i0}^*-\gamma^*=0.$$

which gives the reaction function of foreign firm i

$$q_{i0}^*=\frac{\alpha-\beta(q_0+\sum_{l\neq i}^{N-1}q_{l0}^*)-\gamma^*}{2\beta}.$$

Since all foreign firms are identical

$$\sum_{i=1}^{N-1}q_{i0}^*=\left(N-1\right)q_0^*.$$

The production levels of the exporting firm and a typical foreign firm are

$$q_0 = \frac{S(1-\tau)(\alpha + (N-1)\gamma^*) - N\gamma}{\beta(N+1)S(1-\tau)}$$
$$q_0^* = \frac{S(1-\tau)(\alpha - 2\gamma^*) + \gamma}{\beta(N+1)S(1-\tau)}.$$

The optimal production of the domestic firm is positive for all exchange rate values such that

$$S \geq \frac{N\gamma}{(1-\tau)(\alpha + (N-1)\gamma^*)} = \widetilde{S}.$$

Operating profits when exporting are

$$\pi_0(S) = S(1-\tau)(\alpha - \beta Q_0)q_0 - f - \gamma q_0.$$

Substituting the optimal production values of the domestic and foreign firms yields

$$\begin{aligned}\pi_0(S) &= \left[S(1-\tau)\left(\alpha - \beta\left(\frac{S(1-\tau)(N\alpha - (N-1)\gamma^*) - \gamma}{\beta(N+1)S(1-\tau)}\right)\right) - \gamma\right] \times \\ &\left[\frac{S(1-\tau)(\alpha + (N-1)\gamma^*) - N\gamma}{\beta(N+1)S(1-\tau)}\right] - f \\ &= \frac{1}{\beta(N+1)^2}\left[S(1-\tau)(\alpha + (N-1)\gamma^*)^2\right. \\ &\left. + \frac{(N\gamma)^2}{S(1-\tau)} - 2(\alpha + (N-1)\gamma^*)N\gamma\right] - f.\end{aligned}$$

Market equilibrium when Multinational

The multinational firm maximizes the following function

$$\pi(q_1) = S\left[(\alpha - \beta Q_1)q_1 - f^* - \gamma^* q_1\right].$$

The first order condition is

$$\frac{\partial\pi(q_1)}{\partial q_1} = S(\alpha - \beta Q_1 - \beta q_1 - \gamma^*) = 0$$

and the reaction function of the multinational is

$$q_1 = \frac{\alpha - \beta(\sum_{i=1}^{N-1} q_{i1}^*) - \gamma^*}{2\beta}.$$

Foreign firm i maximizes

$$\pi\left(q_{i1}^*\right) = \left(\alpha - \beta Q_1\right) q_{i1}^* - f^* - \gamma^* q_{i1}^*.$$

Since the objective function is the same as the objective function of the multinational up to an scale factor, the solution is the same. The production levels of the multinational and the local firms are

$$q_1 = q_{11}^* = q_{21}^* = ... = \frac{\alpha - \gamma^*}{\beta\left(N+1\right)}.$$

The operating profit of the multinational firm will be

$$\pi_1\left(S\right) = S\left[\left(\alpha - \beta Q_1\right) q_1 - f^* - \gamma^* q_1\right]$$

$$\begin{aligned}\pi_1\left(S\right) &= S\left[\left(\alpha - \beta\left(\frac{N\left(\alpha - \gamma^*\right)}{\beta\left(N+1\right)}\right) - \gamma^*\right)\left(\frac{\alpha - \gamma^*}{\beta\left(N+1\right)}\right) - f^*\right] \\ &= S\left[\frac{1}{\beta}\left(\frac{\alpha - \gamma^*}{N+1}\right)^2 - f^*\right]\end{aligned}$$

The particular solution to equation (8)

Substituting the following functional form

$$\begin{aligned}h\left(S\right) &= aS + bS^{-1} + c, \\ h'\left(S\right) &= a - bS^{-2}, \\ h''\left(S\right) &= 2bS^{-3}.\end{aligned}$$

in equation (8) we get

$$\sigma^2 S^2 \left(bS^{-3}\right) + \mu S \left(a - bS^{-2}\right) - r \left(aS + bS^{-1} + c\right) =$$
$$-\frac{1}{\beta (N+1)^2} \left[S (1-\tau) (\alpha + (N-1) \gamma^*)^2 \right.$$
$$\left. + \frac{(N\gamma)^2}{S (1-\tau)} - 2 (\alpha + (N-1) \gamma^*) N\gamma \right] + f.$$

Collecting terms in the left hand side we obtain

$$bS^{-1} \left(\sigma^2 - \mu - r\right) + Sa (\mu - r) - rc =$$
$$-\frac{1}{\beta (N+1)^2} \left[S (1-\tau) (\alpha + (N-1) \gamma^*)^2 \right.$$
$$\left. + \frac{(N\gamma)^2}{S (1-\tau)} - 2 (\alpha + (N-1) \gamma^*) N\gamma \right] + f.$$

Equating coefficients accompanying equal powers of the exchange rate we get

$$a = -\frac{(1-\tau) (\alpha + (N-1) \gamma^*)^2}{\beta (N+1)^2 (\mu - r)},$$
$$b = -\frac{(N\gamma)^2}{\beta (N+1)^2 (1-\tau) (\sigma^2 - \mu - r)},$$
$$c = -\frac{1}{r} \left(\frac{2 (\alpha + (N-1) \gamma^*) N\gamma}{\beta (N+1)^2} + f \right).$$

The particular solution to equation (9)

Substituting the following functional form

$$g(S) = eS,$$
$$g'(S) = e,$$
$$g''(S) = 0.$$

in equation (9) we get

$$eS\left(\mu-r\right)=-S\left[\frac{1}{\beta}\left(\frac{\alpha-\gamma^{*}}{N+1}\right)^{2}-f^{*}\right].$$

Equating coefficients of the same powers we get

$$e=\frac{1}{\mu-r}\left[f^{*}-\frac{1}{\beta}\left(\frac{\alpha-\gamma^{*}}{N+1}\right)^{2}\right].$$

The present value of the firm

First we show that the solution to the non homogenous part of the differential equation (8) is the expected present value of the operating profits of the exporting firm. To do so it is convenient to write operating profits as

$$\pi_0\left(S\right)=m_0+m_1S+m_2S^{-1},$$

where

$$\begin{aligned} m_0 &= -\frac{2\left(\alpha+\left(N-1\right)\gamma^{*}\right)N\gamma}{\beta\left(N+1\right)^{2}}-f, \\ m_1 &= \frac{\left(1-\tau\right)\left(\alpha+\left(N-1\right)\gamma^{*}\right)^{2}}{\beta\left(N+1\right)^{2}}, \\ m_2 &= \frac{\left(N\gamma\right)^{2}}{\left(1-\tau\right)\beta\left(N+1\right)^{2}}. \end{aligned}$$

The expected present value of exporting can be written as the sum of three integrals

$$\begin{aligned} E\int_0^{\infty}\pi_0\left(S(t)\right)e^{-rt}dt &= m_0E\int_0^{\infty}e^{-rt}dt+m_1E\int_0^{\infty}S(t)e^{-rt}dt \\ &\quad +m_2E\int_0^{\infty}S(t)^{-1}e^{-rt}dt. \end{aligned} \tag{21}$$

The first integral in the right hand side of the previous equation is very easy to solve as it is not stochastic

$$\int_0^\infty e^{-rt}dt = \frac{1}{r}.$$

The second integral is also easy to solve once we realize that it is not stochastic either, as the expected value makes the integrand deterministic. However, for doing so we have to find $E(S)$. Defining $F = Ln(S)$ and using Itô's lemma we have

$$dF = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dz.$$

Integrating this last equation we get

$$\begin{aligned} F(t) &= F(0) + \int_0^t (\mu - \frac{1}{2}\sigma^2)d\tau + \int_0^t \sigma dz \\ &= F(0) + (\mu - \frac{1}{2}\sigma^2)t + \sigma(z(t) - z(0)). \end{aligned}$$

Since $S = e^F$ we get

$$S(t) = S(0)e^{(\mu-\frac{1}{2}\sigma^2)t+\sigma(z(t)-z(0))}.$$

Hence

$$\begin{aligned} E(S(t)) &= S(0)e^{(\mu-\frac{1}{2}\sigma^2)t}E(e^{\sigma(z(t)-z(0)})) \\ &= S(0)e^{(\mu-\frac{1}{2}\sigma^2)t}e^{\frac{\sigma^2}{2}t} = S(0)e^{\mu t}. \end{aligned}$$

Therefore, the integral we seek is

$$E\int_0^\infty S(t)e^{-rt}dt = S(0)\int_0^\infty e^{(\mu-r)t}dt = \frac{S(0)}{r-\mu}$$

provided $r > \mu$.

To find the solution to the third integral in the right hand side of (21) first define $G = -Ln(S)$ and apply Itô's lemma to get

$$dG = \left(\frac{\sigma^2}{2} - \mu\right) dt - \sigma dz.$$

Integrating this last equation we obtain

$$\begin{aligned} G(t) &= G(0) + \int_0^t \left(\frac{\sigma^2}{2} - \mu\right) d\tau - \int_0^t \sigma dz \\ &= G(0) + \left(\frac{\sigma^2}{2} - \mu\right) t - \sigma(z(t) - z(0)). \end{aligned}$$

Since $1/S = e^G$ we get

$$\frac{1}{S(t)} = \frac{1}{S(0)} e^{\left(\frac{\sigma^2}{2} - \mu\right) t - \sigma(z(t) - z(0))}.$$

Therefore, the expected value is

$$E\left(\frac{1}{S(t)}\right) = \frac{1}{S(0)} e^{\left(\frac{\sigma^2}{2} - \mu\right) t} e^{\frac{\sigma^2}{2} t} = \frac{1}{S(0)} e^{(\sigma^2 - \mu) t}.$$

Hence

$$E \int_0^\infty S(t)^{-1} e^{-rt} dt = \frac{1}{S(0)} \int_0^\infty e^{(\sigma^2 - \mu - r)t} dt = -\frac{1}{S(0)(\sigma^2 - \mu - r)}.$$

Finally we write the present discounted value of profits when exporting as

$$E \int_0^\infty \pi_0 (S) e^{-rt} dt = \frac{m_0}{r} + \frac{m_1 S(0)}{r - \mu} - \frac{m_2}{S(0)(\sigma^2 - \mu - r)}$$

which is exactly the particular solution to the non-homogeneous differential equation (8) given in (11).

Now we show that the expected present value of a the operating profits of the multinational firm is equal to the particular solution to the non-homogeneous equation (9). The expected present value of the operating profits of the multinational firm are

$$E\int_0^\infty \pi_1(S)\, e^{-rt}dt = E\int_0^\infty m_4 S e^{-rt}dt,$$

where

$$m_4 = \frac{1}{\beta}\left(\frac{\alpha-\gamma^*}{N+1}\right)^2 - f^*.$$

Using previous results we have that

$$E\int_0^\infty \pi_1(S)\, e^{-rt}dt = m_4\frac{S(0)}{r-\mu}.$$

which is equal to the particular solution to the non-homogeneous equation (9) as given by (12).

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