Unemployment and growth dynamics: new insights on the hysteresis hypothesis*

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Very Preliminary Version

Abstract

We develop a growth model with unemployment in the labor market due to wage inertia. The lack of immediate adjustment in wages implies that those fiscal policies affecting economic growth have permanent effects on unemployment. We use this model to show that the fiscal policies followed by most of the European countries after the shocks of the 1970's may contribute to support the hysteresis hypotheses, which is explained as the result of the selection between different equilibrium paths.

Key Words: unemployment, hysteresis, multiple equilibria, growth, fiscal policy.

JEL Classification Numbers: E24, E62, O41.

*Raurich is grateful to Spanish Ministry of Education for financial support through grant SEC2003-00306. Sala is grateful to the Spanish Ministry of Education for financial support through DGICYT grant SEC2000-0474. Sorolla is grateful to Spanish Ministry of Education for financial support through DGICYT grant SEC2000-0684.

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1. Introduction

This paper takes a new look at the hysteresis hypothesis and provides new insights to evaluate the European Unemployment Problem. More precisely, the aim is to provide an explanation for some empirical regularities that have not received sufficient attention. These are: i) the close correlation between the unemployment rate trajectory and the growth rate of capital stock; and ii) the existence of two regimes (this being the central feature of the hysteresis hypotheses) in the unemployment rate and the growth rate of capital stock. In contrast with most of the existing literature, we take a dynamic general equilibrium approach and explain these empirical regularities as the result of equilibria selection in an endogenous growth model with wage inertia, where direct taxes are set by the government to balance its budget constraint.

The fact that European labor markets have never recovered the full employment levels which characterized the 1960s and first 1970s remains as one of the main puzzles in economics. To tackle this puzzle, some studies, like Bentolila and Bertola (1990), have pointed to employment protection to argue that Eurosclerosis was the main responsible for the high levels of unemployment in Europe. A more general approach, based on the "natural rate of unemployment" (NRU), asserts that European unemployment raised because the long-run equilibrium unemployment increased; nevertheless, despite the recent successful experiences in some European countries,[1] unemployment rates in the EU are still far away from their 1970s levels. Other studies, have outlined the persistent effects of temporary shocks, referring to the hysteresis hypotheses in the extreme case, when some of these shocks had permanent effects either on employment or the unemployment rate (Blanchard and Summers (1986)).

Within the studies explaining the path of unemployment as a result of hysteresis, we should differentiate between those arguing that temporary shocks have persistent effects on unemployment because the speed of convergence is very low, and those arguing that temporary shocks have persistent effects because these shocks make agents to coordinate into another equilibrium path. Our paper belongs to the latter line of research, and explains the patterns of unemployment as a result of equilibrium selection, i.e. hysteresis.

This hysteresis approach to explain the employment rate starts with the seminal papers by Blanchard and Summers (1986 and 1988) who argued that it was

[1] The UK, Ireland, the Netherlands or Spain seem to have manadged to reduce unemployment along a reduction in the equilibrium unemployment rate.

necessary to go beyond the natural rate hypothesis and concluded that "theories of fragile equilibria [a concept to highlight the sensitive dependence of unemployment on current and past events] are necessary to come to grip with events in Europe". In particular, they claimed that mechanisms that generate either upward-sloping demand curves or downward-sloping supply curves would allow for the possibility of multiple equilibria, making unemployment very sensitive to initial conditions and to current and past shocks.

Despite this claim, the work on multiple equilibria has not played a major role in the literature. Two main contributions in this area are Diamond (1982) and Mortensen (1989), but in the context of search and matching models, which fall well apart from the dynamic general equilibrium approach we propose in this paper. From the more traditional perspective of the demand-supply side analysis, Manning (1990 and 1992) argued in favor of models with multiple equilibria to explain the postwar behavior of unemployment. He supported his theoretical arguments by providing some empirical evidence for the UK unemployment rise in the 1980s, dismissing single equilibrium models and the hysteresis hypothesis. In particular, he found difficulties in identifying the shocks that were responsible for the dramatic increase in the equilibrium rate of unemployment, while claimed the hysteresis hypothesis could not explain the previous fast rise and the posterior fast reduction in unemployment. Thus, in his 1990 paper, he claimed that his simple multiple equilibrium model could provide an explanation of the UK unemployment in the 1980s. Nonetheless, the mainstream literature on unemployment in the 1990s has kept apart from the multiple equilibria perspective and, following the work by Layard, Nickell and Jackman (1991), has focused mainly on the NRU/NAIRU (i.e., a unique unemployment equilibrium rate), leaving also the hysteresis hypothesis a secondary role.

Some work is, of course, being done on the hysteresis hypotheses, mainly on the empirical side. Some examples are Cross (1988 and 1995), and some papers therein that relate the hysteresis hypothesis with the NRU; or, Jaeger and Parkinson (1994) and, recently, Piscitelli, Cross, Grinfeld and Lamba (2000), Hughes Hallet and Piscitelli (2002) and León-Ledesma-McAdam (2003), on the empirical testing of the hysteresis hypotheses.

Part of this literature is related with the finding of multiple equilibria in unemployment rates, generally by the use of Markow regime switching models. For example, in León-Ledesma-McAdam (2003) the presence of a high and low equilibria in most of the Central and Eastern European countries is observed; Akram (1998 and 1999) applies this analysis to the Norwegian case. Finally, Bianchi and Zoega (1998) find that the observed persistence in the unemployment rate of 15 OECD countries is consistent either with multiple equilibria models or with models with an endogenous natural rate.

Given that in this work there is also an empirical bias, the main sets of candidates for explaining hysteresis are still the ones proposed in Blanchard and Summers (1986a and 1986b), a first one pointing to insider-outsider arguments, a second one to capital accumulation (either in the form of physical or human capital) and a third one to fiscal policy. In contrast with the extensive literature on the insider-outsider argument (see, among many others, Lindbeck and Snower (1988 and 2001)), the other two explanations have received little relative attention in the theoretical literature.

On the one hand, Coimbra, Lloyd-Braga and Modesto (2000) and Ortigueira (2001)[2] are among the few exceptions that argue that a low accumulation of capital may explain a persistent high unemployment.[3] They explain hysteresis in the framework of a growth model and as the result of complementarities between employment and capital. These papers show that the existence of multiple equilibria, that could explain the persistence of the shocks, requires strong increasing returns to scale. However, empirical evidence does not support the existence of strong increasing returns to scale (see Basu and Fernald (1997)).

On the other hand, Den Haan (2003) and Rocheteau (1999) show, in the framework of a matching model without capital accumulation, that balanced budget rules may yield multiple steady states. Actually, these authors develop the origi-

[2] Like us, Coimbra, Lloyd-Braga and Modesto (2000) argue in favor of multiple steady states, but with an Overlapping Generations Model with strong increasing returns to scale, which are at odds with the empirical evidence. In turn, our approach also differs from Ortigueira (2001), whose analysis is based on a model of labor search with frictional unemployment and human capital accumulation.

[3] Furthermore, the overcome of the hysteresis hypothesis by the popularity of the NRU/NAIRU has derived in a strong emphasis on the role of two main sets of factors as responsibles for the rise in European unemployment. A first one with institutional variables such as union power, the fiscal wedge or unemployment benefits (Layard, Nickell and Jackman (1991)); and a second one with shocks such as the oil price ones in the 1970s or the interest rates ones in the 1980s (Phelps (1994)). Although new variables are being considered in this literature -asset prices have been found to play an important role (Phelps and Zoega (2001))-, the main argument still relies on the role played by a whole set of institutions and shocks, which have contributed to rise the equilibrium unemployment rate (Nickell (1997), Blanchard and Wolfers (2000)). Since the NAIRU models are static and, thus, investment is imposed to have no permanent effects on unemployment[4], it should come as no surprise that capital stock is ignored in the mainstream analyses of the unemployment problem.

nal argument by Blanchard and Summers (1986b) and show that balanced budget rules may make the effects on unemployment of a temporary negative shock persistent, and eventually, permanent.

In contrast to the theoretical literature, mainly focusing in either a static or a dynamic partial equilibrium framework, and thus overlooking the close relation displayed by the data between capital accumulation and unemployment,[5] our growth model with wage inertia provides a benchmark where the growth rate of capital stock is found to have permanent effects on unemployment: labor demand is continuously shifted up by capital accumulation, thereby causing a permanent effect on the employment rate because the wage, due to its inertia, does not fully adjust. In this framework, we discuss whether either increasing returns in the production function or balanced budget rules may provide an explanation to the hysteresis hypothesis in the patterns of both unemployment and growth.

In particular, we present a simple one-sector growth model that allows for sustained growth because of a linear production technology. For simplicity, we consider a union that sets the wage as a mark-up over a reservation wage, assumed to be a weighted average of past income and, thus, taking account of wage inertia. In this context, the higher economic growth the higher capital accumulation and the more labor demand is shifted up. Hence, employment is enhanced provided the rise in labor demand does not fully translate into wage increases, which happens when wage inertia is sufficiently strong. Moreover, when the production function exhibits increasing returns to scale, more employment implies higher interest rates. If agents are willing to substitute consumption intertemporally, the increase in the interest rate enhances savings and the accumulation of capital and, thus, accelerates economic growth reinforcing the process. It follows that our simple endogenous growth model with wage inertia creates a complementarity between employment and growth (savings) that could result into a high and a low equilibrium characterized by high (and low) savings and employment rates. These two equilibrium paths could correspond to the two regimes found in the literature and that we illustrate in Section 2. However, we show that the equilibrium path of this simple model is unique unless we introduce strong increasing returns

[5] Some empirical literature show that there is a close relationship between these two variables. This is outlined by Rowthorn (1999) who suggests "that a major factor behind persistent unemployment may also be inadequate growth in capital stock". Henry, Karanassou and Snower (2000) point to the importance of the role of capital stock in influencing the UK unemployment trajectory, but it is in Karanassou, Sala and Snower (2003a, 2003b) where a reappraisal of the causes of european unemployment is provided, and capital stock is shown to be an important determinant (if not the leading one) of the movements in the European unemployment rate.

to scale. This strong increasing returns to scale are not supported by the empirical evidence (see Basu and Fernald (1997)). Thus, this simple model fails to explain the employment hysteresis as a result of capital-employment complementarities.

Having outlined the relevance of wage rigidities and increasing returns, it is important to draw attention on another feature of the model. In yielding higher levels of production/income, increases in capital stock and employment rise public revenues via direct taxes. This confers a central role to the link between fiscal policy and economic activity. If we assume that the governments' aim is to maintain a balanced budget constraint, then, along the business cycle, either government spending or taxes must adjust to keep it effectively balanced. Therefore, when government spending is treated as endogenous, the direct tax rates will be constant and, hence, will exhibit an acyclical behavior. In contrast, when direct tax rates are considered endogenous, we expect them to be countercyclical; i.e., to be high in bad times and low in good times.[6] This is simply a result from the fact that government expenditures, such as unemployment benefits, rise in bad times and government revenues shrink in good times.

Interestingly enough, the case in which direct taxes are endogenous introduces a new source of complementarity which may explain the existence of multiple equilibrium paths. Multiple equilibria may exists when agents' expectations are self-fulfilling.. To see how direct taxes may make agents' expectations self-fulfilling, assume that agents coordinate into an expectations of high after tax interest rate. If agents are willing to substitute intertemporally consumption, the savings rate will be large and thus the growth rate of the capital stock and of the labor demand will also be large. When there is wage inertia, this large growth of the labor demand implies a high value of the employment rate. Thus, due to the expectation, economic activity will be large which will imply large government revenues and low government expenditures. Obviously, the endogenous direct tax rate will be low in this economy and hence the net of taxes equilibrium interest rate will be large. This means that endogenous tax rates makes agents' expectation hold in equilibrium. This complementarity explains the existence of an equilibrium path corresponding to an economic regime of high economic activity and, by means of the same arguments, it can also explain the existence of another equilibrium path corresponding to a lower economic activity regime.

In Section 6, we show that this coordination failure may cause the existence of two different equilibrium path converging to different steady states. One of them corresponds to a high regime characterized by high employment, savings

[6] We understand the direct tax rate as the percentage of direct taxes on GDP.

and growth rates, and low direct tax rates. The other one is the low regime, characterized by low employment, growth and savings rates, and high tax rates. Along these two equilibrium paths, government spending as a fraction of income is constant. Thus, both paths belong to the same Laffer curve. In this context, we interpret hysteresis as the result of equilibrium selection between two paths belonging to the same Laffer curve. In contrast, when the tax rate is exogenous, we show in Section 5 that the equilibrium is unique because, in fact, the government selects one equilibrium path by means of setting the tax rate.

From a theoretical perspective, the model allows us to derive a number of conditions under which an economy is more likely to generate hysteresis. These are: i) Strong wage rigidities; ii) Endogenous (countercyclical) tax rates; iii) Large unemployment benefits; and iv) Large willingness to substitute intertemporally consumption. In turn, from an empirical point of view, our model matches remarkably well some observed regularities explained in Section 2. In particular: i) using Kernel density functions, we show that most of the EU economies display high and low regimes in unemployment and the growth rate of capital stock, whereas the US economy displays a unique regime in unemployment; ii) we show that direct taxes seem to have been acyclical in the US economy, in contrast with most EU economies where they have tended to be countercyclical, or even highly countercyclical (like in Spain or France). It is clearly beyond the scope of this paper to provide a global account of the US and EU experiences. Nevertheless, given the appropriate of our model, in the last part of the paper we have conducted a numerical analysis attempting to reproduce some of these stylized facts. In this, way we can illustrate how work the above mentioned conditions for the existence of hysteresis.

From this analysis, we conclude that the shocks suffered by the world economy in the 70s, whose main expression was a permanent downturn in TFP, had different consequences in the US and the EU economies. Our model focuses primarily in the fiscal policy response. In the US, direct tax rates were kept constant and the TFP downturn produced a temporary fall in savings, economic growth and employment, which progressively recovered to reach the original equilibrium. There were no permanent consequences, as the model explains when tax rates are exogenous. In contrast, the EU experience seems to correspond to a case where direct tax rates are endogenous and two equilibrium path exist. In that case, the shocks of the 70's and the resulting TFP downturn may have caused agents to coordinate into a low regime equilibrium, hence keeping permanent the effects of these temporary shocks. In this way, we are able to explain hysteresis as the result of a coordination failure between equilibria belonging to the same Laffer curve. As this coordination failure does not occur when the government selects one equilibrium by means of setting the tax rate, we conclude that a fiscal policy based on exogenous tax rates is a superior fiscal policy that may prevent both economic instability and the convergence into the wrong side of the Laffer curve.

The structure of the paper is the following. Section 2 provides an evaluation of the regime changes in unemployment, which we find closely related with the trajectory of the capital stock growth rate; a countercyclical behavior of the direct tax rates is also identified for most of the European countries. Section 3 describes a simple growth model, whose equilibrium is obtained in Section 4. The equilibrium is characterized in the following two sections, but in two different cases: when the direct tax rates are exogenous (Section 5) and when they are endogenous (Section 6). Using this benchmark, in Section 7 we present a numerical example of a TFP shock, which illustrates under what plausible conditions hysteresis can be generated. Section 8 summarizes our findings and concludes.

2. Empirical evidence underlying our theoretical modelling

Blanchard and Summers (1986a) launched the hysteresis hypothesis as the main explanation for the differences between the labor market performance in Europe and the US in the 1970s and early 1980s. As noted before, they pointed to three main causes of hysteresis: insider power, fiscal policy, and negative shocks in human and physical capital stock. In this section we shed some light on the empirical relevance of the latter argument and argue that there is a close link between changes in the unemployment rate and changes in the growth rate of capital stock. We show this at the aggregate and individual EU level and for the US. The last subsection deals with the fiscal policies and identifies a procyclical pattern in most European countries.

The analysis we undertake next is inspired in Bianchi and Zoega (1998), whose work is based on the estimation of Kernel density functions aiming at the identification of regime changes in the unemployment rate of 15 OECD countries, including all the ones considered here.[7] Notwithstanding their work, and in their own words, "the question of the causes of the mean shifts still arises (...) despite two explanations have been suggested in the literature: (...) changes in equilibrium

[7] They also evaluate the resulting mean shifts accross unemployment regimes and find that when the mean shift is subtracted from the second regime mean (with high unemployment rates), the degree of unemployment persistence is substantially lowered.

unemployment and (...) transitory shocks which may have a persistent effects on unemployment such as hysteresis channels". What our analysis suggests is that regime changes in the direct tax rates should be considered as one of the reasons for the mean regime shifts found in unemployment in Bianchi and Zoega (1998) and that this regime shifts in the unemployment rate are directly related with regime shifts in the growth rate of the capital stock.

As noted before, it is customary to use Markov switching regime models to identify the timing of regime or mean shifts. This implies -first- the need to fix a-priori the number of regimes, some of which -in a second stage- are considered as reflecting business cycles changes, and thus -in a third stage- are removed from the analysis of structural changes. This procedure, usually applied just to unemployment time series, lies far beyond the scope of the exercise we perform in this section, where we just seek to outline the main empirical regularities underlying our theoretical modelling. This is why we proceed in a simpler way.

Our exercise involves two type of decisions, the first one on the **number** of regimes, and the second one on the **type** of regimes. With respect to the first decision, we know that when a time series displays different regimes, the density of the frequency distribution of that series will be multimodal, with the number of modes corresponding to the number of regimes. Thus, as Bianchi and Zoega (1998), we will use a Kernel density analysis to identify regime changes in the time series of the unemployment and the capital stock growth rates. Our identification criteria is the following. We will consider a regime exists when there is a point whose first derivative is zero and whose second derivative is negative. This point indicates the regime mean value, which can be seen as a local maximum (i.e., a point with the highest density). When two or more regimes exist, a 'valley point' (whose first derivative is zero and the second is positive) divides the data points in the sample. Those observations with values above the 'valley point' will belong to the upper regime, whereas those with values below will belong to the lower regime.

With respect to the second type of decision, we consider two type of regimes shifts. First, **temporary**, in response to transitory or persistent shocks. This means that a set of data points remain in the same regime at most during four consecutive periods. Second, **permanent**, in response to irreversible shifts or permanent shocks. These are all shocks that lasted at least five years. This allows to disentangle business cycles movements from structural changes reflected in permanent "mean shifts".[8]

[8]It could be argued that there is some degree of arbitrariness in the characterization of the

Our database is the same than the one used in Karanassou, Sala and Snower (2003a and 2003b): it has annual data on unemployment, business capital stock, GDP and direct taxes, all provided by the OECD, and involves annual information on 11 EU countries starting in the 1960s (Austria, Belgium, Denmark, Germany, Finland, France, Italy, Netherlands, Spain, Sweden and the United Kingdom).

2.1. Unemployment

Figure 1 pictures the sharp contrast between the unemployment rate trajectory in the EU and the US.

[Insert Figure 1]

In Europe there is a neat structural change, placed in 1980 by our Kernel density analysis, which shifts the regime mean upwards from 2.5% to 9.7%.[9] In contrast, the US analysis reveals a unique regime, only altered at the beginning of the 1980s by what seems to have been a one-off shock. The country-specific analysis, presented in Figure 2, gives additional evidence on this matter.[10]

[Insert Figure 2]

These plots are obtained from the kernel density analysis depicted in Figure 3.

[Insert Figure 3]

It seems clear, thus, that the EU has experienced a permanent change, whereas the US series is characterized by a stationary pattern. Next we argue that the European countries experienced a permanent change in capital formation, with a regime mean shift that corresponds to the regime shift in unemployment.

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regime changes, but it seems sensitive to consider temporary every change lasting 4 or less years. Furthermore, it should be noted that our EU aggregate evaluation does not depend on the country-specific results: we perform the whole analysis on the aggregate time series (see Figures 1 and 4) and it turns out to yield very clean results in terms of the judgement on the number and type of shocks. Thus, our criteria affects mainly the individual analysis, whose results are presented in the appendix.

[9] Despite the quantitative results we present are, of course, sensitive to the sample period, it should be taken even as more relevant the clean qualitative picture that emerges from figures 1 to 3.

[10] In Figures 2 and 5 we present only what we consider permanent regime changes (i.e., temporary regime changes are not plotted, as in Figures 1 and 4) using the criteria explained at the beginning of this section.

2.2. Capital stock growth

Given the existence of several particular cases, we refer first to the country-specific results. In particular, the results of the Kernel density analysis for the individual countries, pictured in Figures 4 and 5 below, are presented in Table 1.

[Insert Table 1]

Note that in Finland, Netherlands and UK only one regime is identified, whereas the rest of countries display two regimes.[11] All the regime changes take place in the mid 1970s, when the unemployment rates in these countries started to rise sharply. This matches remarkably well with Bianchi and Zoega's 1998 results and our own analysis on the unemployment rate.

[Insert Figures 4 and 5]

With respect to the aggregate capital stock series for the whole EU, there is no long time-series directly provided by the OECD. Thus, we need to aggregate the series corresponding to the pool of countries under consideration, which involves two important requirements: first, to establish an accurate criterion to assign country weights; second, to avoid any noise derived from exchange rates fluctuations, given that the capital stock series are expressed in national currencies. The connection between capital stock and output point at GDP as the relevant measure to weight the individual capital stock series. Moreover, GDP series are generally available since the 1960s and they allow us to compute a yearly weight. In particular, to reach the second criterion we use a especial series of real GDP in Purchasing Power Parities, which is what better guarantees country comparisons taking into account changes in national prices and exchange rate cross-variations. Since we are not interested in the EU levels of the capital stock, but in its growth rate, what we finally construct is an aggregate series of the growth rate.[12]

[11] In Finland and the Netherlands the reason may be the lack of data in the 1960s (the series start in 1970 and 1969 respectively), which prevents the Kernel density analysis to consider the few data points with high values as a separate regime (see figure 5). In Finland, the unique regime diplays a mean of 2.9%, but from 1970 to 1977 capital stock growth is above 3%. In the Netherlands the regime mean is at 2.3%, but from 1969 to 1979 takes values above 2.5 percentage points in all years except 1976. In turn, the UK displays an exceptional behaviour in the late 1990s at odds with the rest of the countries.

[12] For Austria, Belgium, Denmark, Germany and Italy we have data on capital stock since 1960 (on the growth rate since 1961). Nevertheless, the growth rate of the aggregate capital

[Insert Figure 6]

With the aggregate EU and US series we conduct a Kernel density analysis, exactly in the same way as before, and obtain the results displayed in Figure 6. In Figure 6c a first regime is identified for the EU, lasting from 1963 to 1974, and having a mean capital stock growth rate of 4.9%. The second one starts in 1975 and lasts up to 1999, with a regime mean of 2.7%. The only exceptional data point in this regime occurs in 1991, when the series comes across the German unification consequences, in the form of a sudden rise in the growth rate of capital stock.[13] The analysis for the US yields a different picture. Despite two regimes are identified (Figure 6d), they differ by just 1.1 percentage points, which makes the series oscillate several times between regimes. Following our criterion to qualify the type of shocks, we would identify a high regime mean up to 1985 (with two temporary negative shocks corresponding to the oil price crises), followed by a low regime mean which ends by a temporary upwards shift. The latter corresponds to the anti-inflationist monetary policy of the Volcker era, from 1979 to 1987, which shifted real interest rates upwards.[14]

Beyond this quantitative analysis, the general picture that emerges by comparing Figures 1a and 1b *versus* 6a and 6b is the following. In the EU there is a structural change which is expressed in an upwards unemployment regime shift of 7.2 percentage points that, perhaps taking too far our analysis, corresponds with a 2.2 percentage points reduction in the mean growth rate of physical capital stock. On the contrary, there is no such a structural change in the US unemployment, something that matches well with the oscillatory pattern of the regime shifts in the growth rate of capital stock. What seems clear from this analysis is the appropriateness of a multiple equilibria model for the EU, assigning a relevant role to capital formation.

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stock series starts in 1963 because since 1962 we also have data for France and the UK, and since 1963 for Spain, all countries with substantial weight in the EU. The rest of the countries are progressively taken into account, the weights being amended correspondingly: data for Sweden start in 1965, for the Netherlands in 1968 and for Finland in 1969.

[13] This is clearly a one-off shock as the mean shifts from the low to the high regime just during one year.

[14] To add two more years to the sample period would probably yield us to qualify the last positive shock on the growth rate of capital stock in the US as permanent. Of course, this depends on our own definition of what constitutes a temporary or a permanent shock, but in any case, we would reach the same conclusion: capital formation in the US has not experienced a structural change of the EU sort.

2.3. Fiscal policies and the business cycle

Our theoretical model, presented below, indicates that the different economic performance of the EU and the US may in part be a response to their different fiscal policy behavior. In particular, even if both areas were affected by similar shocks in the 70s, their different response in terms of fiscal policy would have seriously conditioned their subsequent performance.

Figure 7 relates the trajectory of the direct tax rate (as percentage of GDP) to economic growth. As stated before, we interpret a negative relationship of these two series as the result of the implementation of countercyclical tax rates, something that seems to have been the case in most EU countries. In particular, the coexistence of what could be taken as a high economic growth regime mean in the 1960s and first 1970s with a low direct tax rate regime mean (and the opposite in the 1980s and 1990s) is apparent in all the EU areas: southern (Italy and Spain), continental (Austria, Belgium, Germany, Netherlands and France) and northern (Denmark, Sweden and, to a less extent, Finland). The sole exception in the EU is the UK, where these two series display a very mild negative correlation, just as in the US. Furthermore, in the latter case there are signs of procyclical tax rates since the second half of the 1980s.

[Insert Figure 7]

Had we presented Figure 7 in the form of scattered diagrams we would have shown the negative correlation between the direct tax rate and the business cycle. Table 2 presents the estimates of this correlation, which is significant at the 1% significance level in all countries except Germany (at 8%), Finland (6%) and, of course, the UK and the US, where it is not significant.

[Insert Table 2]

Despite we find necessary, in support of our claims, to present this raw data, it is beyond our scope to conduct a full analysis of fiscal cyclicality. Indeed, this is already done in Lane (2003) for the OECD countries, who sustains that "evidence of fiscal procyclicality in fiscal policy has been uncovered in a number of studies" (p. 2661). Despite his analysis targets different spending categories, and does not explicitly deal with direct taxes, he finds that "the United Kingdom and the United States are not representative of the full sample, with these countries displaying more countercyclical fiscal behavior than the average in the sample. In contrast, countries such as Ireland and Portugal exhibit above-average procyclicality in fiscal policy across the range of spending categories" (p. 2669).

It seems, thus, that there is a different fiscal policy pattern in the EU and the US, which leads us to think that the degree of fiscal cyclicality, mainly in the direct tax rates, may be a relevant factor underlying these two areas' different labor market performance. This is taken into account in the theoretical model presented in Section 3.

2.4. Total factor productivity

As a growth model, TFP plays a relevant role as a determinant of employment, savings and economic growth. There is a broad consensus on the fact that the TFP productivity downturn was the main expression of the consequences of the shocks that hitted the advanced economies in the 70's. Since the numerical example illustrating how hysteresis can be generated attains the Spanish economy, we have used the model of Karanassou, Sala and Snower (2002) to estimate this downturn, which is shown in Figure 8.[15]

[Insert Figure 8]

Note that this downturn is expressed both as a downward one-off shift and a change in the slope.

Our model predicts that this downturn leads to a reduction in economic growth, the savings rate and employment, something that the data seems to confirm at least in the Spanish case. In particular, Figures 9a and 9b depict a negative correlation between unemployment and, respectively, savings and economic growth.[16]

[15] The production function estimated in Karanassou, Sala and Snower (2002) takes the standard form: it is a Cobb-Douglas production function where the log of GDP depends on its past values, the log of the two production factors (capital stock and employment) and a linear trend (t). The sole particularity is that this trend is splitted in two terms: t on its own (with a positive sign) and t multiplied by a dummy taking value 1 1978 onwards (the year of the dummy was selected by the AIC criterion). The latter reduces by a third the impact of the trend, meant to capture the influence of technological change. In this context, the TFP (in logs) is defined as the variation in production not accounted by movements in the two production factors. Figure 8 displays its trajectory (in logs), with a downturn in 1978 corresponding to the effect of the above mentioned multiplicative dummy.

[16] The results of the two regressions (t-statistics in parentheses) are the following:

$$u_t = \underset{(4.81)}{45.5} - \underset{(-3.57)}{2.54} S_t \text{ with } R^2 = 0.27 \text{ and sample period 1964-1999,}$$

[Insert Figure 9]

3. The Economy

In this section we construct a simple one sector endogenous growth model where a labor market friction is introduced by means of a monopolistic union. We use the model to provide an explanation of some of the empirical regularities described in the previous section. In particular, we use the model to provide an explanation to the existence of two regimes for the unemployment and the growth rate. We proceed to describe the technology.

3.1. Technology

For simplicity, we assume the following Cobb-Douglas aggregate production function:

$$Y(t) = AK(t)^{\alpha} L(t)^{1-\alpha} \overline{k}(t)^{1-\alpha} \overline{l}(t)^{\beta}, \; A > 0, \; \alpha \in (0,1), \; \beta \in [0,1),$$

where $Y(t)$ is the gross domestic product (GDP), $K(t)$ is the aggregate stock of capital, $L(t)$ is the number of employed workers, $k(t) = \frac{K(t)}{L(t)}$ is the per employed worker stock of capital, $N(t)$ is the number of workers in the economy, $l(t) = \frac{L(t)}{N(t)}$ is the employment rate, $\overline{k}(t)$ is the average stock of capital per employed worker and $\overline{l}(t)$ is the average employment rate in the economy. Note that $\overline{k}(t)$ and $\overline{l}(t)$ are positive externalities. The technological parameter A determines, together with the path of the externalities, the total factor productivity (TFP), and the parameter β provides a measure of the intensity of the externalities accruing from the average employment rate. Note that if $\beta = 0$ the production function exhibits constant returns to scale, whereas it exhibits increasing returns to scale when $\beta > 0$.

Perfect competition and profit maximization imply that the competitive factors payment are

$$r(t) = \alpha AK(t)^{\alpha-1} L(t)^{1-\alpha} \overline{k}(t)^{1-\alpha} \overline{l}(t)^{\beta},$$

$$u_t = \underset{(9.37)}{17.3} - \underset{(-4.08)}{1.47} \Delta y_t \text{ with } R^2 = 0.31 \text{ and sample period 1961-1999,}$$

where u is the unemployment rate, S is the saving ratio and Δy_t is the change in real GDP (all variables expressed in percentage points).

and

$$w(t) = (1-\alpha) AK(t)^{\alpha} L(t)^{-\alpha} \overline{k}(t)^{1-\alpha} \overline{l}(t)^{\beta}.$$

This last equation implicitly defines the non-equilibrium labor demand

$$L^{d}\left(w(t), K(t), \overline{k}(t), \overline{l}(t)\right) = \left(\frac{(1-\alpha) AK(t)^{\alpha} \overline{k}(t)^{1-\alpha} \overline{l}(t)^{\beta}}{w(t)}\right)^{\frac{1}{\alpha}},$$

Because of the externalities accruing from the average stock of capital and the assumptions made on the production function, it exhibits constant returns with respect to both the private production factors and also to the stock of capital along a symmetric equilibrium path, i.e. when $k(t) = \overline{k}(t)$. Thus, we can rewrite the production function in per employed worker terms as follows

$$y(t) = Ak(t)^{\alpha} \overline{k}(t)^{1-\alpha} \overline{l}(t)^{\beta},$$

where $y(t) = \frac{Y(t)}{L(t)}$. Along a symmetric equilibrium

$$y(t) = Ak(t) l(t)^{\beta}.$$

Note that the production function corresponds to an Ak production function when $\beta = 0$. The competitive factor payments along a symmetric equilibrium are

$$r(t) = \alpha A l(t)^{\beta}, \tag{3.1}$$

and

$$w(t) = (1-\alpha) Ak(t) l(t)^{\beta}. \tag{3.2}$$

The last equation determines the equilibrium labor demand, which can be rewritten as follows

$$l^{d}\left(\left(w(t), \widetilde{k}(t)\right)\right) = \left(\frac{(1-\alpha) A\widetilde{k}(t)}{w(t)}\right)^{\frac{1}{1-\beta}}, \tag{3.3}$$

where $\widetilde{k}(t) = \frac{K(t)}{N(t)}$ is the per capita stock of capital.

Note that the labor demand positively depends on the per capita stock of capital. Thus, the growth of the stock of capital rises the labor demand, which causes an increase of the employment rate when there is wage inertia. In the following section, we show, in a simple model of the labor market, that economic growth enhances the employment rate when there is wage inertia.

3.2. Labor market

We assume firm-level wage setting, where a firm-level union sets the wage in order to maximize:

$$\underset{w(t)}{Max}\; V = \left[(1-\tau(t))\, w(t) - w^s(t)\right]^{\gamma} L^d\left(w(t), K(t), \overline{k}(t), \overline{l}(t)\right),$$

where $w^s(t)$ is a reference wage and $\tau(t)$ is the direct tax rate.[17] The solution to this maximization problem characterizes the wage equation

$$w(t) = \frac{w(t)^s}{(1-\tau(t))\left(1+\frac{\gamma}{\varepsilon(t)}\right)},$$

where $\varepsilon(t)$ is the inverse of the price elasticity of the labor demand. When the union does not take into account the externality, $\varepsilon(t) = -\frac{1}{\alpha}$ and the wage equation simplifies as follows[18]

$$w(t) = \frac{w(t)^s}{(1-\tau)(1-\alpha\gamma)}. \tag{3.4}$$

We assume that the reference wage is given by the following weighted average of the workers' past labor income:

$$w^s(t) = w^s(0)\, e^{-\theta t} + \theta \int_0^t e^{-\theta(t-i)} x(i)\, di, \tag{3.5}$$

where $w^s(0)$ is the initial wage reference, $x(t)$ is the workers' average labor income and $\theta > 0$ provides a measure of the rate of wage adjustment.[19] Note that the higher θ, the lower is the weight of past average labor income in determining the reference wage, that is, the lower is the wage inertia.[20] Actually, if the parameter θ diverges to infinite then the reference wage will coincide with the current average income. In this case, there is no wage inertia. If we assume further that the current

[17] Note that the unions take as given the labor demand. Thus, the labor demand model is a right-to-manage model.

[18] Alternativelly, one could assume a national level union that sets the wage taking into account capital and labor externalities, that is considering the equilibrium labor demand $L^d\left(w(t), \widetilde{k}(t)\right)$. In this case the wage equation is $w(t) = \frac{w(t)^s}{(1-\tau)(1-(1-\beta)\gamma)}$.

[19] The relevant economic agent is the family. This explains why the average income determines the reference wage.

[20] The reference wage is a controversial variable. Blanchard and Wolfers (2000) argue that it depends on the unemployment benefit, past wages, social benefits among other variables.

average income is proportional to per capita GDP then the wage set by the unions will rise with the economic growth rate. In this case with no wage inertia, the increases in the labor demand due to economic growth will fully translate into wage increases that will prevent employment growth. In contrast, when there is wage inertia, the increases in the labor demand will not fully translate into wage increases and hence economic growth will cause employment growth. It follows that the relation between economic growth and the employment rate will depend on wage inertia. Further, this positive relation will also occur in the long run, as sustained growth implies that the labor demand permanently grows and, hence, wage inertia will limit the wage adjustment even in the long run.

It is important to note that there is an initial condition on $w^s(0)$, as this variable is determined by past average labor income. Moreover, $w^s(0)$ determines the initial wage that is set by the unions, $w(0)$. Finally, given the initial wage and the initial stock of capital, the initial employment rate is obtained from the equilibrium labor demand sets, $l(0) = l^d(w(0), k(0))$. Thus, in this model the employment rate is a state variable.

Differentiate with respect to time (3.5) and we obtain

$$\dot{w}^s(t) = \theta(x(t) - w^s(t)), \tag{3.6}$$

where the average labor income is

$$x(t) = (1-\tau(t))\, l(t)\, w(t) + \lambda(1-\tau(t))(1-l(t))\, w(t) + pw(t), \ \ \lambda > 0, \ \ p < 0, \tag{3.7}$$

where $\lambda(1-\tau(t))\, w(t)$ are the unemployment benefits, and $pw(t)$ amounts to a tax on the wage.[21] From now on we will assume that $\lambda(1-\tau(t)) + p > 0$ since otherwise the labor income of the unemployed workers would be negative.

We proceed to obtain the equilibrium path of the employment rate. To this end, we first combine (3.6) with (3.7) to obtain

$$\dot{w}^s(t) = \theta([(1-\tau(t))(l(t) + (1-l(t))\lambda) + p]\, w(t) - w^s(t)),$$

and use (3.4) to obtain

$$\frac{\dot{w}^s(t)}{w^s(t)} \equiv \xi(l(t), \tau(t)) = \theta\left[\frac{l(t) + (1-l(t))\lambda}{1-\alpha\gamma} + \frac{p}{(1-\tau(t))(1-\alpha\gamma)} - 1\right].$$

[21] In our model, these taxes amount to any tax that does not modify the mark-up set by the unions. We modelled them as proportional to the wage just for the sake of simplicity.

We log-differentiate with respect to time (3.4) and we obtain

$$\frac{\dot{w}^s(t)}{w^s(t)} = \frac{\dot{w}(t)}{w(t)} - \frac{\dot{\tau}(t)}{1-\tau(t)} = \xi(l(t), \tau(t)),$$

where $\xi(l(t), \tau(t))$ is the growth rate of the after tax wage. Note that the growth rate of the after tax wage coincides with the growth rate of the reservation wage.

We differentiate with respect to time (3.2) and we obtain

$$\frac{\dot{w}(t)}{w(t)} = \frac{\dot{k}(t)}{k(t)} + \beta \frac{\dot{l}(t)}{l(t)}.$$

Combining with the previous equation, we obtain

$$\beta \frac{\dot{l}(t)}{l(t)} = \frac{\dot{\tau}(t)}{1-\tau(t)} + \xi(l(t), \tau(t)) - \frac{\dot{k}(t)}{k(t)}. \tag{3.8}$$

Thus, the growth of the employment rate depends on fiscal policy, on the growth of the gross wage and on the accumulation of the capital stock. In the following section, we characterize the accumulation of the capital stock in a simple infinite horizon representative agent model.

3.3. Families

We assume that there is a unique infinitely lived dynasty in the economy. Let $N(t)$ be the number of members of this dynasty that inelastically supply one unit of labor. Thus, for simplicity, we assume that the aggregate labor supply is equal to $N(t)$. The dynasty maximizes the discounted sum of the utility of each member

$$U = \int_0^\infty e^{-(\rho-n)t} \left(\frac{\widetilde{c}(t)^{1-\sigma} - 1}{1-\sigma} \right) dt, \ \rho - n > 0, \ \sigma > 0,$$

subject to the budget constraint

$$\widetilde{c}(t) + \dot{\widetilde{k}}(t) = ((1-\tau(t)) r(t) - n) \widetilde{k}(t) + x(t),$$

where $\widetilde{c}(t) = \frac{C(t)}{N(t)}$ is the per capita level of consumption, the parameter $\rho > 0$ is the subjective discount rate, n is the constant population growth rate and $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution. Note that consumers' revenues accrue from capital income and from average labor income.

We introduce the average labor income because we assume a large dynasty. In this way, we avoid the problems associated with heterogeneity that would make the model analytically intractable..

The solution to the families maximization problem characterizes the growth rate of the per capita consumption

$$\frac{\dot{\widetilde{c}}(t)}{\widetilde{c}(t)} = \frac{(1-\tau(t))\, r(t) - \rho}{\sigma},$$

and the transversality condition

$$\lim_{t\to\infty} K(t)\, e^{-\rho t} C(t)^{-\sigma} = 0. \tag{3.9}$$

Let us denote the growth of the per capita consumption by $\mu(t) = \frac{\dot{\widetilde{c}}(t)}{\widetilde{c}(t)}$. Then, by using (3.1), we obtain

$$\mu(l(t), \tau(t)) = \frac{(1-\tau(t))\, \alpha A l(t)^{\beta} - \rho}{\sigma}. \tag{3.10}$$

Thus, the consumption growth rate depends on the employment rate and on the direct tax rate.

We have shown that both the consumption growth rate and the growth of the employment rate depend on fiscal policy. In the following section, we conclude the description of the economy by means of characterizing the government budget constraint.

3.4. Government

We assume that the government follows a balanced budget rule. The government budget constraint is given by the following equation:

$$\tau(t)\, Y(t) - pN(t)\, w(t) = G(t) + (N(t) - L(t))\, \lambda (1-\tau(t))\, w(t).$$

Government revenues accruing from taxes are used to finance non-productive government spending $G(t)$ and the unemployment benefit. Let us denote by $g(t) = \frac{G(t)}{Y(t)}$ the fraction of GDP devoted to government spending. Then, the government budget constraint can be rewritten as follows

$$\tau(t) - g(t) = ((1 - l(t))\, \lambda (1-\tau(t)) + p)\, \frac{w(t)}{\widetilde{y}(t)},$$

where $\widetilde{y}(t) = \frac{Y(t)}{N(t)}$ is the per capita GDP. Note that $Y(t) = \widetilde{y}(t) N(t) = y(t) L(t)$. Thus,

$$\widetilde{y}(t) = y(t) l(t) = Ak(t) l(t)^{\beta+1}$$

and $\frac{w(t)}{\widetilde{y}(t)} = \frac{1-\alpha}{l(t)}$.

By using the government budget constraint it follows that if $g(t) = g$ is constant and exogenous and the government sets the value of the direct tax rate to balance its budget constraint in each period, then the government budget constraint determines the path of the endogenous direct tax rate as the following function of the employment rate:

$$\tau(l(t)) = \frac{gl(t) + ((1 - l(t))\lambda + p)(1 - \alpha)}{l(t) + (1 - l(t))\lambda(1 - \alpha)}. \tag{3.11}$$

Note that

$$\tau'(l(t)) = (1 - \alpha)\frac{(g - 1)\lambda - [1 - \lambda(1 - \alpha)]p}{(l(t) + (1 - l(t))\lambda(1 - \alpha))^2} < 0,$$

if $(g - 1)\lambda - [1 - \lambda(1 - \alpha)]p < 0$. From now one, we assume that this condition is satisfied so that $\tau'(l(t)) < 0$, which means that the tax rate is countercyclical.. Note also that parameter constraints must be introduced in order to guarantee that $\tau(l) \in (0, 1)$.

By using the government budget constraint, we can also obtain the value of $g(t)$ that balances the government budget constraint when $\tau(t) = \tau$ is constant and exogenous. This path of government spending is also a function of the employment rate

$$g(l(t)) = \tau - ((1 - l(t))\lambda(1 - \tau) + p)\left(\frac{1 - \alpha}{l(t)}\right). \tag{3.12}$$

In this section we have obtained the equations that describe the economy. In the following section, we derive the equations that characterize the equilibrium path.

4. Equilibrium

In this section we first derive an equation that drives the evolution of the savings rate and that summarizes the consumers' behavior. Then, we obtain an equation that drives the growth of the employment rate and that results from the labor market frictions.

4.1. Savings rate

The resource constraint in this economy is

$$C(t) + G(t) + S(t) = Y(t),$$

where $S(t)$ are the aggregate savings of the economy that coincide with investment. Let $s'(t) = \frac{S(t)}{Y(t)}$ be the savings rate then

$$\frac{C(t)}{Y(t)} = 1 - s'(t) - g(t).$$

It will be useful to define the fraction of income that is not consumed by the families as $s(t) = s'(t) + g(t)$. Then,

$$s(t) = 1 - \frac{C(t)}{Y(t)} = 1 - \frac{\frac{C(t)}{N(t)}}{\frac{Y(t)}{N(t)}} = 1 - \frac{\widetilde{c}(t)}{\widetilde{y}(t)} = 1 - \frac{\widetilde{c}(t)}{Ak(t)\, l(t)^{\beta+1}},$$

which can be rewritten as

$$\frac{\widetilde{c}(t)}{k(t)} = (1 - s(t))\, Al(t)^{\beta+1}.$$

Note that $K(t) = \widetilde{k}(t)\, N(t) = k(t)\, L(t)$. Then, $\widetilde{k}(t) = k(t)\, l(t)$ and

$$\frac{\widetilde{c}(t)}{\widetilde{k}(t)} = (1 - s(t))\, Al(t)^{\beta}.$$

Differentiate with respect to time this equation and we obtain

$$\dot{s}(t) = (1 - s(t)) \left[\beta \left(\frac{\dot{l}(t)}{l(t)} \right) - \left(\mu(t) - g_{\widetilde{k}}(t) \right) \right]. \tag{4.1}$$

The accumulation of capital is obtained from the resource constraint

$$C(t) + \dot{K}(t) = (1 - g(t))\, Y(t),$$

where we assume for simplicity that private capital does not depreciate. The resource constraint in per capita terms is

$$\widetilde{c}(t) + \dot{\widetilde{k}}(t) - n\widetilde{k}(t) = (1 - g(t))\, \widetilde{y}(t),$$

where $\widetilde{k}(t) = \frac{K(t)}{N(t)}$. From this equation we obtain the growth of the per capita stock of capital

$$g_{\widetilde{k}}(t) = (1 - g(t)) \frac{\widetilde{y}(t)}{\widetilde{k}(t)} - \frac{\widetilde{c}}{\widetilde{k}(t)} - n,$$

and

$$g_{\widetilde{k}}(t) = (s(t) - g(t)) Al(t)^{\beta} - n. \tag{4.2}$$

Combining (3.10), (4.1) and (4.2) we obtain a differential equation that drives the path of the savings rate

$$\dot{s}(t) = (1 - s(t)) \left[\beta \left(\frac{\dot{l}(t)}{l(t)} \right) - \mu(t) + (s(t) - g(t)) Al(t)^{\beta} - n \right], \tag{4.3}$$

$$\dot{s}(t) = \widetilde{s}\left(s(t), l(t), \tau(t), g(t), \dot{l}(t) \right).$$

Note that this equation summarizes the consumers' behavior and the resource constraint of the economy. In the following section, we use the equations characterizing the labor market to obtain the differential equation that drives the path of the employment rate.

4.2. Employment rate

From (3.8) and $\frac{\dot{k}(t)}{k(t)} = g_{\widetilde{k}}(t) - \frac{\dot{l}(t)}{l(t)}$, we obtain

$$(1 - \beta) \frac{\dot{l}(t)}{l(t)} = g_{\widetilde{k}}(t) - \frac{\dot{\tau}(t)}{1 - \tau(t)} - \xi(l(t), \tau(t)) \ .$$

Thus, the growth of the employment rate depends on the difference between the growth of the per capita stock of capital and the growth of the wage before taxes. Note that the growth of the per capita stock of capital drives the growth of the labor demand as follows from (3.3) and the growth of the wage before taxes provides a measure of the rise in the labor cost. Thus, the previous equation implies that the employment rate grows when the rise in the labor demand is larger than the rise in the cost of one unit of labor. By using (4.2), we obtain the differential equation that drives the equilibrium path of the employment rate

$$\dot{l}(t) = \left[\frac{l(t)}{1 - \beta} \right] \left[(s(t) - g(t)) Al(t)^{\beta} - n - \xi(l(t), \tau(t)) - \frac{\dot{\tau}(t)}{1 - \tau(t)} \right]. \tag{4.4}$$

$$\dot{l}(t) = \widetilde{l}(s(t), l(t), \tau(t), g(t), \dot{\tau}(t))$$

Note that (4.3) and (4.4) depend on the nature of fiscal policy. In other words, the equations characterizing the equilibrium depend on the tax rate being endogenous or exogenous. This distinction is important, as we identify economies with acyclical tax rates as an example of an equilibrium with exogenous tax rates and we identify economies with countercyclical tax rates as an example of an equilibrium with endogenous taxes. According to the empirical evidence provided in Section 2, the behavior of the direct tax rates in the US is acyclical, whereas the behavior of these taxes in most of the European economies is countercyclical. In the following section we describe the equilibrium path of an economy with exogenous tax rates and in Section 6 we describe the equilibrium path of an economy with endogenous tax rates.

5. The equilibrium path when tax rates are exogenous

In this section we characterize the equilibrium path of an economy with exogenous tax rates. Thus, we assume that the tax rate is constant which means that $\tau(t) = \tau$ and hence $\dot{\tau}(l) = 0$. As a consequence, (4.4) simplify as follows

$$\dot{l}(t) = \widetilde{l}(s(t), l(t), g(t)) = \left[\frac{l(t)}{1-\beta}\right]\left[(s(t) - g(t)) Al(t)^{\beta} - n - \xi(l(t), \tau)\right], \tag{5.1}$$

and (4.3) can be rewritten as

$$\begin{aligned} \dot{s}(t) &= \widetilde{s}(s(t), l(t), g(t)) \\ &= (1 - s(t))\left[\frac{(s(t) - g(t)) Al(t)^{\beta} - n - \beta\xi(l(t), \tau)}{1-\beta} - \mu(t)\right] \end{aligned} \tag{5.2}$$

Next, we define the equilibrium of this economy as follows.

Definition 5.1. Given $\{l_0, k_0; \tau, p, \lambda\}$ an equilibrium with exogenous tax rates is defined by $\{l(t), s(t), g(t)\}_{t=0}^{\infty}$ such that solve (3.12), (5.1), and (5.2), satisfy equation (3.9) and the following constraints: $l(t) \in (0,1)$, $s(t) - g(t) \in (0,1)$ and $g(t) \in (0,1)$, for all $t \geq 0$.

In what follows we characterize the path of the dynamic equilibrium. We first show that there is a unique Balanced Growth Path (BGP) unless strong externalities are assumed. We define a BGP as a path along which $l(t)$ and $s(t)$ remain constant, and consumption, capital and GDP growth at the same constant growth rate. By using $\dot{l}(t) = 0$ and $\dot{s}(t) = 0$, it is straight forward to show that the employment rate along a BGP must satisfy the following equation:

$$Q(l) = \xi(l) - \mu(l) = 0.$$

Thus, along a BGP the long run economic growth rate coincides with the growth rate of the wage. In this simple model, this long run growth rate corresponds to the growth rate of per capita consumption, which is equal to the growth rate of the per capita stock of capital and, as follows from (3.3), it is also equal to the growth rate of the labor demand. It follows that the employment rate attains a BGP when the growth of the labor demand and of the wage coincide. From this equation, it follows that if $\beta = 0$ then the only candidate to BGP equilibrium is

$$l^* = \left(\frac{1-\alpha\gamma}{1-\lambda}\right)\left(\frac{\mu}{\theta}+1\right) - \left(\frac{p+\lambda(1-\tau)}{(1-\tau)(1-\lambda)}\right), \tag{5.3}$$

where

$$\mu = \frac{(1-\tau)\alpha A - \rho}{\sigma}.$$

Note that the economic growth rate is constant when $\beta = 0$, i.e.$\mu(l) = \mu$. Note also that economic growth can only increase the long run employment rate when there is wage inertia, i.e. when θ does not diverge to infinite.

In contrast, two BGP equilibrium may arise when $\beta > 0$. As explained in the definition, these BGP must satisfy the following constraints: $l \in (0,1)$, $g(l) \in (0,1)$ and $s(l) - g(l) \in (0,1)$. First, note that $\dot{s}(t) = 0$ implies that

$$s(l) - g(l) = \frac{\mu+n}{A}.$$

Thus, $s(l) - g(l) \in (0,1)$ when $\mu + n < A$. Next, note that if $g(l) > 0$ then $s(l) > 0$ and if $s(l) < 1$ then $g < 1$. Thus, it is enough to impose that $g(l) > 0$ and $s(l) < 1$. From (3.12) we obtain $g(l) \in (0,1)$ if and only if $l \in \left(\underline{l}, \bar{l}\right)$ where

$$\underline{l} = \frac{(\lambda(1-\tau)+p)(1-\alpha)}{(\tau+\lambda(1-\tau)(1-\alpha))},$$

and

$$\bar{l} = \frac{1+(\lambda(1-\tau)+p)(1-\alpha)}{\tau+\lambda(1-\tau)(1-\alpha)}.$$

The following proposition characterizes the BGP of this economy.

Proposition 5.2. Assume that $\mu + n < A$.

a). If $\beta = 0$, then l^* is the unique BGP when $l^* \in \left(\max\{0, \underline{l}\}, \min\left\{1, \bar{l}\right\}\right)$. There is no BGP otherwise.

b). If $\beta > 0$, then there is a unique BGP when $Q(0)\,Q(1) < 0$, and $Q(\underline{l})\,Q\left(\bar{l}\right) < 0$. There are two BGP when $min\left\{Q(0), Q(1), Q(\underline{l}), Q\left(\bar{l}\right)\right\} > 0$, $Q(l_{\mathsf{min}}) < 0$ and $l_{\mathsf{min}} \in \left(\underline{l}, \bar{l}\right)$, where l_{min} is such that $Q'(l_{\mathsf{min}}) = 0$. There is no BGP otherwise.

Proof. The proof follows from the definition of the function $Q(l)$ and the constraints on the savings rate and government spending as a fraction of GDP.

This proposition shows that two BGP may arise when there are positive externalities. As the existence of multiple BGP is an obvious requirement for hysteresis, we conclude that in this model hysteresis can only arise when the production function exhibits positive externalities that cause a strong complementarity that makes agents' expectations be satisfied in equilibrium. To see how expectations are self-fulfilling, assume that agents' coordinate into an expectations of a larger interest rate. Accordingly, they increase savings which implies an acceleration of the accumulation of capital and in the growth of the labor demand. When there is wage inertia, the growth of the labor demand causes the rise in the employment rate, which results into an increase in the equilibrium interest rate when $\beta > 0$. Thus, expectations will be fulfilled in equilibrium. Note that if $\beta = 0$ the interest rate is constant and thus the increases in the employment rate will not increase the interest rate. It follows that in this case there is no complementarity and the equilibrium will be unique.

We have shown that multiple BGP may only arise when the complementarity is sufficiently strong which requires large aggregate increasing returns to scale, i.e. β must be positive and large. Actually, numerical examples show that multiple BGP may only arise for large value of the returns to scale, that are much larger than the values obtained in the empirical literature. Basu and Fernald (1997) provide evidence of returns to scale that are no larger than 1.03, whereas in this model two BGP can only arise for values of β larger than 0.2 which implies that two BGP may only arise when the returns to scale are larger than 1.2.

We conclude that, under plausible parameter values, the equilibrium of this growth model with an exogenous tax rate has a unique BGP and, hence, the equilibrium cannot explain the patterns of the employment rate in the European economies. In what follows we assume that $\beta = 0$ so that the production function exhibits aggregate constant returns to scale. Along this unique BGP, the employment rate is l^*, the growth rate is μ and the savings rate is $\frac{\mu+n}{A}$.

Proposition 5.3. Assume that $\beta = 0$. Then,

	A	ρ	σ	θ	λ	γ	τ	p	n
l^*	+	-	-	-	-	-	-	-	0
μ	+	-	-	0	0	0	-	0	0
$s(l^*) - g$	?	-	-	0	0	0	-	0	+
$g(l^*)$	+	-	-	-	-	-	+	-	0

Proof. The proof follows from the BGP value of the variables.

As in any AK growth model, the long run growth rate increases with TFP, the share of capital income in national income, α, and the intertemporal elasticity of substitution, $\frac{1}{\sigma}$, and it decreases with the subjective discount rate. When there is wage inertia, the employment rate positively depends on the economic growth rate, which explains the effects of these parameters on the employment rate. Moreover, if there is positive growth, the employment rate increases with the wage rigidity, which is negatively related with the parameter θ. As it is usual, the employment rate decreases with the replacement ratio, λ, with the weight of unions to the wage gap, γ, and with the direct tax rates, τ. The other tax rates that reduce the average workers' income decrease the reservation wage and, hence, increase the employment rate. The effects on the savings rate follow from the effects on the growth rate with two exceptions. First, the positive effect of the population growth rate, n, which is due to the need of increasing the accumulation of capital in order to maintain the per capita stock of capital. And, second, the ambiguous effect of TFP on the savings rate which is explained by the coexistence of a wealth and a substitution effect.[22] Finally, government spending increases with the employment rate. This positive relation explains most of the effects of the parameters on government spending.

As we have shown in Section 2, the direct tax rate has been acyclical in the US economy during the last forty years. We interpret this as evidence that in the US the direct tax rate is exogenous. In Table 3 we use the model with exogenous taxes to simulate the main variables and parameters of the US economy. We use this simulation as the benchmark economy with exogenous tax rates. Also in Table 3 we quantify the effect of some parameter increases in the main variables of this economy.

[Insert Table 3]

[22] The substitution effect dominates the wealth effect when $n < \frac{\rho}{\sigma}$, which seems the plausible assumption.

Next, we proceed to discuss the stability properties of the BGP equilibrium. Remember that $s(t)$ is a control variable and $l(t)$ is a state variable. It follows that the steady state exhibits saddle path stability when the determinant of the Jacobian matrix is negative. The results on stability are summarized in the following proposition.

Proposition 5.4. Assume that $\beta = 0$. The unique BGP is saddle path stable and the path of the dynamic equilibrium is locally unique.

Proof. See the appendix.

Figure 10 displays the phase diagram of this economy.[23] Note that the savings rate are constant along the equilibrium path as in any Ak model. In contrast, employment exhibits a transition to the BGP due to the wage inertia.

[Insert Figure 10]

In Figure 10 it is shown that there is no relation between the savings rate and the employment rate along the equilibrium path. This occurs because employment does not modify the interest rate, which is constant, and hence does not agents' decisions on savings.

In the following section, government spending as a fraction of GDP will be constant. We will show that the equilibrium displays a positive correlation between the savings rate and the employment rate. This correlation will show up the complementarity between employment and capital due to the endogenous tax rates.

6. The equilibrium path when tax rates are endogenous

In this section we characterize the equilibrium path of an economy with endogenous tax rates. We assume that government spending as a fraction of GDP is constant, i.e. $g(t) = g$. From (3.11), it follows that $\tau(t) = \tau(l(t))$ and thus

$$\dot{\tau}(t) = \tau'(l(t))\,\dot{l}(t).$$

We use the previous equation to rewrite (4.4) as

$$\dot{l}(t) = \widetilde{l}(s(t), l(t), \tau(t)) = l(t)\left(\frac{(s(t) - g(t))A - n - \xi(l(t), \tau(t))}{1 + \frac{\tau'(l(t))l(t)}{1-\tau(t)}}\right), \quad (6.1)$$

[23] See the appendix for a discussion on the construction of this phase diagram.

and (4.3) simplify as follows

$$\dot{s}(t) = \widetilde{s}(s(t), l(t), \tau(t)) = (1 - s(t))\left[(s(t) - g)A - n - \mu(\tau(t))\right]. \qquad (6.2)$$

Using these two equations, we define the equilibrium of this economy as follows.

Definition 6.1. Given $\{l_{0,}k_0; g, p, \lambda\}$ an equilibrium with endogenous tax rates is characterized by $\{l(t), s(t), \tau(t)\}_{t=0}^{\infty}$ such that solve equations (3.11), (6.1), and (6.2), and satisfy (3.9) and the following constraints: $l(t) \in (0,1)$, $s(t) - g \in (0,1)$, and $\tau(t) \in (0,1)$ for all $t \geq 0$.

We first characterize the BGP by means of assuming that $\dot{l}(t) = \dot{s}(t) = 0$. These equations imply that a BGP is characterized by

$$Q(l) = \xi(l, \tau(l)) - \mu(\tau(l)) = 0,$$

where $\tau(l)$ is obtained from the government budget constraint (3.11). As in the previous section, along a BGP the economic growth rate coincides with the rate of growth of the wage. The only difference with the previous section is that the tax rate is endogenous, which makes the function $Q(l)$ be a third order polynomial that may have three real roots within the relevant domain, i.e. the close interval $[0,1]$. These three roots will be the BGP when the associated savings and tax rate belong also to the close interval $[0,1]$. The proposition below provides necessary and sufficient conditions that guarantee that there are three BGP.[24]

Proposition 6.2. Let $l_{\max}$ and $l_{\min}$ be such that $Q'(l_{\max}) = Q'(l_{\min}) = 0$ and

$$l' = \frac{(1-\alpha)\left(p + \lambda \min\left\{\frac{(A-n)\sigma+\rho}{\alpha A}, 1\right\}\right)}{1 - g - \min\left\{\frac{(A-n)\sigma+\rho}{\alpha A}, 1\right\}(1 - \lambda(1-\alpha))}.$$

Then, there are three BGP if and only if $min\{Q(1), Q(l_{\max}), Q(l')\} > 0$, $Q(l_{\min}) < 0$, $l_{\max} > 0$ and $l_{\min} < \min\{1, l'\}$.

Proof. See the appendix.

Multiple BGP arise in this economy because the endogenous tax rates introduce a complementarity between the employment and the savings decisions that

[24] Conditions that guarantee the existence of two or one equilibria are not provided in this proposition since, as we will show, hysteresis will only arise if there are three BGP.

make agents' expectations be self-fulfilling.. We proceed to explain this complementarity. Assume that agents coordinate into an expectations of high after tax interest rate. If agents are willing to substitute intertemporally consumption, the savings rate will be large and thus the growth rate of the capital stock and of the labor demand will also be large. When there is wage inertia, this large growth of the labor demand implies a high value of the employment rate. Thus, due to the expectation, economic activity will be large which will imply large government revenues and low government expenditures. Obviously, the endogenous direct tax rate will be low in this economy and hence the net of taxes equilibrium interest rate will be large. This means that endogenous tax rates makes agents' expectation hold in equilibrium. This complementarity explains the existence of an equilibrium path corresponding to an economic regime of high economic activity. The same arguments apply to explain the existence of another equilibrium path corresponding to a lower economic activity regime.

We denote these three roots by l_1, l_2, and l_3. Without loss of generality, we assume that $l_1 < l_2 < l_3$. In the proof of Proposition 6.1 we have shown that $Q(0) < 0$ which implies that if there are two BGP, they will correspond to l_1 and l_2 and if there is a unique BGP, it will correspond to l_1. This means that the stability properties and the comparative static results will hold in each BGP, regardless of the number of BGP equilibria. Note also that along each BGP the tax rate is obtained from (3.11) as a function of the employment rate, $\tau(l_i)$, for $i = 1, 2, 3$. From (3.10), it follows that the economic growth rate is

$$\mu(l_i) = \frac{(1 - \tau(l_i))\,\alpha A - \rho}{\sigma}$$

and, finally, the savings rate is obtained from $\dot{s}(t) = 0$ as

$$s(l_i) - g = \frac{\mu(l_i) + n}{A}.$$

Because the tax rate is negatively related with the employment rate, the economic growth rate and the savings rate will be positively related. Thus, the following relations between the BGP will be observed: $\mu(l_1) < \mu(l_2) < \mu(l_3)$; $s(l_1) < s(l_2) < s(l_3)$, and $\tau(l_1) > \tau(l_2) > \tau(l_3)$.

In order to show that three BGP may arise, in Table 4 we provide a numerical example based on the Spanish economy. Since the second BGP is unstable, we identify the first BGP with the low regime of the Spanish economy and the third

BGP with the high regime. We observe that the model is able to replicate the value of the employment, savings and tax rate in the two regimes.[25]

[Insert Table 4]

In what follows we obtain first the effects of the parameters along the BGP 1 and 3 and then we characterize the stability of each BGP. We will not consider the effects of the parameters on BGP 2 because this BGP is unstable.

Proposition 6.3. Let $i = 1, 3$ and assume that $l_1 < l_3$. Then,

	A	ρ	σ	θ	γ	λ	p	g	n
l_i	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	0
τ_i	$-$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	0
μ_i	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	0
$s_i - g$	?	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$

.

Proof. See the appendix.

Note that the effects of the parameters in BGP1 and 3 are similar to the effects obtained in the BGP with exogenous tax rates. The intuitions behind these effects are also close.

Proposition 6.4. The BGP 1 and 3 exhibit saddle path stability whereas BGP 2 may be either unstable or locally stable.

Proof. See the Appendix.

Numerical simulations show that the instability of BGP 2 seems to be a robust result. Thus, from now on we assume that this BGP is unstable.

It can be shown that if there is a unique BGP, it will correspond to BGP 1. Hence, the equilibrium path will be unique. When there are two BGP, they will correspond to BGP 1 and 2. In this case, the equilibrium path may not be unique. In particular, it is not unique when there is a stable limit cycle surrounding BGP 2. In this case, either the equilibrium converges to BGP 1 or to the limit cycle. However, in this case, the coordination from one equilibrium path to another one does not explain the shift from the high regime to a low regime. In order to

[25] The model has problems in providing the values of the growth rate. This may be due to the assumption of zero depreciation of the stock of capital or the assumption of CRTS.

explain this shift, the equilibrium path should converge to different BGP. This happens when there are three BGP. In this case, agents can choose an equilibrium path that converges to BGP 1 or another path that converges to BGP 3.

The previous results on stability imply that hysteresis can only be explained in this model when there are three BGP. In the following proposition we provide the necessary conditions for the existence of three BGP which allow us to show up how the savings decisions, fiscal policy and the labor market institutions interact to explain the existence of these three BGP.

Proposition 6.5. Three BGP will not exists when either $\theta \to \infty$, $\lambda = 0$, $\gamma < \overline{\gamma}$, $\sigma \to \infty$ or $g \notin \left(\underline{g}, \overline{g}\right)$.

Proof. See the appendix.

Thus, the existence of three BGP requires that there is wage inertia, which explains the positive effect of economic growth on employment; that the weight of the wage gap in the unions' objective function is sufficiently large, so that there is a large markup; and that the government provides a positive and sufficiently large unemployment benefit. This three elements explain the labor market rigidities that are necessary for the existence of multiple BGP since they prevent the wage to shift with the labor demand. Note also that the intertemporal elasticity of substitution must be sufficiently large since the complementarity requires that the savings rate increases with the interest rate. Finally, government spending must belong to a given interval. In order to explain how government spending affects the number of BGP, we proceed to construct the long run Laffer curve.

Along the BGP, both the exogenous and the endogenous tax rate economies are characterized by the same two equations: the government budget constraint and the equality between the growth rate of the wage and the economic growth rate. In (5.3), we have shown that this equality happens when $l^* = l^* (\tau)$. From the government budget constraint (3.12), we obtain

$$g = g\left(l^*\left(\tau\right), \tau\right),$$

which is a Laffer curve, relating the tax rate with the fraction of GDP devoted to government spending. This Laffer curve is displayed in Figure 11. Note that in the exogenous tax rate economy, given a value of the tax rate we obtain a value of government spending and, thus, a unique BGP. In contrast, in the economy with endogenous tax rates, multiple BGP may exist given a value of government spending. These BGP correspond to a high tax rate and low economic activity regime and to a low tax rate and high economic activity regime. Note the these BGP belong to the same Laffer curve.

[Insert Figure 11]

From Figure 11 it follows that three BGP may only exists when government spending belongs to a given interval. Otherwise, when government spending is too large, then it can only be financed by means of a large tax rate and when government spending is too low, it can only be financed by means of low a tax rate. Thus, there is a unique equilibrium.

When tax rates are endogenous, agents' may coordinate, depending on their expectation on the tax rate, between different equilibrium paths that belong to the same long run Laffer curve. If they expect tax rates to be large (small), the economy will converge into a low (high) regime that makes the equilibrium tax rates be large (small). In this way agents' expectations are self-fulfilling.. In contrast, when tax rates are endogenous, the government selects the equilibrium path by means of setting the tax rate.

In what follows we study the transitional dynamics.. To this end, Figure 12 displays the phase diagram when there are three BGP. Note that given an initial value of the employment rate agents may coordinate, by means of their savings decisions, in either an equilibrium path driving towards the high regime (BGP 3) or in an equilibrium path driving towards the low regime (BGP 1).

[Insert Figure 12]

Note that the policy functions driving towards the BGP 1 and 3 have a positive slope. Thus, if the economy converges into one of these two BGP the equilibrium will exhibit a positive relationship between the employment and the savings rate. The equations of these policy functions are

$$s_i(t) = (l_i(t) - l_i^*)\left(\frac{\lambda_{i,1} - (1 - s_i)\,A}{\frac{(1-s_i)\alpha A\tau'(l_i)}{\sigma}}\right) + s_i^*,\ i = 1, 3.$$

Note also that the slope of the of policy function is positive because we have assumed that $\tau'(l_i) < 0$. Otherwise, the slope would be negative. The intuition behind this positive slope that implies a positive correlation between the employment rate and the savings rate is as follows. An increase in the employment rate implies a reduction in government expenditures and an increase in government revenues. As a consequence, the tax rate decreases with the employment rate. This reduction implies that the net of tax interest rate increases which rises the savings rate. The rise in the savings rate implies a larger accumulation of capital and a larger growth rate. The larger accumulation of capital rises the labor demand and, because of the wage inertia, the employment rate increases.

7. Dynamic effects of a reduction in TFP

In this section we study the effects of a reduction in TFP that we simply identify with a jump downwards in the parameter A. We show that in an economy with exogenous tax rates this reduction does not imply hysteresis, whereas it may cause hysteresis when the tax rate is endogenous.

We first consider the effects of a reduction in the parameter A in the economy with exogenous tax rates. We use the phase diagram in order to show the transition. To this end, we use the phase lines provided in the appendix in order to show how they shift when there is a reduction in A[26]

[Insert Figure 13]

Figure 13 shows the transition. Note that the reduction in the TFP causes a reduction in the employment rate and a reduction in the savings rate. The reduction in the savings rates is explained by the decrease in the interest rate. The reduction in the employment rate occurs because TFP implies a decrease in the labor demand which results into a lower employment rate when there is wage inertia. Actually, if there were no wage inertia the reduction in the labor demand would translate fully into a reduction in the wage and no effect on the employment rate would occur. The transition of the employment rate is explained by the wage inertial and the transition in the savings rate is due to the endogenous tax rates.

[Insert Figure 14]

Figure 14 displays the effects of a 5% reduction in the parameter A in the benchmark economy when we assume that this economy is initially in the BGP.

[26] Note that $\frac{\partial \widetilde{s_1}}{\partial A} = -\left(\frac{n-\frac{\rho}{\sigma}}{A^2}\right) < 0$ since $n > \frac{\rho}{\sigma}$ and $\frac{\partial \widetilde{s_2}}{\partial A} = -\left(\frac{\xi(l)+n}{A^2}\right) < 0$. It is obvious that $\left|\frac{\partial \widetilde{s_1}}{\partial A}\right| < \left|\frac{\partial \widetilde{s_2}}{\partial A}\right|$.

This figure provides the effects on the employment rate, the path of government spending, the growth and the savings rate. Note that the effects on these variables would be transitory when the shock is not permanent. We conclude that if tax rates are exogenous, the equilibrium does not exhibit hysteresis.

We next consider the effects of the same reduction in the parameter A in the economy with endogenous tax rates. Again, we use the phase diagram in order to show the transition.[27]

[Insert Figure 15]

Figure 15 displays the phase diagram when we assume that the equilibrium is initially in the BGP 3. The reduction in TFP opens the possibilities to coordination. In the phase diagram it is displayed to different equilibrium paths. In one of them, agents choose an initial small reduction in the savings rate that makes the economy converge back to the BGP 3. In the other one, agents choose a large initial reduction in the savings rate which places the economy in the policy function converging towards BGP 1.

[Insert Figure 16]

Figure 16 shows numerically these two equilibrium paths by means of using the benchmark economy discusses in Table 4 and assuming that initially the economy is in the BGP corresponding to the high regime. We introduce, as a shock, a 1% reduction in TFP. The figures show that the long run effect of a reduction in TFP depends on the initial jump in the savings rate that depends, in turn, on agents' expectations. When these expectations make agents coordinate into another equilibrium path, temporal shocks will cause permanent effects, i.e. hysteresis.

We explain hysteresis as the result of the coordination between different equilibrium paths that converge to different BGP. In order to explain this coordination failure, note that the reduction in TFP has a direct negative effect in the gross interest rate that makes agents be willing to reduce savings. However, when the tax rate is endogenous, the effect on the after tax interest rate of a reduction in

[27] In order to construct the phase diagram we must consider the effects on the phase lines of a reduction in the parameter A. First, $\frac{\partial \widehat{s_1}}{\partial A} = -\left(\frac{n-\frac{\rho}{\sigma}}{A^2}\right) < 0$ since $n > \frac{\rho}{\sigma}$. Next, $\frac{\partial \widehat{s_2}}{\partial A} = -\left(\frac{\xi(l)+n}{A^2}\right) < 0$. Note that $\left|\frac{\partial \widehat{s_1}}{\partial A}\right| < \left|\frac{\partial \widehat{s_2}}{\partial A}\right|$.

TFP depends on agents expectations. In particular, if agents coordinate into an expectations of low tax rates, they will expect a small reduction of the after tax interest rate. As a consequence, they will choose a small initial reduction in the savings rate that will imply a small reduction in economic growth. This small reduction in economic growth implies a small decrease in the labor demand and a small decline in the employment rate. In this case, the equilibrium converges back to the same BGP which has a lower employment, savings and growth rates. Because employment and capital are large, government expenditures as a fraction of GDP will be low and, thus, the required equilibrium tax rate will be small. Expectation will be fulfilled in equilibrium.

However, agents may also coordinate into an expectations of high tax rates, which means that agents expect a large reduction in the after tax interest rate. These expectation make agents choose a large reduction in the savings rate that causes a large reduction in the growth rate and, hence, the employment rate suffers a large decline. Because of the strong reduction in employment, government expenditures as a fraction of GDP will be large which imply that the equilibrium tax rate will also be large. Again, expectations are fulfilled in equilibrium. When agents coordinate in this equilibrium with high tax rates, a temporal reduction in TFP may have permanent effects.

According to our model, both the US and the European economies suffer a similar shock in TFP. The different patterns of the variables and the differences in the persistence of the shock are explained by differences in fiscal policy. In the US, the direct tax rates keep constant and employment and growth suffer a temporal decline. In Europe, tax rates exhibits a countercyclical behavior and employment and growth suffer a persistent decline.

8. Concluding remarks

In this paper we develop a growth model with a non-competitive labor market. We use the model to show that complementarities between employment and capital yield the possibility of multiple equilibria. We show endogenous tax rates may cause a complementarity that explains these multiple equilibrium paths. In this case, the equilibrium path is the result of a coordination between equilibria with high tax rates and low employment and savings rates and equilibria with low tax rates and high economic activity. These different equilibrium paths belong to the same Laffer curve.

The implied path of the variables of this model may explain the behavior of the variables in both the US and the European economies. On the one hand, in the US, direct tax rates are acyclical which means that they can be interpreted as exogenous. In this case, the model implies that a temporary reduction in TFP causes a temporary reduction in the employment rate, the savings rate and the growth rate as the one shown in the US data. On the other hand, in most European countries the direct tax rates are countercyclical which may imply that these tax rates are, in fact, endogenous. In this case, the model predicts the possibility of hysteresis. That is, a temporary shock in TFP may imply a permanent reduction in the path of employment, growth and savings, depending on agents' expectations. This permanent reduction is explained because agents coordinate into an equilibrium with high tax rate, low employment and savings rate. In other words, agents choose the wrong side of the Laffer curve.

When tax rates are endogenous, agents may coordinate on either side of the Laffer curve. This failure in the coordination causes economic instability and, furthermore, agents may coordinate in an equilibrium path that converges in the wrong side of the Laffer curve (the low regime). In contrast, when the government sets the tax rate, the government selects the equilibrium path. In this way, prevent economic instability and may place the economy in the right side of the Laffer curve. Thus, according to this model, to set the tax rate is a superior fiscal policy. An open question is to study if the introduction of public deficits can move the economy to a path that converges to the right side of the Laffer curve, without decreasing government spending along the transition towards the BGP.

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Appendix

Proof of Proposition 5.3

The BGP exhibits saddle path stability when the determinant of the Jacobian matrix is negative. In order to obtain this determinant, first we compute the elements of the Jacobian matrix

$$J = \begin{pmatrix} \widetilde{l}_l & \widetilde{l}_s \\ \widetilde{s}_l & \widetilde{s}_s \end{pmatrix} = \begin{pmatrix} -g'(l)\, lA - l\xi'(l) & Al \\ -(1-s)\, g'(l)\, A & (1-s)\, A \end{pmatrix}.$$

The determinant of the Jacobian matrix is

$$Det\,(J) = -(1-s)\, Al\xi'(l) = -(1-s)\, Al\theta \left(\frac{1-\lambda}{1-\alpha\gamma}\right) < 0.$$

■

Phase diagram of the economy with exogenous tax rates

Assume that $\beta = 0$. The phase lines are

$$\widetilde{s}_1 = \frac{\mu + n}{A} + g(l),$$

if $\dot{s}(t) = 0$ and

$$\widetilde{s}_2 = \frac{\xi(l) + n}{A} + g(l),$$

if $\dot{l}(t) = 0$. Note that

$$\frac{\partial \widetilde{s}_1}{\partial l} = g'(l) = \left(\frac{1-\alpha}{l^2}\right)(p + \lambda(1-\tau)) > 0,$$

$$\frac{\partial^2 \widetilde{s}_1}{\partial l^2} = g''(l) = -\left(\frac{1-\alpha}{l^3}\right)(p + \lambda(1-\tau)) < 0,$$

$$\widetilde{s}_1(0) = \frac{\mu + n}{A} + g(0) \to -\infty,$$

$$\widetilde{s}_1(1) = \frac{\mu + n}{A} + g(1) = \frac{\mu + n}{A} + \tau - (1-\alpha)\, p > 0,$$

$$\frac{\partial \widetilde{s}_2}{\partial l} = \frac{\xi'(l)}{A} + g'(l) = \left(\frac{\theta}{A}\right)\left(\frac{1-\lambda}{1-\alpha\gamma}\right) + \left(\frac{1-\alpha}{l^2}\right)(p + \lambda(1-\tau)) > 0,$$

$$\frac{\partial^2 \widetilde{s}_2}{\partial^2 l} = -\left(\frac{1-\alpha}{l^3}\right)(p + \lambda(1-\tau)) < 0,$$

$$\widetilde{s}_2(0) = \frac{\xi(0)+n}{A} + g(0) \rightarrow -\infty,$$

and

$$\widetilde{s}_2(1) = \frac{\xi(1)+n}{A} + g(1) \underbrace{>}_{\xi'(l)>0} \frac{\mu+n}{A} + \tau - (1-\alpha)p = \widetilde{s}_1(1).$$

Thus, both $\widetilde{s}_1$ and $\widetilde{s}_2$ are increasing, concave, diverge to minus infinite when $l \rightarrow 0$, and $\widetilde{s}_2(1) > \widetilde{s}_1(1)$. Using these results on the two functions, it follows that the phase diagram is as it is displayed in Figure 10.

In what follows we obtain analytically this policy function relation $s(t)$ with $l(t)$. To this end, we use the following linear solution to the system of two differential equations:

$$\begin{aligned} l(t) &= A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} + l^*, \\ s(t) &= B_1 e^{\lambda_1 t} + B_2 e^{\lambda_2 t} + s^*, \end{aligned}$$

where the eigenvalues are $\lambda_1 < 0$ and $\lambda_2 > 0$. By construction the equation of the saddle path is

$$s(t) = (l(t) - l^*)\left(\frac{B_1}{A_1}\right) + s^*,$$

where $\frac{B_1}{A_1}$ is the eigenvector. The relation between the eigenvectors that corresponds to the slope of the policy function is given by

$$\frac{A_1}{B_1} = \frac{(1-s)A - \lambda_1}{(1-s)g'(l)A} > 0.$$

■

Proof of Proposition 6.1

First, we obtain the function $Q(l)$

$$Q(l) = \left(\frac{l(1-\lambda)}{1-\alpha\gamma} + \frac{\lambda}{1-\alpha\gamma} + \frac{\rho}{\sigma\theta} - 1\right)(1-\tau) + \frac{p}{(1-\alpha\gamma)} - \frac{(1-\tau)^2 \alpha A}{\sigma\theta} \quad \text{(A1)}$$

and combining with (3.11) we obtain

$$
\begin{aligned}
Q(l) &= \left(\frac{l(1-\lambda)+\lambda}{1-\alpha\gamma}+\frac{\rho}{\sigma\theta}-1\right)[(1-g)l-p(1-\alpha)][l+(1-l)\lambda(1-\alpha)] \\
&\quad +\left(\frac{p}{1-\alpha\gamma}\right)[l+(1-l)\lambda(1-\alpha)]^2-\frac{\alpha A}{\sigma\theta}[(1-g)l-p(1-\alpha)]^2
\end{aligned}
$$

First, it can be shown that

$$Q(0)=\frac{p}{\lambda}\left(1+\frac{\mu(0)}{\theta}\right)<0.$$

and

$$
\begin{aligned}
Q(1) &= \left(\frac{1}{1-\alpha\gamma}+\frac{\rho}{\sigma\theta}-1\right)[(1-g)-p(1-\alpha)] \\
&\quad +\frac{p}{1-\alpha\gamma}-\frac{\alpha A}{\sigma\theta}[(1-g)-p(1-\alpha)]^2 .
\end{aligned}
$$

It can also be shown that

$$Q'(l)=q_2 l^2+q_1 l+q_0,$$

where

$$q_2=3(1-(1-\alpha)\lambda)(1-g)\frac{(1-\lambda)}{1-\alpha\gamma}>0,$$

$$
\begin{aligned}
q_1 &= \frac{(1-\lambda)}{1-\alpha\gamma}(1-g)(1-\alpha)\lambda-p(1-\alpha)\frac{(1-\lambda)}{1-\alpha\gamma}(1-(1-\alpha)\lambda)+ \\
&\quad \frac{1-\lambda}{1-\alpha\gamma}(1-g)(1-\alpha)\lambda+(1-(1-\alpha)\lambda)(1-g)\left[\frac{\lambda}{1-\alpha\gamma}-1+\frac{\rho}{\theta\sigma}\right]+ \\
&\quad -p(1-\alpha)\frac{(1-\lambda)}{1-\alpha\gamma}(1-(1-\alpha)\lambda) \\
&\quad +(1-(1-\alpha)\lambda)(1-g)\left[\frac{\lambda}{1-\alpha\gamma}-1+\frac{\rho}{\theta\sigma}\right]+ \\
&\quad \left(\frac{p}{1-\alpha\gamma}\right)2(1-(1-\alpha)\lambda)^2-\left(\frac{\alpha A}{\theta\sigma}\right)2(1-g)^2,
\end{aligned}
$$

$$
\begin{aligned}
q_0 &= -p(1-\alpha)\frac{(1-\lambda)}{1-\alpha\gamma}(1-\alpha)\lambda+\left[\frac{\lambda}{1-\alpha\gamma}-1+\frac{\rho}{\theta\sigma}\right](1-g)(1-\alpha)\lambda+ \\
&\quad -p(1-\alpha)\left[\frac{\lambda}{1-\alpha\gamma}-1+\frac{\rho}{\theta\sigma}\right](1-(1-\alpha)\lambda)+ \\
&\quad \left(\frac{p}{1-\alpha\gamma}\right)2(1-\alpha)\lambda(1-(1-\alpha)\lambda)+\left(\frac{\alpha A}{\theta\sigma}\right)2[p(1-\alpha)](1-g).
\end{aligned}
$$

Thus, $Q'(l)$ is a second order convex polynomial ($q_2 > 0$) with two roots given by

$$
\begin{aligned}
l_{\min} &= \frac{1}{2q_2}\left(-q_1+\sqrt{(q_1^2-4q_2q_0)}\right), \\
l_{\max} &= \frac{1}{2q_2}\left(-q_1-\sqrt{(q_1^2-4q_2q_0)}\right).
\end{aligned}
$$

From the previous arguments, it follows that $Q(l)$ may only have three roots if $Q(1) > 0$, $Q(l_{\min}) < 0$, $Q(l_{\max}) > 0$, and $0 < l_{\max} < l_{\min} < 1$.

We must obtain conditions that guarantee that the solutions to $Q(l)$ satisfy $s-g \in (0,1)$ and $\tau \in (0,1)$. Note that

$$
s-g=\frac{(1-\tau)\alpha A-\rho}{A\sigma}+\frac{n}{A}
$$

which is positive if

$$
(1-\tau)\alpha A > \rho-\sigma n
$$

Since we assume $\rho-\sigma n < 0$, the previous inequality is satisfied when $\tau < 1$. Next, $s-g < 1$ when

$$
1-\tau < \frac{(A-n)\sigma+\rho}{\alpha A}.
$$

Thus, the tax rate and the savings rate belong to $(0,1)$ when

$$
0 < 1-\tau < \min\left\{\frac{(A-n)\sigma+\rho}{\alpha A},1\right\}.
$$

From (3.11), we obtain that

$$
0 < \frac{(1-g)l-p(1-\alpha)}{l+(1-l)\lambda(1-\alpha)} < \min\left\{\frac{(A-n)\sigma+\rho}{\alpha A},1\right\}.
$$

Note that the first inequality is always satisfied, whereas the second inequality is satisfied when

$$l < \frac{(1-\alpha)\left(p+\lambda \min\left\{\frac{(A-n)\sigma+\rho}{\alpha A},1\right\}\right)}{1-g-\min\left\{\frac{(A-n)\sigma+\rho}{\alpha A},1\right\}(1-\lambda(1-\alpha))} = l'.$$

Thus, if $l_3 < l'$ the three solutions of $Q(l)$ will be BGP. From the form of the polynomial $Q(l)$ note that this happens when $Q(l') > 0$ and $l' > l_{\mathsf{min}}$. ■

Proof of Proposition 6.2

We use $Q(l)$ to obtain

$$\frac{\partial Q(l)}{\partial A} = -\left(\frac{\alpha}{\theta\sigma}\right)\left[(1-g)l - p(1-\alpha)\right]^2 < 0,$$

$$\begin{aligned}\frac{\partial Q(l)}{\partial \sigma} &= -\frac{\rho}{\theta\sigma^2}\left[(1-g)l-p(1-\alpha)\right]\left[(1-(1-\alpha)\lambda)l+(1-\alpha)\lambda\right] \\ &\quad +\left(\frac{\alpha A}{\theta\sigma^2}\right)\left[(1-g)l-p(1-\alpha)\right]^2.\end{aligned}$$

From the budget constraint it follows that

$$(1-(1-\alpha)\lambda)l+(1-\alpha)\lambda = \frac{(1-g)l-p(1-\alpha)}{1-\tau},$$

and

$$\frac{\partial Q(l)}{\partial \sigma} = \frac{\left[(1-g)l-p(1-\alpha)\right]^2\left[\alpha(1-\tau)A-\rho\right]}{(1-\tau)\theta\sigma^2} > 0.$$

Following the same procedure we obtain

$$\begin{aligned}\frac{\partial Q(l)}{\partial \theta} &= -\frac{\rho}{\theta^2\sigma}\left[(1-g)l-p(1-\alpha)\right]\left[(1-(1-\alpha)\lambda)l+(1-\alpha)\lambda\right] \\ &\quad +\left(\frac{\alpha A}{\theta^2\sigma}\right)\left[(1-g)l-p(1-\alpha)\right]^2 \\ &= \frac{\left[(1-g)l-p(1-\alpha)\right]^2\left[\alpha(1-\tau)A-\rho\right]}{(1-\tau)\sigma\theta^2} > 0.\end{aligned}$$

Next,

$$\frac{\partial P(l)}{\partial \rho} = \left[\frac{1}{\theta\sigma}\right] \left[(1-g)\,l - p\,(1-\alpha)\right] \left[(1-(1-\alpha)\,\lambda)\,l + (1-\alpha)\,\lambda\right] > 0.$$

$$\begin{aligned} \frac{\partial Q(l)}{\partial p} &= \left[\frac{l\,(1-\lambda)+\lambda}{1-\alpha\gamma} - 1 + \frac{\rho}{\theta\sigma}\right] \left[-(1-\alpha)\right] \left[(1-(1-\alpha)\,\lambda)\,l + (1-\alpha)\,\lambda\right] \\ &+ \left(\frac{1}{1-\alpha\gamma}\right) \left[(1-(1-\alpha)\,\lambda)\,l + (1-\alpha)\,\lambda\right]^2 \\ &+2\left(\frac{\alpha A}{\theta\sigma}\right) \left[(1-g)\,l - s_2\,(1-\alpha)\right](1-\alpha) \\ &= \left[(1-(1-\alpha)\,\lambda)\,l + (1-\alpha)\,\lambda\right] \left[\left(\tfrac{1}{1-\alpha\gamma}\right)\alpha l + (1-\alpha)\left(1+\tfrac{2\alpha(1-\tau)A-\rho}{\theta\sigma}\right)\right] > 0. \end{aligned}$$

$$\begin{aligned} &\frac{\partial Q(l)}{\partial \gamma} \\ &= \alpha\left[\frac{l\,(1-\lambda)+\lambda}{(1-\alpha\gamma)^2}\right] \left[(1-g)\,l - p\,(1-\alpha)\right] \left[(1-(1-\alpha)\,\lambda)\,l + (1-\alpha)\,\lambda\right] \\ &\quad +\alpha\left(\frac{p}{(1-\alpha\gamma)^2}\right) \left[(1-(1-\alpha)\,\lambda)\,l + (1-\alpha)\,\lambda\right]^2 = \\ &= \alpha\frac{\left[(1-(1-\alpha)\,\lambda)\,l + (1-\alpha)\,\lambda\right]^2}{(1-\alpha\gamma)^2} \left[l\,(1-\lambda)(1-\tau) + \lambda\,(1-\tau) + p\right] > 0 \end{aligned}$$

because we assume that $\lambda(1-\tau)+p>0$.

Next, we use (A1) to obtain

$$\frac{\partial Q(l)}{\partial g} = \left[\frac{2\,(1-\tau)\,\alpha A}{\sigma\theta} - \left(\frac{l\,(1-\lambda)}{1-\alpha\gamma} + \frac{\lambda}{1-\alpha\gamma} + \frac{\rho}{\sigma\theta} - 1\right)\right]\frac{\partial \tau}{\partial g}.$$

Using the equation characterizing the BGP, we obtain

$$\begin{aligned} \frac{\partial Q(l)}{\partial g} &= \left[\frac{2\,(1-\tau)\,\alpha A}{\sigma\theta} - \frac{(1-\tau)\,\alpha A}{\sigma\theta} + \frac{p}{(1-\alpha\gamma)(1-\tau)}\right]\frac{\partial \tau}{\partial g} = \\ &= \left[\frac{(1-\tau)\,\alpha A}{\sigma\theta} + \frac{p}{(1-\alpha\gamma)(1-\tau)}\right]\underbrace{\frac{\partial \tau}{\partial g}}_{+} > 0. \end{aligned}$$

Next, we use again (A1) to obtain

$$\begin{aligned}\frac{\partial Q(l)}{\partial\lambda} &= \left(\frac{1-l}{1-\alpha\gamma}\right)(1-\tau)+\left(\frac{(1-\tau)\alpha A}{\sigma\theta}+\frac{p}{(1-\alpha\gamma)(1-\tau)}\right)\left(\frac{\partial\tau}{\partial\lambda}\right)\\ &> \left(\frac{1-l}{1-\alpha\gamma}\right)(1-\tau)+\frac{p}{(1-\alpha\gamma)(1-\tau)}\left(\frac{\partial\tau}{\partial\lambda}\right).\end{aligned}$$

Remember that $\lambda(1-\tau)>-p$. Then,

$$\frac{\partial Q(l)}{\partial\lambda}>\left(\frac{1-l}{1-\alpha\gamma}\right)(1-\tau)-\frac{\lambda}{1-\alpha\gamma}\left(\frac{\partial\tau}{\partial\lambda}\right)>0$$

if

$$(1-l)(1-\tau)>\lambda\left(\frac{\partial\tau}{\partial\lambda}\right),$$

where

$$\frac{\partial\tau}{\partial\lambda}=\frac{(1-l)(1-\alpha)[l(1-g)-p(1-\alpha)]}{(l+(1-l)\lambda(1-\alpha))^2}>0.$$

Thus, $\frac{\partial Q(l)}{\partial\lambda}>0$ if

$$\begin{aligned}(1-l)(1-\tau)&>\lambda\left(\frac{(1-l)(1-\alpha)[l(1-g)-p(1-\alpha)]}{(l+(1-l)\lambda(1-\alpha))^2}\right),\\ (l+(1-l)\lambda(1-\alpha))^2(1-\tau)&>\lambda(1-\alpha)[l(1-g)-p(1-\alpha)],\\ (l+(1-l)\lambda(1-\alpha))((1-g)l-p(1-\alpha))&>\lambda(1-\alpha)[l(1-g)-p(1-\alpha)],\\ 1-\lambda(1-\alpha)&>0.\end{aligned}$$

Remember that $Q'(l_1)>0$, $Q'(l_2)<0$, and $Q'(l_3)>0$. Using the Implicit Function Theorem, the results for the employment rate in Proposition 6.2 follow. Using (3.11), we obtain the results for the tax rate and using (3.10) and (4.1) we obtain the results for the growth and savings rate, respectively. ■

Proof of Proposition 6.3

In order to discuss the stability of each BGP, we obtain the elements of the Jacobian matrix formed by equations (6.2) and (6.1)

$$J=\begin{pmatrix}\frac{\partial\tilde{l}}{\partial l} & \frac{\partial\tilde{l}}{\partial s}\\ \frac{\partial\tilde{s}}{\partial l} & \frac{\partial\tilde{s}}{\partial s}\end{pmatrix}=\begin{pmatrix}-\frac{l\xi'(l,\tau(l))}{1+\frac{\tau'(l(t))l}{1-\tau(l)}} & \frac{lA}{1+\frac{\tau'(l(t))l}{1-\tau(l)}}\\ \frac{(1-s)\alpha A\tau'(l)}{\sigma} & (1-s)A\end{pmatrix}$$

The determinant of the Jacobian matrix is

$$\begin{aligned} Det\left(J\right) &= -\left(\frac{l\xi'\left(l,\tau\left(l\right)\right)}{1+\frac{\tau'(l(t))l}{1-\tau(l)}}\right)\left(1-s\right)A-\left(\frac{lA}{1+\frac{\tau'(l(t))l}{1-\tau(l)}}\right)\frac{\left(1-s\right)\alpha A\tau'\left(l\right)}{\sigma} = \\ &= -\left(\frac{\left(1-s\right)lA}{1+\frac{\tau'(l(t))l}{1-\tau(l)}}\right)\underbrace{\left[\xi'\left(l,\tau\left(l\right)\right)+\frac{\alpha A\tau'\left(l\right)}{\sigma}\right]}_{Q'(l)}. \end{aligned}$$

Note that

$$1+\frac{l\tau'\left(t\right)}{1-\tau\left(t\right)} = \frac{(1-g)l(1-(1-\alpha)\lambda)l-p(1-\alpha)[2(1-(1-\alpha)\lambda)l+(1-\alpha)\lambda]}{[(1-g)l-p(1-\alpha)][(1-(1-\alpha)\lambda)l+(1-\alpha)\lambda]} > 0.$$

It follows that the determinant has a sign that it is opposite to the sign of the slope of the function $Q\left(l\right)$. As we have shown that $Q\left(0\right) < 0$ it must be that $Q'\left(l_1\right) > 0$, $Q'\left(l_2\right) < 0$ and $Q'\left(l_3\right) > 0$. It follows that the determinant is negative at the BGP 1 and 3, and it is positive at BGP 2. In this second BGP, stability depends on the sign of the trace which is given by

$$Tr = -\frac{l\xi'\left(l,\tau\left(l\right)\right)}{1+\frac{\tau'(l(t))l}{1-\tau(l)}}+\left(1-s\right)A.$$

The second BGP is unstable when $Tr\left(l_2,s_2\right) > 0$ and it is locally stable when $Tr\left(l_2,s_2\right) < 0$. ■

Proof of Proposition 6.4

The proof follows directly from $Q\left(l\right)=0$. First, it is straightforward to show that $Q\left(l\right)$ is a second order polynomial when $\sigma\to\infty$, or $\theta\to\infty$. To prove that it is a second order polynomial when $\lambda\to 0$ note that in this case

$$\begin{aligned} Q\left(l\right) &= \left[\frac{l}{1-\alpha\gamma}-1+\frac{\rho}{\theta\sigma}\right]\left[\left(1-g\right)l-p\left(1-\alpha\right)\right]l \\ &+\left(\frac{p}{1-\alpha\gamma}\right)l^2-\left(\frac{\alpha A}{\theta\sigma}\right)\left[\left(1-g\right)l-p\left(1-\alpha\right)\right]^2 \end{aligned}$$

From the budget constraint it follows that

$$\left(1-g\right)l-p\left(1-\alpha\right)=\left(1-\tau\right)l.$$

Then,

$$Q(l) = \left[\frac{l}{1-\alpha\gamma} - 1 + \frac{\rho}{\theta\sigma}\right](1-\tau)\, l^2 + \left(\frac{p}{1-\alpha\gamma}\right) l^2 - \left(\frac{\alpha A}{\theta\sigma}\right) l^2 (1-\tau)^2$$

$$Q(l) = \left[\frac{l}{1-\alpha\gamma} - 1 + \frac{\rho}{\theta\sigma}\right](1-\tau) + \left(\frac{p}{1-\alpha\gamma}\right) - \left(\frac{\alpha A}{\theta\sigma}\right)(1-\tau)^2 .$$

$$Q(l) = \left[\frac{l}{1-\alpha\gamma} - 1 + \frac{\rho}{\theta\sigma}\right]\left(1-g-\frac{p(1-\alpha)}{l}\right) + \left(\frac{p}{1-\alpha\gamma}\right) - \left(\frac{\alpha A}{\theta\sigma}\right)\left(1-g-\frac{p(1-\alpha)}{l}\right)^2 .$$

Note there are two roots at most.

Next, we proceed to prove that three BGP will not exists when $\gamma < \overline{\gamma}$. To this end, note that three BGP will not exists when $Q(1) < 0$. This happens when

$$\left(\frac{1}{1-\alpha\gamma}\right)[(1-g)+p\alpha] < \frac{\alpha A}{\sigma\theta}\left[\underbrace{(1-g)-p(1-\alpha)}_{1-\tau(1)}\right]^2 - \left(\frac{\rho}{\sigma\theta}-1\right)[(1-g)-p(1-\alpha)]$$

$$\left(\frac{1}{1-\alpha\gamma}\right)[(1-g)+p\alpha] < (1-\tau(1))\left(\underbrace{\frac{(1-\tau(1))\,\alpha A-\rho}{\sigma\theta}}_{\frac{\mu(1)}{\theta}} + 1\right)$$

$$\gamma < \frac{1}{\alpha} - \frac{(1-g)+p\alpha}{\alpha(1-\tau(1))\left(\frac{\mu(1)}{\theta}+1\right)} = \overline{\gamma}.$$

It may also happen when $g \notin \left(\underline{g}, \overline{g}\right)$. To see this, note that

$$Q(1) = -\frac{\alpha A}{\sigma\theta}(1-\tau(1))^2\left(\frac{1}{1-\alpha\gamma}+\frac{\rho}{\sigma\theta}-1\right)(1-\tau(1)) + \frac{p}{1-\alpha\gamma} < 0$$

if $\tau_1 \notin (\underline{\tau_1}, \overline{\tau_1})$ where

$$1-\underline{\tau}, \underline{\tau} = \frac{\left(\frac{l(1-\lambda)}{1-\alpha\gamma}+\frac{\lambda}{1-\alpha\gamma}+\frac{\rho}{\sigma\theta}-1\right) \pm \sqrt{\left(\frac{l(1-\lambda)+\lambda}{1-\alpha\gamma}+\frac{\rho}{\sigma\theta}-1\right)^2 + \frac{4p\alpha A}{(1-\alpha\gamma)\sigma\theta}}}{2\frac{\alpha A}{\sigma\theta}},$$

and

$$\underline{g},\overline{g}=1-p\left(1-\alpha\right)-\frac{\left(\frac{l(1-\lambda)}{1-\alpha\gamma}+\frac{\lambda}{1-\alpha\gamma}+\frac{\rho}{\sigma\theta}-1\right)\pm\sqrt{\left(\frac{l(1-\lambda)+\lambda}{1-\alpha\gamma}+\frac{\rho}{\sigma\theta}-1\right)^{2}+\frac{4p\alpha A}{(1-\alpha\gamma)\sigma\theta}}}{2\frac{\alpha A}{\sigma\theta}}.$$

If $g\notin\left(\underline{g},\overline{g}\right)$ then $Q\left(1\right)<0$ and three BGP will not exists. ■

Phase diagram of the equilibrium with endogenous tax rates

The phase lines are if $\dot{s}\left(t\right)=0$

$$\widehat{s}_{1}-g=\frac{\mu\left(l\right)+n}{A}=\frac{\alpha\left(1-\tau\left(l\right)\right)}{\sigma}-\frac{\rho}{A\sigma}+\frac{n}{A}.$$

And if $\dot{l}\left(t\right)=0$

$$\widehat{s}_{2}-g=\frac{\xi\left(l\right)+n}{A}=\frac{\theta}{A}\left[\frac{l\left(1-\lambda\right)+\lambda}{1-\alpha\gamma}+\frac{p}{\left(1-\tau\right)\left(1-\alpha\gamma\right)}-1\right]+\frac{n}{A}.$$

Note that

$$\frac{\partial\widehat{s}_{1}}{\partial l}=-\frac{\alpha\tau'\left(l\right)}{\sigma}>0$$

as we assume that $\tau'\left(l\right)<0$. Moreover,

$$\frac{\partial^{2}\widehat{s}_{1}}{\partial^{2}l}=-\frac{\alpha\tau''\left(l\right)}{\sigma},$$

where

$$\tau''\left(l\right)=\left(1-\alpha\right)\left(1-\left(1-\alpha\right)\lambda\right)\left(\frac{\left(1-g\right)\lambda+p\left(1-\left(1-\alpha\right)\lambda\right)}{\left[\left(1-\left(1-\alpha\right)\lambda\right)l+\left(1-\alpha\right)\lambda\right]^{3}}\right)>0$$

because $\left(1-g\right)\lambda+p\left(1-\left(1-\alpha\right)\lambda\right)>0$ since we impose that $\tau'\left(l\right)<0$. Thus, $\frac{\partial^{2}\widehat{s_{1}}}{\partial^{2}l}<0$ and $\widehat{s}_{1}$ is increasing and concave. Next,

$$\widehat{s}_{1}\left(0\right)-g=\frac{\mu\left(0\right)+n}{A}=-\frac{\alpha p}{\sigma\lambda}-\frac{\rho}{A\sigma}+\frac{n}{A}>0.$$

$$\widehat{s}_{1}\left(1\right)-g=\frac{\mu\left(1\right)+n}{A}=-\frac{\alpha}{\sigma}\left(\left(1-g\right)-p\left(1-\alpha\right)\right)-\frac{\rho}{A\sigma}+\frac{n}{A}>0.$$

We proceed to characterize the second phase line

$$\frac{\partial \widehat{s}_2}{\partial l} = \frac{\xi'(l)}{A} = \frac{\theta}{A(1-\alpha\gamma)}\left[1-\lambda+\frac{\tau'(l)\,p}{(1-\tau)^2}\right] > 0,$$

$$\frac{\partial_2^2 \widehat{s}}{\partial^2 l} = \frac{\xi''(l)}{A} = \underbrace{\left(\frac{\theta p}{A(1-\alpha\gamma)}\right)}_{-}\underbrace{\left[\frac{\tau''(l)}{(1-\tau)^2}+\frac{2(\tau'(l))^2}{(1-\tau)^3}\right]}_{+} < 0,$$

$$\widehat{s}_2(0) - g = \frac{\xi(0)+n}{A} = \frac{n-\theta}{A},$$

$$\widehat{s}_2(1) - g = \frac{\xi(1)+n}{A} = \frac{\theta\left[\frac{1}{1-\alpha\gamma}+\frac{p}{((1-g)-p(1-\alpha))(1-\alpha\gamma)}-1\right]+n}{A},$$

Note that $\widehat{s}_2(0) - g < \widehat{s}_1(0) - g$ because

$$\widehat{s}_2(0) < \widehat{s}_1(0)$$

$$\frac{n-\theta}{A} < -\frac{\alpha p}{\sigma\lambda} - \frac{\rho}{A\sigma} + \frac{n}{A}$$

$$\theta > \frac{\alpha A p}{\sigma\lambda} + \frac{\rho}{\sigma} > -\frac{\alpha A(1-\tau)-\rho}{\sigma} = -\mu$$

$$\theta + \mu > 0.$$

Thus, $s_1(l)$ and $s_2(l)$ are increasing and concave and $\widehat{s}_2(0) < \widehat{s}_1(0)$. Using these two functions, we can construct the phase diagram that is displayed in Figure 12.

In what follows we obtain analytically the equations of the saddle path. To this end, we use the linear solution to the differential equations that is given by

$$\begin{aligned} l_i(t) &= A_{i,1}e^{\lambda_{i,1}t} + A_{i,2}e^{\lambda_{i,2}t} + l_i^*, \\ s_i(t) &= B_{i,1}e^{\lambda_{i,1}t} + B_{i,2}e^{\lambda_{i,2}t} + s_i^*, \end{aligned}$$

where $i = 1, 3$ and $\lambda_{i,1} < 0$ and $\lambda_{i,2} > 0$ are the eigenvalues. By construction the saddle path is

$$\begin{aligned} l_i(t) &= A_{i,1}e^{\lambda_{i,1}t} + l_i^*, \\ s_i(t) &= B_{i,1}e^{\lambda_{i,1}t} + s_i^*, \end{aligned}$$

and

$$s_i(t) = (l_i(t) - l_i^*)\left(\frac{B_{i,1}}{A_{i,1}}\right) + s_i^*,$$

where $\frac{B_{i,1}}{A_{i,1}}$ is the slope of the saddle path. This slope is obtained from the relation between the eigenvectors

$$\frac{A_{i,1}}{B_{i,1}} = \frac{\lambda_{i,1} - (1 - s_i) A}{\frac{(1-s_i)\alpha A \tau'(l_i)}{\sigma}} > 0,$$

which is positive because by assumption $\tau'(l) < 0$. Finally, the two policy functions are

$$s_i(t) = (l_i(t) - l_i^*)\left(\frac{\lambda_{i,1} - (1 - s_i) A}{\frac{(1-s_i)\alpha A \tau'(l_i)}{\sigma}}\right) + s_i^*, \ i = 1, 3.$$

Tables

Table 1: Regimes in the EU capital stock growth rates. 1960-1999.
Means in annual growth rates expressed in percentage points

	Regime 1		Regime 2	
	Means	Years	Means	Years
Austria	7.0	1961-77	4.2	1978-99
Belgium	4.4	1961-78	2.8	1979-99
Denmark	5.2	1961-74	3.2	1975-99
Germany	6.1	1964-71	3.0	1972-99
Finland	2.9	1970-99		
France	4.3	1963-77	2.4	1978-99
Italy	4.8	1961-74	3.0	1975-99
Netherlands	2.3	1969-99		
Spain	11.2	1964-75	4.3	1976-99
Sweden	4.7	1966-76	2.4	1977-99
United Kingdom	1.7	1963-99		

Table 2: Covariation in the direct tax rate and the business cycle.

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Based on the regression: $\tau_t^\delta = \alpha + \beta \Delta y_t$
where τ^d is the ratio $\frac{\text{direct taxes}}{\text{GDP}}$ and Δy is GDP growth

	β	t-stat.	period		β	t-stat.	period
Austria	-0.37	-3.72*	1964-99	Italy	-1.14	-4.84*	1961-99
Belgium	-0.56	-3.15*	1970-99	Netherlands	-0.31	-2.73*	1969-99
Denmark	-1.04	-2.87*	1961-99	Spain	-0.56	-2.73*	1964-99
Germany	-0.10	-1.79**	1961-99	Sweden	-0.44	-2.70*	1961-99
Finland	-0.19	-1.92**	1970-99	UK	-0.02	-0.10***	1963-99
France	-0.45	-3.49*	1964-99	US	-0.01	-0.14***	1961-99

Note: * stands for significant at 1% *** stands for non-significant
** stands for significant between 5% and 10%

Table 3. The Economy with exogenous tax rates

The data of the US economy shows the following values of the parameters: $\alpha = 0.34$, $n = 1.06\%$, $\tau = 13.23\%$, $\lambda = 0.5$. Following the existing literature, we assume that $\rho = 0.045$. The parameter $A = .53$ is such that $s = 8.26\%$. The parameter $\sigma = 3.35$ is such that $\mu = 3.37\%$. The parameter $p = -15.4\%$ is such $g = 22.29\%$ when $l = 94.4\%$. The parameter $\theta = .165$ is such that $l = 94.4\%$ when $\gamma = 1$.[28]

$\alpha = 0.34, \rho = 0.045, \sigma = 3.35, n = 1.06\%, A = .53,$ $\lambda = 0.5, \theta = .165, \tau = 13.23\%, p = -15.37\%, \gamma = 1$		
Benchmark	$\triangle A = \%5$	$\triangle \tau = \%5$
$l = 94.4\%$	$l = 96.27\%$	$l = 94.38\%$
$\mu = 3.37\%$	$\mu = 3.6\%$	$\mu = 3.33\%$
$s = 8.26\%$	$s = 8.28\%$	$s = 8.19\%$
$g = 22.29\%$	$g = 22.67\%$	$g = 22.96\%$
Saddle Path	*Saddle Path*	*Saddle Path*
$\triangle p = \%5$	$\triangle \lambda = \%5$	
$l = 96.17\%$	$l = 94.1\%$	
$\mu = 3.37\%$	$\mu = 3.37\%$	
$s = 8.26\%$	$s = 8.25\%$	
$g = 23.18\%$	$g = 22.96\%$	
Saddle Path	*Saddle Path*	

[28] α is the share of capital income on national income and it is obtained from Garofalo and Yamarik (2002). g and τ are the average ratio during the period 1960-1999 of government spending to GDP and of direct taxes to GDP, respectively. These data is obtained from OECD, Economic Outlook. Using the same source, we obtain n and μ as the average population and GDP growth rates during the same period. Also from the same source, we obtain s as the average fraction of family income not devoted to consumption. Finally, λ is the replacement ratio that we obtain from Bover, Arellano and Bentolila (2002).

Table 4. The economy with endogenous tax rates

The value of A and σ are such that $s_1 = 12.42\%$ and $s_3 = 14.51\%$. p is such that $\tau_3 = 10.12\%$. The value of the parameters λ, γ and θ are such that the steady state values of l are within the range of observed values. The values of α, n, g correspond to the average value of this parameters during the period. The parameter ρ takes values standard in the literature.[29]

$\alpha = 0.47, \rho = 0.045, n = 0.69\%, g = 17.47\%, \sigma = 1.4686,$
$A = .14526, p = -.26969, \lambda = 0.91505,\ \gamma = .95051, \theta = .047274.$

	Spanish Economy		Model		
	Low Regim	High Regim	BGP1	BGP2	BGP3
l	80.84%	96.39%	79.16%	89.82%	95.44%
s	12.42%	14.51%	12.20%	13.52%	14.15%
τ	10.12%	3.59%	10.8%	6.68%	4.69%
μ	2.48%	6.52%	1.08%	1.27%	1.36%
			Saddle Path	*Unstable*	*Saddle Path*

[29] The share of capital income on national income is obtained from Karanassou, Sala and Snower (2002). The values of the other variables and parameters are obtained from the sources discussed in Footnote 27.

Figures

Figure 1. Unemployment in the EU and the US

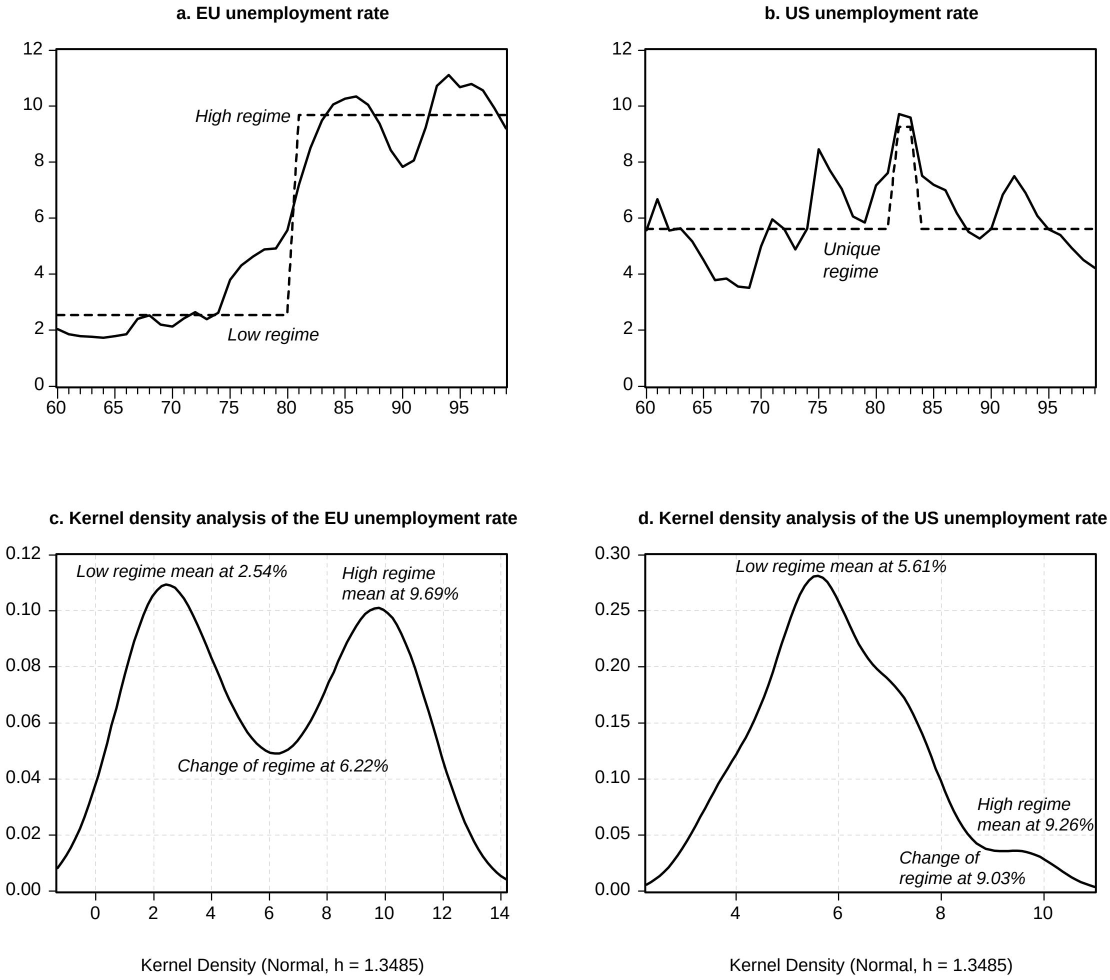


Figure 2. Unemployment rate: actual series and regime means in the EU countries

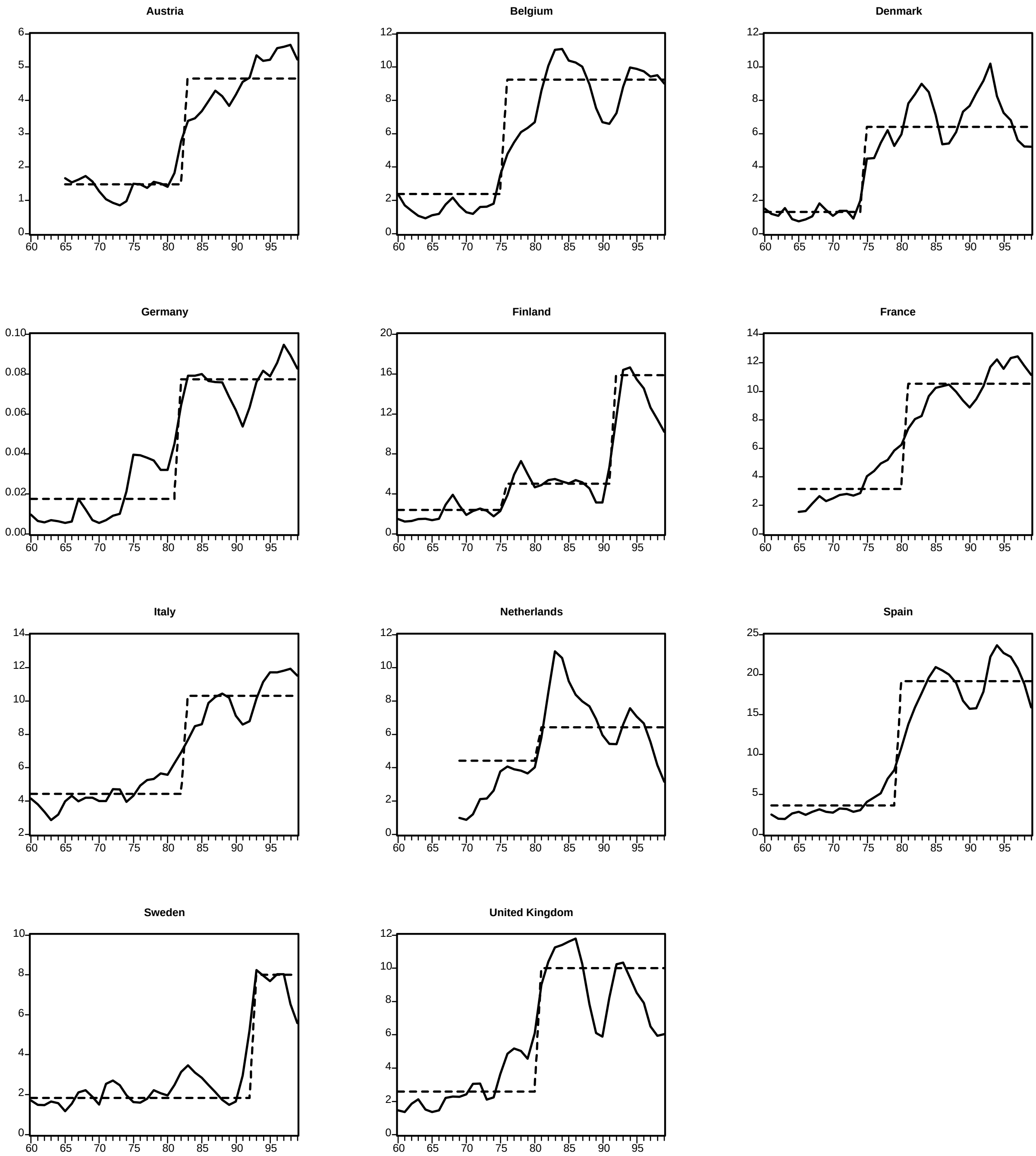


Figure 3. Kernel density analysis of the EU countries unemployment rate

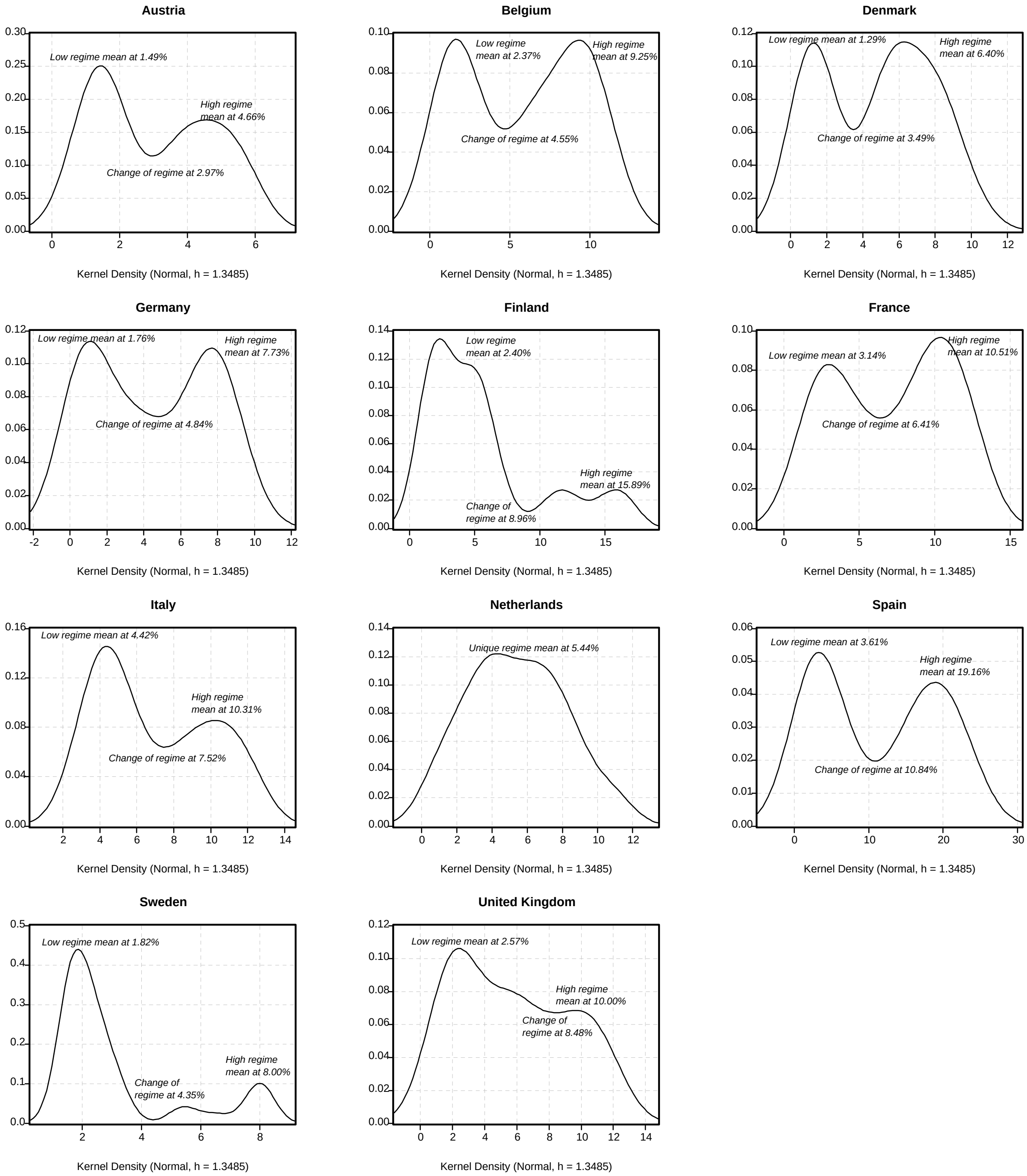


Figure 4. Capital stock growth rate: actual series and regime means in the EU countries

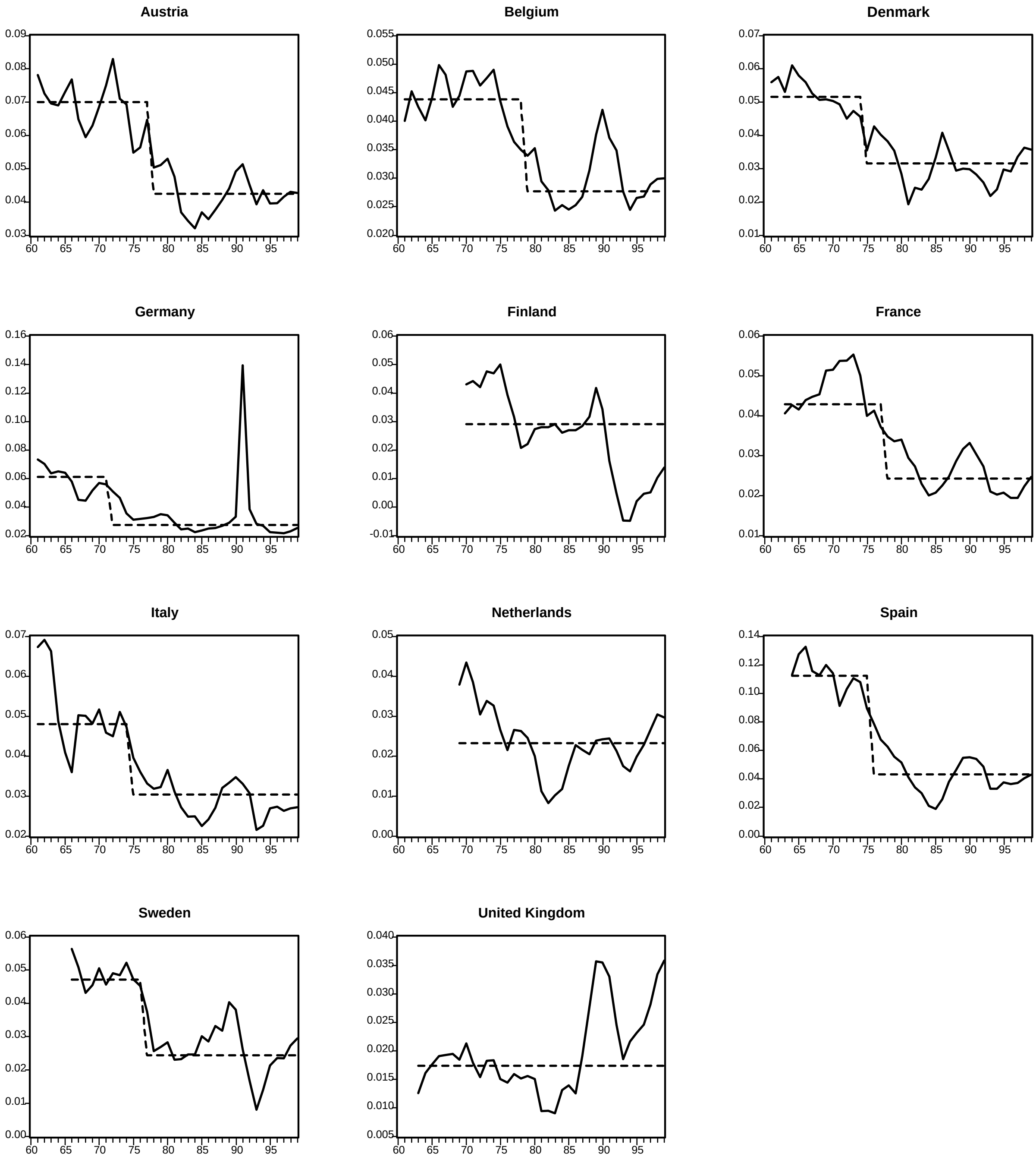


Figure 5. Kernel density analysis of the EU countries capital stock growth

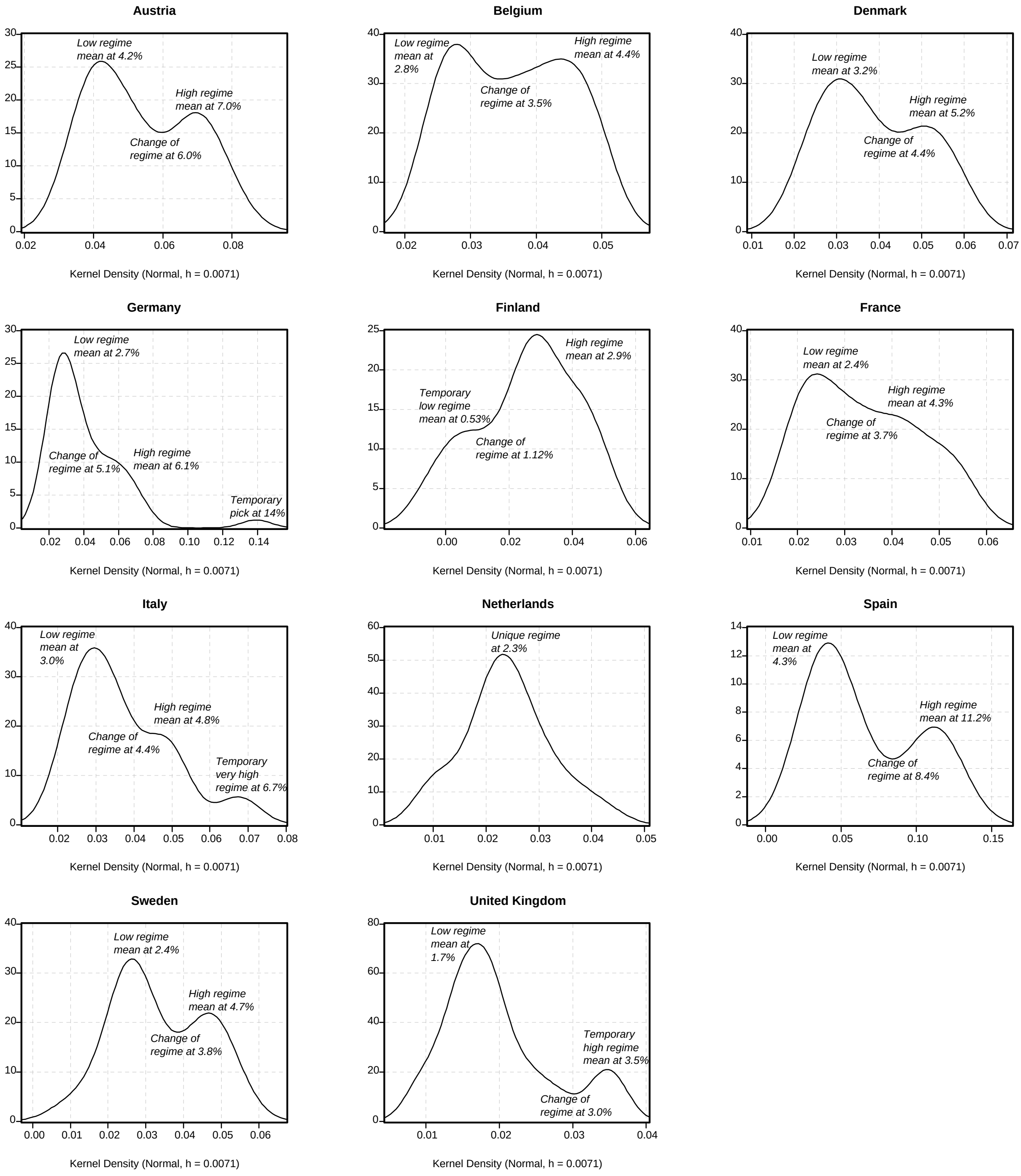


Figure 6. Capital stock growth in the EU and the US

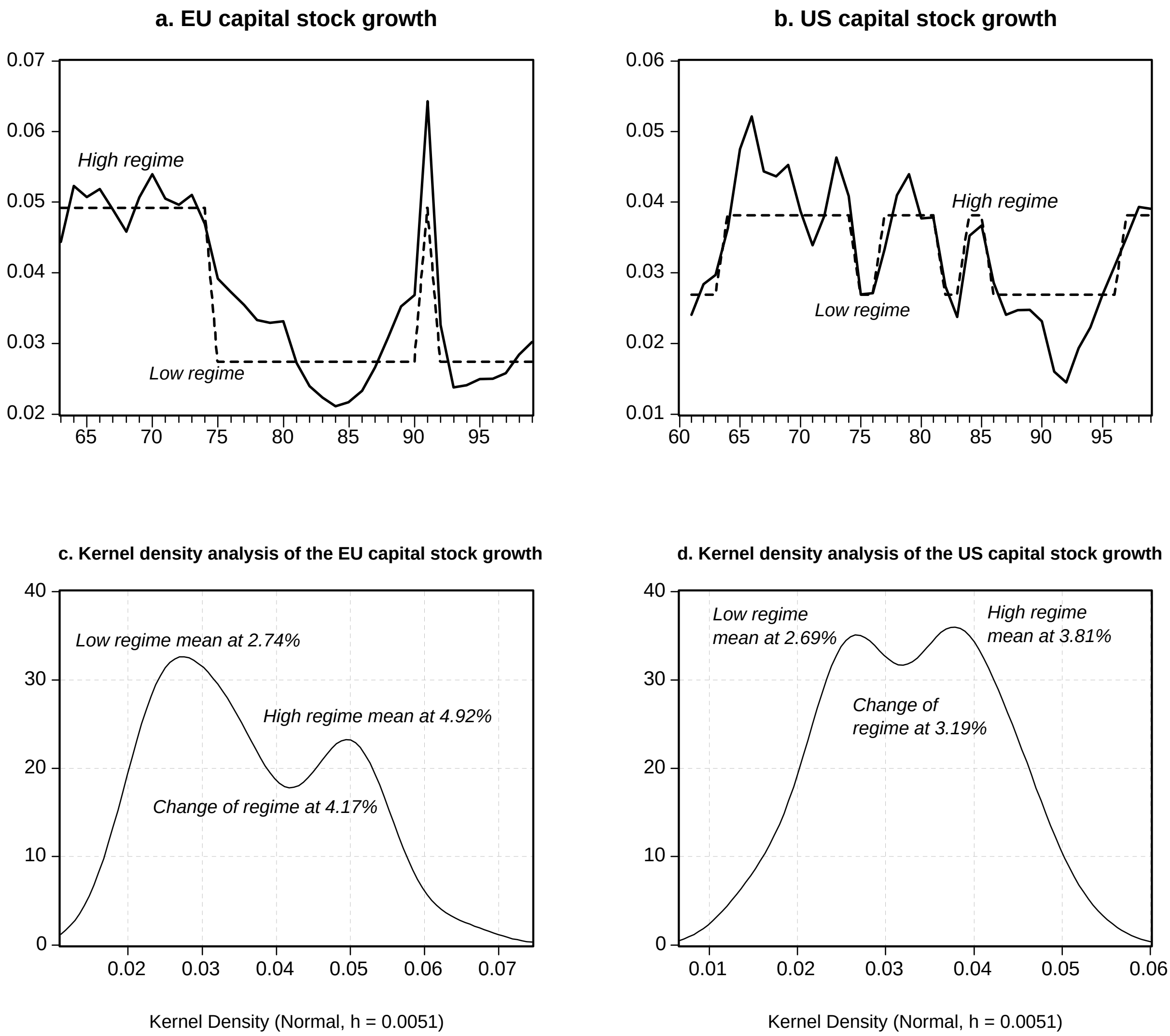


Figure 7. Direct taxes and business cycles in the EU and the US

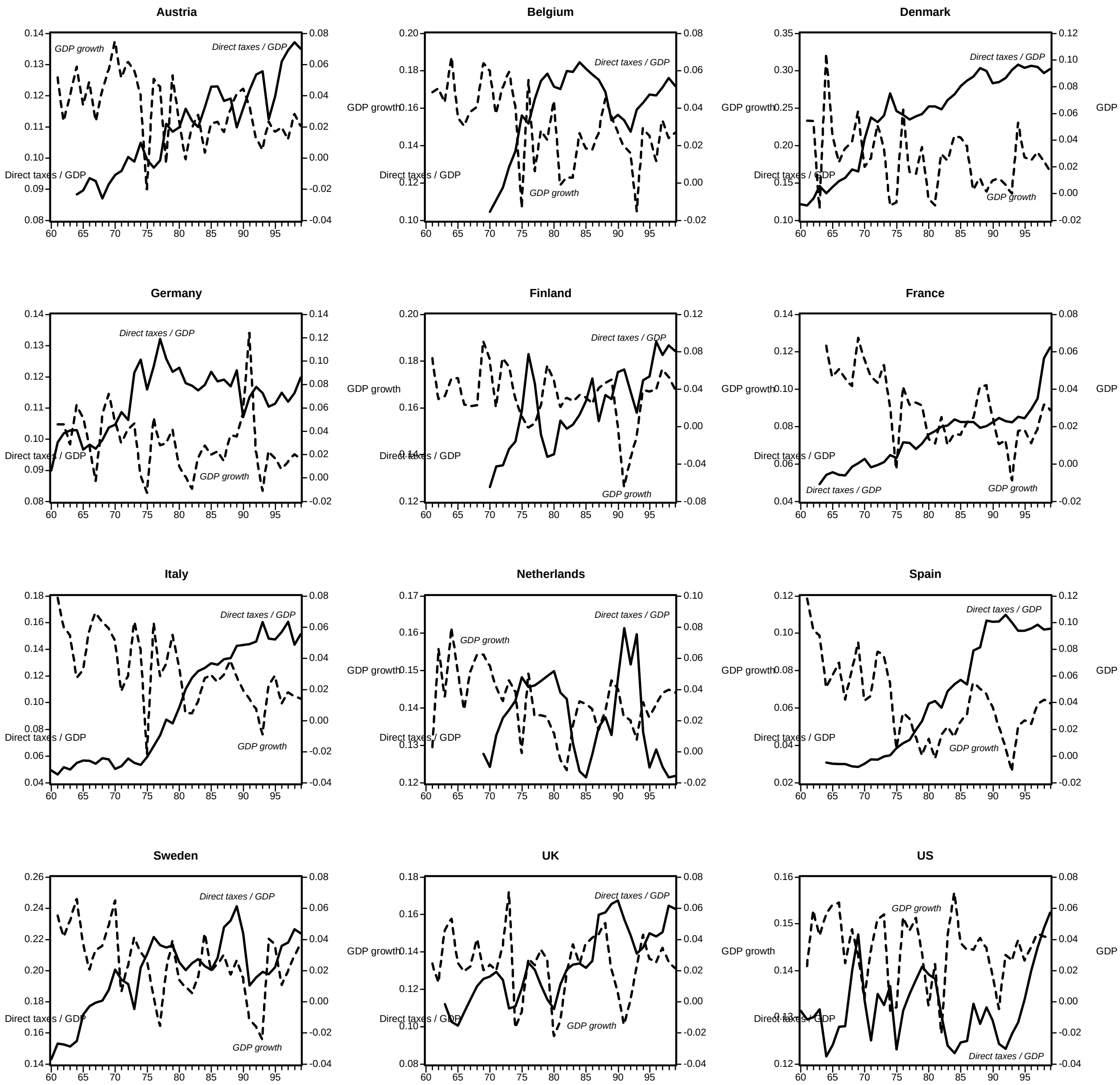


Figure 8. Total Factor Productivity in Spain

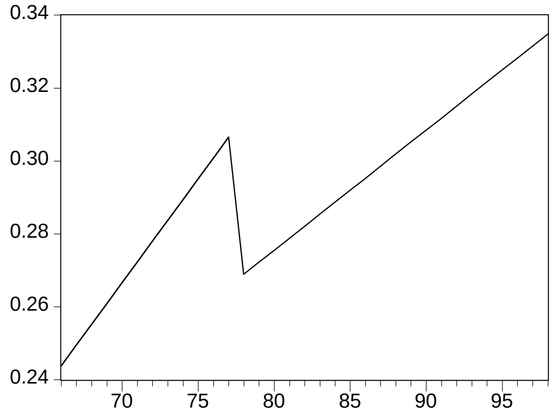
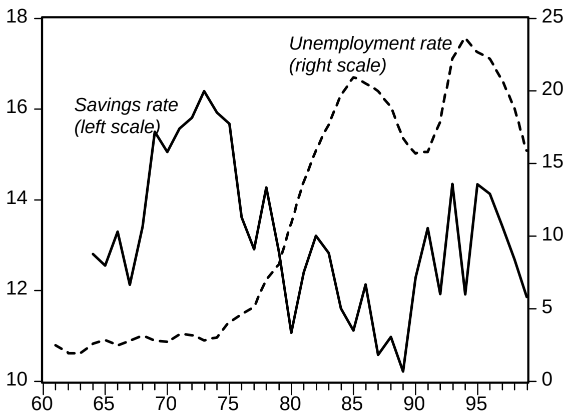


Figure 9. Unemployment, savings and economic growth in Spain

a. Savings and unemployment



b. Economic growth and unemployment

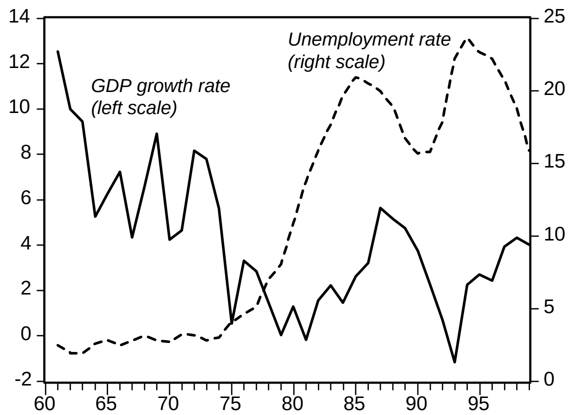


Figure 10. Phase diagram of the economy with exogenous tax rates

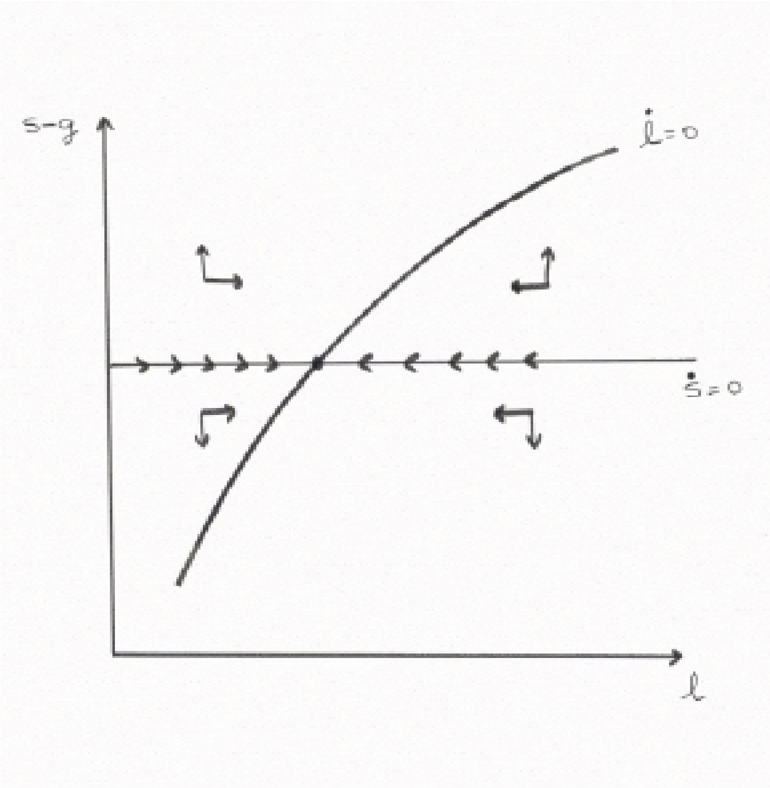


Figure 11. Laffer Curve with unemployment and wage rigidities

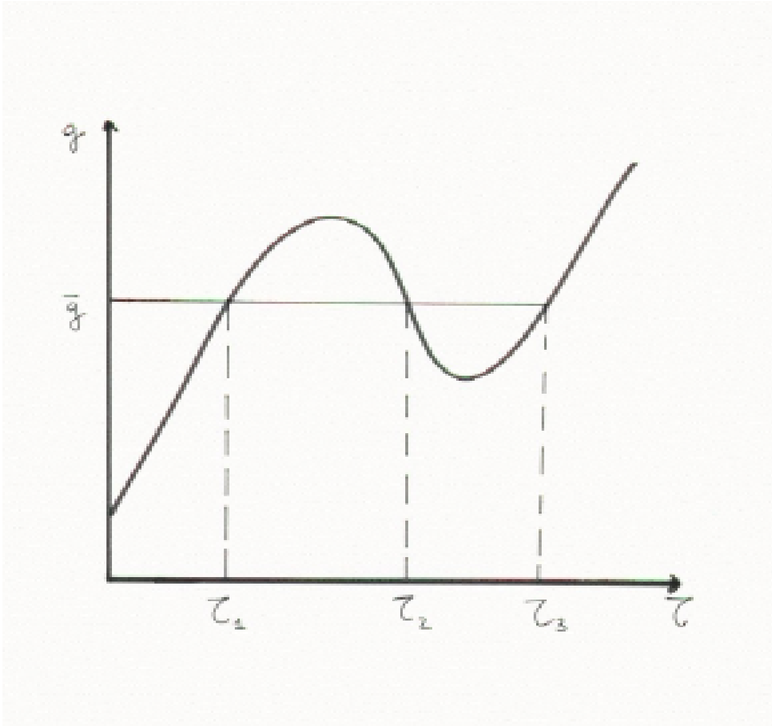


Figure 12. Phase diagram of the economy with endogenous tax rates

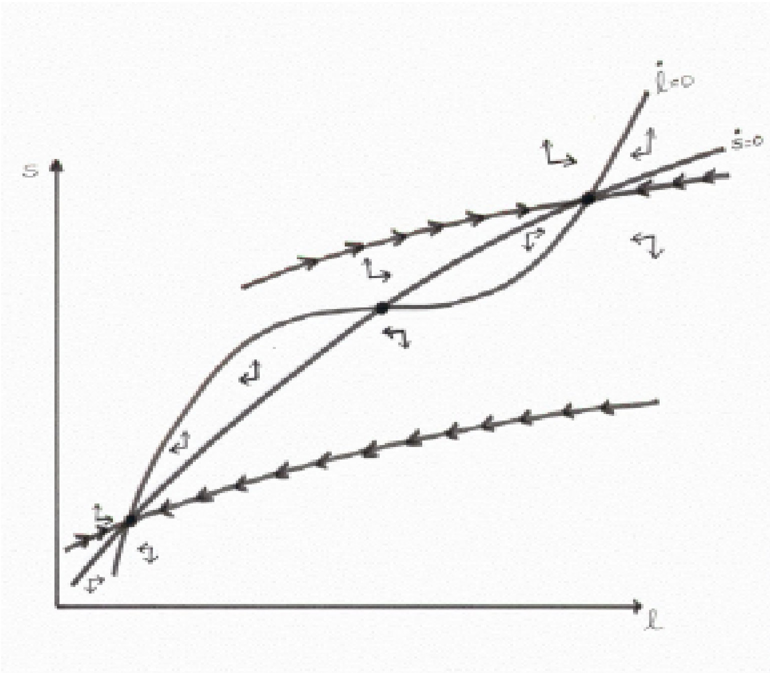


Figure 13. The dynamic effects of a reduction in TFP in an economy with exogenous tax rates

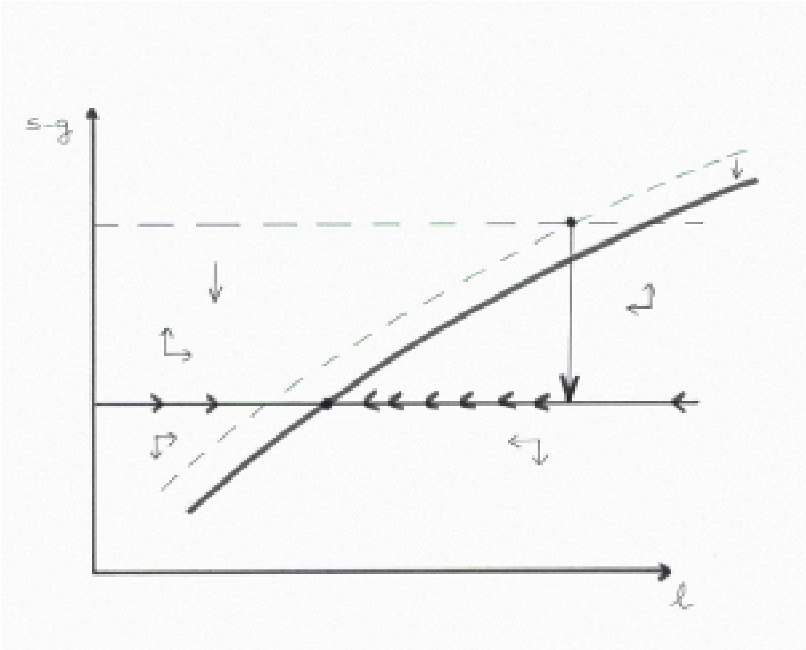
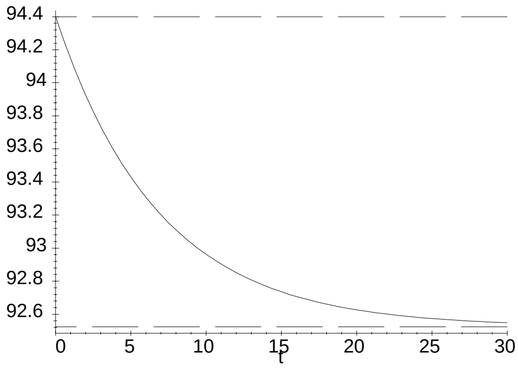
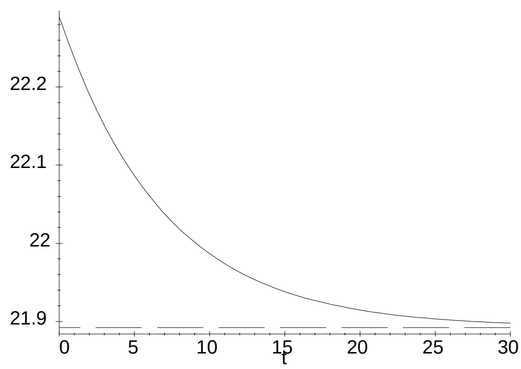


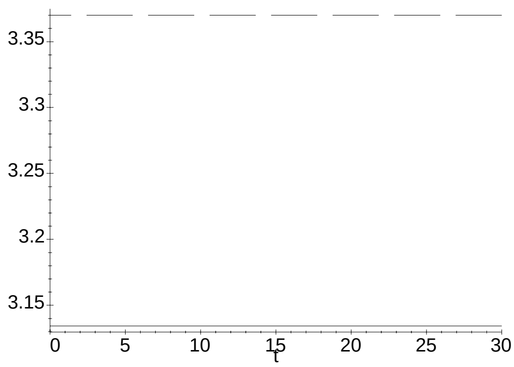
Figure 14. The path of the variables in an economy with exogenous tax rates when $\triangle A = -5\%$



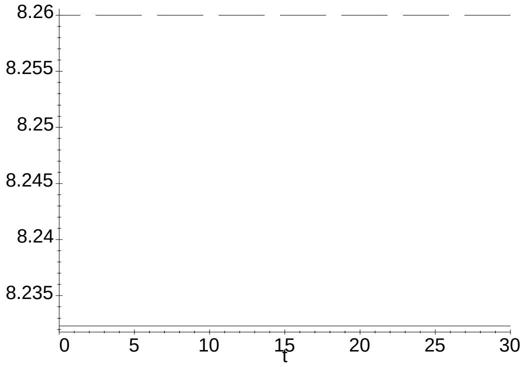
The path of the employment rate



The path of government spending



The path of the growth rate



The path of the savings rate

Figure 15. The dynamic effects of a reduction in TFP in an economy with endogenous tax rates

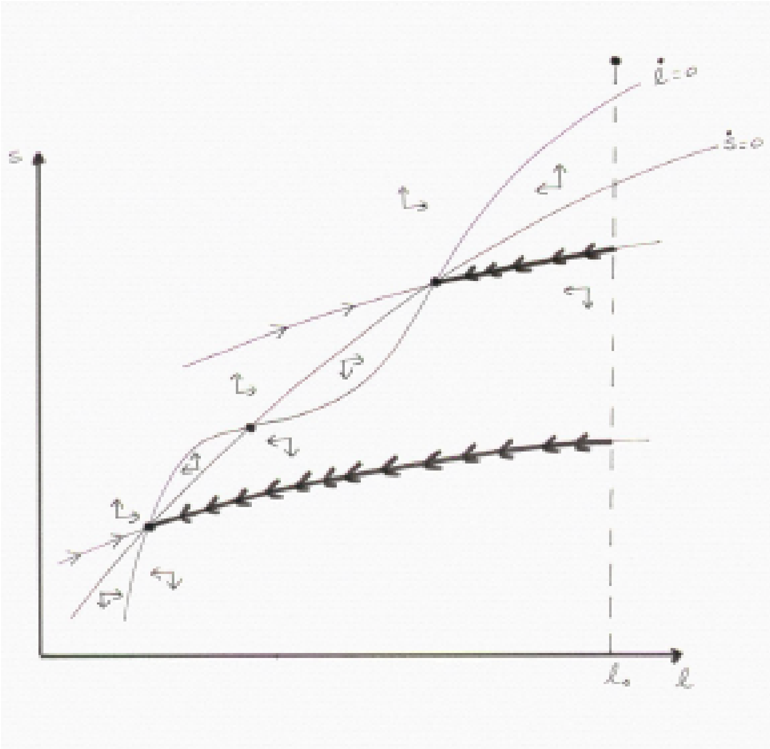
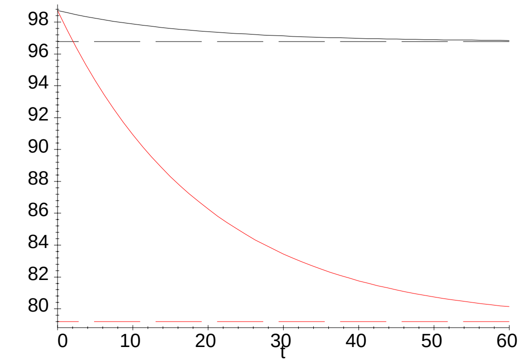
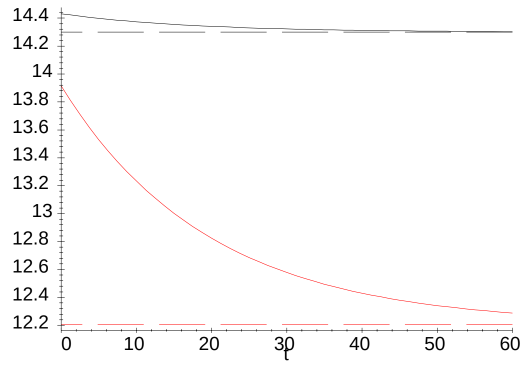


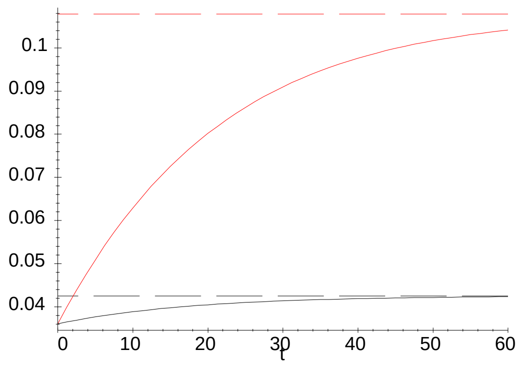
Figure 16. The path of the variables in an economy with endogenous tax rates when $\triangle A = -1\%$



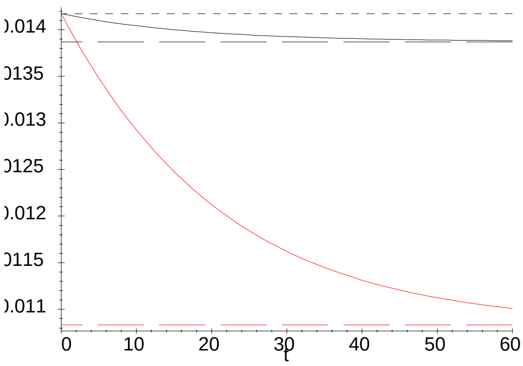
The path of the employment rate



The path of the savings rate



The path of the tax rate



The path of the growth rate