

# Forecasting Value-At-Risk with a Parsimonious Portfolio Spillover GARCH (PS-GARCH) Model\*

Michael McAleer and Bernardo da Veiga

School of Economics and Commerce  
University of Western Australia

Updated: November 2005

**Abstract:** Accurate modelling of volatility (or risk) is important in finance, particularly as it relates to the modelling and forecasting of Value-at-Risk (VaR) thresholds. As financial applications typically deal with a portfolio of assets and risk, there are several multivariate GARCH models which specify the risk of one asset as depending on its own past as well as the past behaviour of other assets. Multivariate effects, whereby the risk of a given asset depends on the previous risk of any other asset, are termed spillover effects. In this paper we analyse the importance of considering spillover effects when forecasting financial volatility. The forecasting performance of the VARMA-GARCH model of Ling and McAleer (2003), which includes spillover effects from all assets, the CCC model of Bollerslev (1990), which includes no spillovers, and a new Portfolio Spillover GARCH (PS-GARCH) model, which accommodates aggregate spillovers parsimoniously and hence avoids the so-called curse of dimensionality, are compared using a VaR example for a portfolio containing four international stock market indices. The empirical results suggest that spillover effects are statistically significant. However, the VaR threshold forecasts are generally found to be insensitive to the inclusion of spillover effects in any of the multivariate models considered.

**Keywords and phrases:** Volatility, Value-at-Risk (VaR) thresholds, multivariate GARCH, conditional correlations, parsimonious portfolio spillovers, forecasting VaR.

**JEL classifications:** C32, C51, C53, F37, G15, G21

\* The authors wish to thank two referees, Dave Allen, Manabu Asai, Felix Chan, Clive Granger, Suhejla Hoti, Marcelo Medeiros, Timo Terasvirta, Alvaro Veiga, seminar participants at the Institute of Economics, Academia Sinica, Taiwan, Chiang Mai University, Fondazione Eni Enrico Mattei - Milan, Griffith University, Hiroshima University, Kyoto University, La Trobe University, Ling Tung University, Macquarie University, Pontifical Catholic University - Rio de Janeiro, Brazil, Queensland University of Technology, Reserve Bank of New Zealand, University of Canterbury, University of New South Wales, University of Padua, University of Queensland, University of Venice "Ca' Foscari", and Yokohama National University, and participants at the International Conference on Simulation and Modeling, Bangkok, Thailand, January 2005, Econometric Study Group Meeting, Christchurch, New Zealand, March 2005, Inaugural Meeting of the Econometric Society of Thailand, June 2005, and Financial Engineering and Risk Management Workshop, Shanghai, July 2005, for helpful comments and suggestions. The first author is grateful for the financial support of the Australian Research Council. The second author acknowledges a University Postgraduate Award and an International Postgraduate Research Scholarship at the University of Western Australia.

## 1. Introduction

Accurate modelling of volatility (or risk) is of paramount importance in finance. As risk is unobservable, several modelling procedures have been developed to measure and forecast risk. The Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986) have subsequently led to a family of autoregressive conditional volatility models. The success of GARCH models can be attributed largely to their ability to capture several stylised facts of financial returns, such as time-varying volatility, persistence and clustering of volatility, and asymmetric reactions to positive and negative shocks of equal magnitude. This has also contributed to the modelling and forecasting of Value-at-Risk (VaR) thresholds.

As financial applications typically deal with a portfolio of assets and risks, there are several multivariate GARCH models which specify the risk of one asset as depending dynamically on its own past risk as well as on the past risk of other assets (see McAleer (2005) for a discussion of a variety of univariate and multivariate conditional and stochastic volatility models). A volatility spillover is defined as the impact of any previous volatility of asset  $i$  on the current volatility of asset  $j$ , for any  $i \neq j$ . A similar definition applies for returns spillovers. da Veiga and McAleer (2005) showed that the multivariate VARMA-GARCH model of Ling and McAleer (2003) and VARMA-Asymmetric GARCH (or VARMA-AGARCH) model of Hoti et al. (2003) provided superior volatility and VaR threshold forecasts than their nested univariate counterparts, namely the GARCH model of Bollerslev (1986) and the GJR model of Glosten, Jagannathan and Runkle (1992), respectively.

Multivariate extensions have great intuitive and empirical appeal as they enable modelling of the relationship between subsets of the portfolio and allow for scenario and sensitivity analyses. Moreover, their structural and asymptotic properties have been well established, especially for multivariate GARCH models (for further details, see Ling and McAleer (2003) and Hoti et al. (2003), which extend the results for a range of univariate GARCH models in Ling and McAleer (2002a, b)). However, the practical usefulness of

this result can be affected by the computational difficulties in estimating the VARMA-GARCH and VARMA-AGARCH models for a large number of assets, as the number of parameters to be estimated can increase dramatically with the number of assets, and hence spillover effects.

Several parsimonious multivariate models have been proposed to deal with the over-parameterization problem. The CCC model of Bollerslev (1990), the Dynamic Conditional Correlation (DCC) model of Engle (2002), and the Varying Conditional Correlation (VCC) model of Tse and Tsui (2002) use a two-step estimation procedure to facilitate estimation. McAleer et al. (2005) extended these conditional correlation models by specifying the shocks to returns as being time dependent, and established the structural and asymptotic properties of the more general model. The Orthogonal GARCH (O-GARCH) model of Alexander (2001) uses principal component analysis to reduce the number of parameters to be estimated.

The need to develop volatility models to estimate accurately large covariance matrices has become especially relevant following the 1995 amendment to the Basel Accord, whereby banks were permitted to use internal models to calculate their VaR thresholds. This amendment was a reaction to widespread criticism that the ‘Standardized’ approach, which banks were originally required to use in calculating their VaR thresholds, led to excessively conservative forecasts. Excessive conservatism has a negative impact on the profitability of banks as higher capital charges are subsequently required. While the amendment was designed to reward institutions with superior risk management systems, a backtesting procedure, whereby the realized returns are compared with the VaR forecasts, was introduced to assess the quality of the internal models. Banks using models that lead to a greater number of violations than can reasonably be expected, given the confidence level, are required to hold higher levels of capital (see the discussion in Section 5 and Table 4 for the penalties imposed under the Basel Accord). If a bank’s VaR forecasts are violated more than 9 times in a financial year, the bank may be required to adopt the ‘Standardized’ approach. The imposition of such a penalty is severe as it has an impact on the profitability of the bank directly through higher capital charges, may

damage the bank's reputation, and may also lead to the imposition of a more stringent external model to forecast the VaR thresholds.

In this paper we investigate the importance of including spillover effects when modelling and forecasting financial volatility. We compare the forecasted conditional variances produced by the VARMA-GARCH model of Ling and McAleer (2003), in which the conditional variance of asset  $i$  is specified to depend dynamically on past squared unconditional shocks and past conditional variances of each asset in the portfolio, with the forecasted conditional variances produced by the CCC model of Bollerslev (1990), where the conditional variance of asset  $i$  is specified to depend only on the squared unconditional shocks and past conditional variances of asset  $i$ . We also develop a new Portfolio Spillover GARCH (PS-GARCH) model, which allows spillover effects to be included in a more parsimonious manner. The parsimonious nature of the PS-GARCH model is of critical importance to practitioners as the model can be estimated for any number of assets, while several other multivariate models can be estimated only for a reasonably small number of assets. This parsimonious nature avoids the so-called curse of dimensionality that can render many multivariate models impractical in empirical applications. This parsimonious model is found to yield volatility and VaR threshold forecasts that are very similar to those of the VARMA-GARCH model. Using the taxonomy proposed in Bauwens et al. (2005), both the PS-GARCH and VARMA-GARCH models are nonlinear multivariate extensions of the standard univariate GARCH model.

The plan of the paper is as follows. Section 2 presents the new PS-GARCH model, discusses alternative multivariate GARCH models with and without spillover effects, and presents a simple two-step estimation method for PS-GARCH. The data for four international stock market indices are discussed in Section 3, the volatility and conditional correlation forecasts produced by alternative multivariate GARCH models are examined in Section 4, the economic significance of the VaR threshold forecasts arising from the alternative multivariate GARCH models is analysed in Section 5, and some concluding remarks are given in Section 6.

## 2. Models and Estimation

This section proposes a parsimonious and computationally convenient PS-GARCH model which captures aggregate portfolio spillover effects, and discusses the structural and statistical properties of the model. The new model is compared with two constant conditional correlation models, one of which models spillover effects from each of the other assets in the portfolio and another which has no spillover effects.

### 2.1 PS-GARCH

Let the vector of returns on  $m$  ( $\geq 2$ ) financial assets be given by

$$Y_t = E(Y_t | F_{t-1}) + \varepsilon_t \quad (1)$$

where the conditional mean of the returns follows a VARMA process:

$$\Phi(L)(Y_t - \mu) = \Psi(L)\varepsilon_t \quad (2)$$

The return on the portfolio consisting of the  $m$  assets is denoted as:

$$Y_{p,t} = E\left(\sum_{i=1}^m x_{i,t} y_{i,t} | F_{t-1}\right) + \varepsilon_{p,t} \quad (3)$$

where  $y_{i,t}$  denotes the return on asset  $i$  at time  $t$  and  $x_{i,t}$  denotes the portfolio weight of asset  $i$  at time  $t$ , such that:

$$\sum_{i=1}^m x_{i,t} = 1 \quad \forall t. \quad (4)$$

The portfolio spillover GARCH (PS-GARCH) model assumes that the returns on the portfolio follow an ARMA process, and that the conditional volatility of the portfolio can be approximated by a GARCH process, as follows:

$$\Phi(L)(Y_{p,t} - \mu_p) = \Psi(L)\varepsilon_{p,t} \quad (5)$$

$$\varepsilon_t = D_t \eta_t \quad (6)$$

$$\varepsilon_{p,t} = h_{p,t}^{1/2} \eta_{p,t} \quad (7)$$

$$h_{p,t} = \omega_p + \sum_{k=1}^r \alpha_{p,k} \varepsilon_{p,t-k}^2 + \sum_{l=1}^s \beta_{p,l} h_{p,t-l} \quad (8)$$

$$H_t = \omega + \sum_{k=1}^r A_k \bar{\varepsilon}_{t-k} + \sum_{k=1}^r C_k I(\eta_{t-k}) \bar{\varepsilon}_{t-k} + \sum_{l=1}^s B_l H_{t-l} + \sum_{k=1}^r G_k \hat{\varepsilon}_{p,t-k}^2 + \sum_{l=1}^s K_l \hat{h}_{p,t-l} \quad (9)$$

where  $H_t = (h_{1t}, \dots, h_{mt})'$ ,  $\omega = (\omega_1, \dots, \omega_m)'$ ,  $D_t = \text{diag}(h_{it}^{1/2})$ ,  $\eta_t = (\eta_{1t}, \dots, \eta_{mt})'$ ,  $\bar{\varepsilon}_t = (\varepsilon_{1t}^2, \dots, \varepsilon_{mt}^2)'$ , and  $\hat{\varepsilon}_{p,t-k}^2$  and  $\hat{h}_{p,t-l}$  are the fitted values from (5) and (8), respectively. The  $m \times m$  matrices  $A_k$ ,  $B_l$  and  $C_k$  are diagonal, with typical elements  $\alpha_{ii}$ ,  $\beta_{ii}$  and  $\gamma_{ii}$ , respectively,  $G_k = (g_1, \dots, g_m)'$ ,  $K_l = (k_1, \dots, k_m)'$ ,  $I(\eta_t) = \text{diag}(I(\eta_{it}))$  is an  $m \times m$  diagonal matrix,  $\Phi(L) = I_m - \Phi_1 L - \dots - \Phi_p L^p$  and  $\Psi(L) = I_m - \Psi_1 L - \dots - \Psi_q L^q$  are polynomials in  $L$ , the lag operator,  $F_t$  is the past information available to time  $t$ ,  $I_m$  is the  $m \times m$  identity matrix, and  $I(\eta_{it})$  is an indicator function, given as:

$$I(\eta_{it}) = \begin{cases} 1, & \varepsilon_{it} \leq 0 \\ 0, & \varepsilon_{it} > 0. \end{cases} \quad (10)$$

The indicator function distinguishes between the effects of positive and negative shocks of equal magnitude on conditional volatility. Portfolio spillovers arise when  $G_k$  and  $K_l$  are not null matrices.

Using (6), the conditional covariance matrix for the PS-GARCH model is given by  $Q_t = D_t \Gamma D_t$ , for which the matrix of conditional correlations is given by  $E(\eta_t \eta_t') = \Gamma$ . The matrix  $\Gamma$  is the constant conditional correlation matrix of the unconditional shocks which is, by definition, equivalent to the constant conditional correlation matrix of the conditional shocks.

## 2.2 VARMA-GARCH

The VARMA-GARCH model of Ling and McAleer (2003), which assumes symmetry in the effects of positive and negative shocks on conditional volatility, is given by:

$$Y_t = E(Y_t | F_{t-1}) + \varepsilon_t \quad (11)$$

$$\Phi(L)(Y_t - \mu) = \Psi(L)\varepsilon_t \quad (12)$$

$$\varepsilon_t = D_t \eta_t \quad (13)$$

$$H_t = \omega + \sum_{k=1}^r A_k \bar{\varepsilon}_{t-k} + \sum_{l=1}^s B_l H_{t-l} \quad (14)$$

where  $H_t = (h_{it}, \dots, h_{mt})'$ ,  $\omega = (\omega_1, \dots, \omega_m)'$ ,  $D_t = \text{diag}(h_{i,t}^{1/2})$ ,  $\eta_t = (\eta_{i,t}, \dots, \eta_{mt})'$ ,  $\bar{\varepsilon}_t = (\varepsilon_{it}^2, \dots, \varepsilon_{mt}^2)'$ ,  $A_k$  and  $B_l$  are  $m \times m$  matrices with typical elements  $\alpha_{ij}$  and  $\beta_{ij}$ , respectively, for  $i, j = 1, \dots, m$ ,  $I(\eta_t) = \text{diag}(I(\eta_{it}))$  is an  $m \times m$  matrix,  $\Phi(L) = I_m - \Phi_1 L - \dots - \Phi_p L^p$  and  $\Psi(L) = I_m - \Psi_1 L - \dots - \Psi_q L^q$  are polynomials in  $L$ , the lag operator, and  $F_t$  is the past information available to time  $t$ . Spillover effects are given in the conditional volatility for each asset in the portfolio, specifically where  $A_k$  and  $B_l$  are not diagonal matrices. Based on equation (13), the VARMA-GARCH model also assumes that the matrix of conditional correlations is given by  $E(\eta_t \eta_t') = \Gamma$ .

An extension of the VARMA-GARCH model is the VARMA-AGARCH model of Hoti et al. (2002), which captures the asymmetric spillover effects from each of the other assets in the portfolio. The VARMA-AGARCH model is also a multivariate extension of the univariate GJR model.

### 2.3 CCC

The VARMA-GARCH, VARMA-AGARCH and PS-GARCH models have several popular constant conditional correlation univariate and multivariate models as special cases. If the model given by equation (14) is restricted so that  $A_k$  and  $B_l$  are diagonal matrices, the VARMA-GARCH model reduces to:

$$h_{it} = \omega_i + \sum_{k=1}^r \alpha_i \varepsilon_{i,t-k} + \sum_{l=1}^s \beta_i h_{i,t-l} \quad (15)$$

which is the constant conditional correlation (CCC) model of Bollerslev (1990). The CCC model also assumes that the matrix of conditional correlations is given by  $E(\eta_t \eta_t') = \Gamma$ . As given in equation (15), the CCC model does not have volatility spillover effects across different financial assets, and hence is intrinsically univariate in nature. Moreover, CCC also does not capture the asymmetric effects of positive and negative shocks on conditional volatility.

### 2.4 Estimation

The parameters in models (11), (14), (15) can be obtained by maximum likelihood estimation (MLE) using a joint normal density, namely:

$$\hat{\theta} = \arg \min_{\theta} \frac{1}{2} \sum_{t=1}^n (\log |Q_t| + \varepsilon_t' Q_t^{-1} \varepsilon_t) \quad (16)$$



where  $\theta$  denotes the vector of parameters to be estimated in the conditional log-likelihood function, and  $|Q_t|$  denotes the determinant of  $Q_t$ , the conditional covariance matrix. When  $\eta_t$  does not follow a joint multivariate normal distribution, equation (16) is defined as the Quasi-MLE (QMLE).

The models described above can also be estimated using the following simple two-step estimation procedure:

- (1) For each financial index return series, the univariate GARCH (1,1) model with an AR(1) conditional mean specification is estimated, and the unconditional shocks and standardized residuals of all  $m$  returns are saved.
- (2) For the portfolio returns, as defined by equation (3), the univariate GARCH (1,1) model with VARMA(1,1) conditional mean specification is estimated, and the unconditional shocks and standardized residuals are saved.
- (3) For each financial returns series, the univariate VARMA(1,1)-GARCH(1,1) model is estimated, including the lagged squared unconditional shocks and the lagged conditional variances of the remaining  $m-1$  assets. The standardized residuals of the  $m-1$  financial returns are saved.
- (4) For each financial returns series, the VARMA(1,1)-PS-GARCH(1,1) model is estimated, including the lagged squared unconditional shocks and the lagged conditional variances from step (2). The standardized residuals of all  $m$  financial returns are saved.
- (5) For each returns series, the constant conditional correlation matrices of the VARMA(1,1)-GARCH(1,1) model are estimated by direct computation using the standardized residuals from step (3). Bollerslev's (1990) CCC matrix is estimated directly using the standardized residuals from step (1). Finally, the constant conditional correlation matrix of the PS-GARCH model is estimated using the standardized residuals from step (4).

The tests of spillover and asymmetric effects are valid under the null hypothesis of independent (that is, no spillovers) and symmetric effects, so that steps (3) and (4) are

valid under the joint null hypothesis. The primary purpose of the structural and asymptotic theory derived in Ling and McAleer (2003) is to demonstrate that such testing is statistically valid.

Using extensions of the structural and asymptotic properties derived in Ling and McAleer (2003), Hoti et al. (2002) and McAleer et al. (2005), it can be shown that the QMLE of the parameters in the PS-GARCH model are consistent and asymptotically normal in the absence of normality in the standardized shocks  $\eta_{p,t}$  in (7) (the proof is available on request).

The VARMA-GARCH and VARMA-AGARCH models are available as pre-programmed options in, for example, the RATS 6 econometric software package. In this paper, estimation was undertaken using the EVIEWS 5.1 econometric software package, although the results were very similar using RATS 6.

### **3. Data**

The data used in the empirical application are daily prices measured at 16:00 Greenwich Mean Time (GMT) for four international stock market indices (henceforth referred to as synchronous data), namely S&P500 (USA), FTSE100 (UK), CAC40 (France), and SMI (Switzerland). New York and London are widely regarded as the two most important global markets, while Paris and Zurich are selected for purposes of examining spillovers using synchronous data. All prices are expressed in US dollars. The data were obtained from DataStream for the period 3 August 1990 to 5 November 2004, which yields 3720 observations. At the time the data were collected, this period was the longest for which data on all four variables were available. The rationale for employing daily synchronous data in modelling stock returns and volatility transmission is four-fold.

First, the Efficient Markets Hypothesis would suggest that information is quickly and efficiently incorporated into stock prices. While information generated yesterday may be

significant in explaining stock price changes today, it is less likely that news generated last month would have any explanatory power today.

Second, it has been argued by Engle et al. (1990) that volatility is caused by the arrival of unexpected news and that volatility clustering is the result of investors reacting differently to news. The use of daily data may help in modelling the interaction between the heterogeneity of investor responses in different markets.

Third, studies that use close-to-close non-synchronous returns suffer from the non-synchronicity problem, as highlighted in Scholes and Williams (1977). In particular, these studies cannot distinguish a spillover from a contemporaneous correlation when markets with common trading hours are analysed. Kahya (1997) and Burns et al. (1998) also observe that, if cross market correlations are positive, the use of close-to-close returns for non-synchronous markets will underestimate the true correlations, and hence underestimate the true risk associated with a portfolio of such assets.

Finally, the use of synchronous data allows the system to be written in a simultaneous equations form, which can be estimated jointly. Such joint estimation of the parameters eliminates potential econometric problems associated with generated regressors, in which unobserved variables are obtained (or generated) through estimation of auxiliary regression models (see, for example, Pagan (1984) and Oxley and McAleer (1993, 1994)), improves efficiency in estimation, increases the power of the test for cross-market spillovers, and analyses market interactions simultaneously. This allows all the relationships to be tested jointly. Joint estimation is also consistent with the notion that spillovers are the impact of global news on each market.

The synchronous returns for each market  $i$  at time  $t$  ( $R_{i,t}$ ) are defined as:

$$R_{it} = \log(P_{i,t} / P_{i,t-1}) \quad (17)$$

where  $P_{i,t}$  is the price in market  $i$  at time  $t$ , as recorded at 16:00 GMT.

The descriptive statistics for the synchronous returns of the four indexes are given in Table 1. All series have similar means and medians at close to zero, minima which vary between -10.251 and -5.533, and maxima that range between 5.771 and 10.356. Although the four standard deviations vary slightly, the coefficients of variation (CoV) are quite different, ranging from 30.97 for S&P500 to 67.30 for CAC40. The skewness differs among all four series, but the kurtosis is reasonably similar for all series. The Jarque-Bera test strongly rejects the null hypothesis of normally distributed returns, which may be due to the presence of extreme observations. As each of the series displays a high degree of kurtosis, this would seem to indicate the existence of extreme observations. Each of the returns series exhibits clustering, which needs to be captured by an appropriate time series model.

**[Insert Table 1 here]**

Several definitions of volatility are available in the literature. This paper adopts the measure of volatility proposed in Franses and van Dijk (2000), where the *true* volatility of returns is defined as:

$$V_{i,t} = (R_{i,t} - E(R_{i,t} | F_{t-1}))^2 \quad (18)$$

where  $F_{t-1}$  is the information set at time  $t-1$ .

The plots of the volatilities of the synchronous returns are given in Figures 1a-d. Each of the series exhibits clustering, which needs to be captured by an appropriate time series model. The volatility of all series appears to be high during the early 1990's, followed by a quiet period from the end of 1992 to the beginning of 1997. Finally, the volatility of all series appears to increase dramatically around 1997, due in large part to the Asian economic and financial crises. This increase in volatility persists until the end of the

period, and is likely to have been affected by the September 11, 2001 terrorist attacks and the conflicts in Afghanistan and Iraq.

**[Insert Figures 1a-d here]**

The descriptive statistics for the volatility of the synchronous returns of the four indexes, although not reported here, indicate that CAC40 displays the highest mean (median) volatility at 2.029 (0.665), while FTSE100 has the lowest mean (median) volatility at 1.357 (0.425). The maxima of the four volatility series differ substantially, with SMI displaying the highest maxima and S&P500 displaying the lowest. Although the four standard deviations vary, the coefficients of variation (CoV) are similar. All series are highly skewed. As each of the series displays a high degree of kurtosis, this would seem to indicate the existence of extreme observations.

#### **4. Value-at-Risk**

Formally, a VaR threshold is the lower bound of a confidence interval for the mean. Suppose that interest lies in modelling the random variable  $Y_t$ , which can be decomposed as follows:

$$Y_t = E(Y_t | F_{t-1}) + \varepsilon_t \quad (19)$$

This decomposition suggests that  $Y_t$  is comprised of a predictable component,  $E(Y_t | F_{t-1})$ , which is the conditional mean, and a random component,  $\varepsilon_t$ . The variability of  $Y_t$ , and hence its distribution, is determined entirely by the variability of  $\varepsilon_t$ . If it is assumed that  $\varepsilon_t$  follows a distribution such that:

$$\varepsilon_t \sim D(\mu_t, \sigma_t) \quad (20)$$

where  $\mu_t$  and  $\sigma_t$  are the unconditional mean and standard deviation of  $\varepsilon_t$ , respectively, these can be estimated using a variety of parametric and/or non-parametric methods. The procedure used in this paper is discussed in Section 3. The VaR threshold for  $Y_t$  can be calculated as:

$$VaR_t = E(Y_t | F_{t-1}) - \alpha \sigma_t \quad (21)$$

where  $\alpha$  is the critical value from the distribution of  $\varepsilon_t$  to obtain the appropriate confidence level. Alternatively,  $\sigma_t$  can be replaced by alternative estimates of the conditional variance to obtain an appropriate VaR (see Section 2 above).

## 5. Forecasts

The purpose of this section is to compare the volatility and conditional correlation forecasts produced by the CCC model of Bollerslev (1990), the VARMA-GARCH model of Ling and McAleer (2003), and the new PS-GARCH model proposed in this paper. A rolling window approach is used to forecast the 1-day ahead conditional correlations and conditional variances. The sample ranges from 3 August 1990 to 5 November 2004. In order to strike a balance between efficiency in estimation and a viable number of rolling regressions, the rolling window size is set at 2000 for all four data sets, which leads to a forecasting period from 6 April 1998 to 5 November 2004.

**[Insert Figure 2 here]**

Figures 1a-d plot the forecasted volatilities using the three models for an equally weighted portfolio containing S&P500, FTSE100, CAC40 and SMI. Table 2 shows the correlations between the three sets of forecasts. The volatility forecasts produced by all models are remarkably similar, with correlation coefficients of the volatility forecasts ranging from 0.987 to 0.993.

**[Insert Table 2 here]**

The forecasted conditional correlations and the correlation of the conditional correlation forecasts are given in Figures 3-8 and Table 3, respectively. The conditional correlation forecasts are virtually identical for all three models, with correlation coefficients ranging from 0.996 to 0.999. This result suggests that for applications where the required inputs are the forecasts of the conditional variances and/or the conditional correlation matrix, all three models considered above yield very similar results.

**[Insert Figures 3-8 here]**

**[Insert Table 3 here]**

## **6. Economic Significance**

The 1988 Basel Capital Accord, which was originally concluded between the central banks from the Group of Ten (G10) countries, and has since been adopted by over 100 countries, sets minimum capital requirements which must be met by banks to guard against credit and market risks. The market risk capital requirements are a function of the forecasted VaR thresholds (see Section 4 above). The Basel Accord stipulates that the daily capital charge must be set at the higher of the previous day's VaR or the average VaR over the last 60 business days multiplied by a factor  $k$ . The multiplicative factor  $k$  is set by the local regulators, but must not be lower than 3.

In 1995, the 1988 Basel Accord was amended to allow banks to use internal models to determine their VaR. However, banks wishing to use internal models must demonstrate that the models are sound. Furthermore, the Basel Accord imposes penalties in the form of a higher multiplicative factor  $k$  on banks which use models that lead to a greater number of violations than would reasonably be expected given the specified confidence level of 1%. Table 4 shows the penalties imposed for a given number of violations for 250 business days.

In certain cases, where the number of violations is deemed to be excessively large, regulators may penalize banks even further by requiring that their internal models be reviewed. In circumstances where the internal models are found to be inadequate, banks can be required to adopt the standardized method originally proposed in 1993 by the Basel Accord. The standardized method suffers from several drawbacks, the most noticeable of which is its systematic overestimation of risk, which stems from the assumption of perfect correlation across different risk factors. Overestimating risk leads to higher capital charges which negatively impact both the profitability and reputation of the bank.

**[Insert Table 4 here]**

The economic significance of the various models proposed above is highlighted by forecasting VaR thresholds using the PS-GARCH, VARMA-GARCH and CCC models (see Jorion (2000) for a detailed discussion of VaR). In order to simplify the analysis, it is assumed that the portfolio returns are normally distributed, with equal and constant weights. We control for exchange rate risk by converting all prices to a common currency, namely the US Dollar. We use the forecasted variances and correlations from Section 4 to produce VaR forecasts for the period 6 May 1998 to 5 November 2004. The backtesting procedure is used to test the soundness of the models by comparing the realised and forecasted losses (see Basel Committee (1988, 1995, 1996) for further details).

Figures 9-11 show the VaR forecasts and realized returns for each empirical model considered. Both the CCC and PS-GARCH VaR forecasts violate the thresholds 7 times from 1720 forecasts, while the VARMA-GARCH model leads to 6 violations from 1720 forecasts.

Table 5 shows that the mean daily capital charge, which is a function of both the penalty and the forecasted VaR, implied by PS-GARCH is the largest at 9.180%, followed by VARMA-GARCH at 9.051% and CCC at 9.009%. A high capital charge is undesirable,



other things equal, as it reduces profitability. Table 5 also shows that CCC leads to violations that are approximately 10% greater in terms of mean absolute deviations, at 0.498, than the VARMA-GARCH and PS-GARCH models, at 0.454 and 0.442, respectively. This is particularly important because large violations, on average, may lead to bank failures, as the capital requirements implied by the VaR threshold forecasts may be insufficient to cover the realized losses. Finally, CCC also leads to the largest maximum violation.

**[Insert Figures 9-11 here]**

**[Insert Table 5 here]**

## **7. Conclusion**

Accurate modelling of volatility (or risk) is important in finance, particularly as it relates to the modelling and forecasting of Value-at-Risk (VaR) thresholds. As financial applications typically deal with a portfolio of assets and risks, there are several multivariate GARCH models which specify the risk of one asset as depending dynamically on its own past, as well as the past of other assets. These models are typically computationally demanding, due to the large number of parameters to be estimated, and can be impossible to estimate for a large number of assets.

The need to create volatility models that can be used to estimate large covariance matrices has become especially relevant following the 1995 amendment to the Basel Accord, whereby banks are permitted to use internal models to calculate their VaR thresholds. While the amendment was designed to reward institutions with superior risk management systems, a backtesting procedure in which the realized returns are compared with the VaR forecasts, was introduced to assess the quality of the internal models. Banks using models that lead to a greater number of violations than can reasonably be expected, given the confidence level, are penalized by having to hold higher levels of capital. The imposition of penalties is severe as it has an impact on the profitability of the bank

directly through higher capital charges, may damage the banks reputation, and may also lead to the imposition of a more stringent external model to forecast the VaR thresholds.

This paper examined various conditional volatility models for purposes of forecasting financial volatility and VaR thresholds. Two constant conditional correlation models for estimating the conditional variances and covariances are the CCC model of Bollerslev (1990) and the VARMA-GARCH model of Ling and McAleer (2003). Although the VARMA-GARCH model accommodates spillover effects from the returns shocks of all assets in the portfolio, which are typically estimated to be significantly different from zero, the forecasts of the conditional volatility and VaR thresholds produced by the VARMA-GARCH model are very similar to those produced by the CCC model.

Finally, the paper also developed a new parsimonious and computationally convenient Portfolio Spillover GARCH (PS-GARCH) model, which allowed spillover effects to be included parsimoniously. The PS-GARCH model was found to yield volatility and VaR threshold forecasts that were very similar to those of the CCC and VARMA-GARCH models. Therefore, although the empirical results suggest that spillover effects are statistically significant, the VaR threshold forecasts are generally found to be insensitive to the inclusion of spillover effects in the multivariate models considered.

## References

- Alexander, C.O. (2001), "Orthogonal GARCH", in C.O. Alexander (ed.), *Mastering Risk, Volume II*, FT-Prentice Hall, pp. 21-38.
- Basel Committee on Banking Supervision, (1988), "International Convergence of Capital Measurement and Capital Standards", BIS, Basel, Switzerland.
- Basel Committee on Banking Supervision, (1995), "An Internal Model-Based Approach to Market Risk Capital Requirements", BIS, Basel, Switzerland.

- Basel Committee on Banking Supervision, (1996), "Supervisory Framework for the Use of "Backtesting" in Conjunction with the Internal Model-Based Approach to Market Risk Capital Requirements", BIS, Basel, Switzerland.
- Bauwens, L., S. Laurent and V.K. Rombouts. (2005), "Multivariate GARCH Models: A Survey", to appear in *Journal of Applied Econometrics*.
- Bollerslev, T. (1986), "Generalised Autoregressive Conditional Heteroscedasticity", *Journal of Econometrics*, 31, 307-327.
- Bollerslev, T. (1990), "Modelling the Coherence in Short-Run Nominal Exchange Rate: A Multivariate Generalized ARCH Approach", *Review of Economics and Statistics*, 72, 498-505.
- Burns, P, R. Engle and J. Mezrich (1998), "Correlations and Volatilities of Asynchronous Data", *Journal of Derivatives*, 5, 7-18.
- Engle, R.F. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica*, 50, 987-1007.
- Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models", *Journal of Business and Economic Statistics*, 20, 339-350.
- Engle, R.F., T. Ito and W. Lin (1990), "Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market", *Econometrica*, 58, 525-542.
- Franses, P.H. and D. van Dijk (2000), *Nonlinear Time Series Models in Empirical Finance*, Cambridge, Cambridge University Press.
- Glosten, L.R., R. Jagannathan, and D.E. Runkle (1992), "On the Relation between the Expected Value and Volatility of the Nominal Excess Return on Stocks", *Journal of Finance*, 46, 1779-1801.
- Hoti, S., F. Chan and M. McAleer (2002), "Structure and Asymptotic Theory for Multivariate Asymmetric Volatility: Empirical Evidence for Country Risk Ratings", invited paper presented to the Australasian Meeting of the Econometric Society, Brisbane, Australia, July 2002.
- Jorion, P. (2000), *Value at Risk: The New Benchmark for Managing Financial Risk*, McGraw-Hill, New York.
- Kahya, E. (1997), "Correlation of Returns in Non-Contemporaneous Markets", *Multinational Finance Journal*, 1, 123-135.

- Ling, S. and M. McAleer (2002a), “Stationarity and the Existence of Moments of a Family of GARCH Processes”, *Journal of Econometrics*, 106, 109-117.
- Ling, S. and M. McAleer (2002b), “Necessary and Sufficient Moment Conditions for the GARCH(r,s) and Asymmetric Power GARCH(r,s) Models”, *Econometric Theory*, 18, 722-729.
- Ling, S. and M. McAleer (2003), “Asymptotic Theory for a Vector ARMA-GARCH Model”, *Econometric Theory*, 19, 278-308.
- McAleer, M. (2005), “Automated Inference and Learning in Modeling Financial Volatility”, *Econometric Theory*, 21, 232-261.
- McAleer, M., F. Chan, S. Hoti and O. Lieberman (2005), “Generalized Autoregressive Conditional Correlation”, unpublished paper, School of Economics and Commerce, University of Western Australia.
- Oxley, L. and M. McAleer (1993), “Econometric Issues in Macroeconomic Models with Generated Regressors”, *Journal of Economic Surveys*, 7, 1-40.
- Oxley, L. and M. McAleer (1994), “Testing the Rational Expectations Hypothesis in Macroeconometric Models with Unobserved Variables”, in L. Oxley et al. (eds.), *Surveys in Econometrics*, Blackwell, Oxford, 1994, pp. 299-349.
- Pagan, A.R. (1984), “Econometric Issues in the Analysis of Regressions with Generated Regressors”, *International Economic Review*, 25, 221-247.
- Scholes, M., and J. Williams (1977), “Estimating Betas from Nonsynchronous Data”, *Journal of Financial Economics*, 5, 309-327.
- Tse, Y.K. and A.K.C. Tsui (2002), “A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time-Varying Correlations”, *Journal of Business and Economic Statistics*, 20, 351-362.
- da Veiga, B. and M. McAleer (2005), “Single Index and Portfolio Methods for Modelling and Forecasting Value-at-Risk”, unpublished paper, School of Economics and Commerce, University of Western Australia.

Table 1: Descriptive Statistics for Returns

Statistics	S&P500	FTSE100	CAC40	SMI
Mean	0.033	0.020	0.020	0.036
Median	0.029	0.013	0.043	0.037
Maximum	5.771	8.336	10.356	7.049
Minimum	-5.533	-5.681	-10.251	-9.134
Std. Dev.	1.022	1.067	1.346	1.164
Skewness	-0.018	0.118	0.015	-0.120
Kurtosis	6.160	6.254	7.391	7.044
CoV	30.97	53.35	67.30	32.33
Jarque-Bera	1548.4	1649.5	2989.0	2543.4

Table 2: Correlations Between Conditional Volatility Forecasts for the Portfolio

CCC	VARMA-GARCH	PS-GARCH
1	0.987	0.993
	1	0.991
		1

Table 3: Correlations of Rolling Conditional Correlation Forecasts Between Pairs of Indexes

S&P500 and FTSE100			S&P500 and CAC40		
CCC	VARMA-GARCH	PS-GARCH	CCC	VARMA-GARCH	PS-GARCH
1	0.996	0.999	1	0.996	0.998
	1	0.997		1	0.997
		1			1
S&P500 and SMI			FTSE100 and CAC40		
CCC	VARMA-GARCH	PS-GARCH	CCC	VARMA-GARCH	PS-GARCH
1	0.995	0.999	1	0.992	0.996
	1	0.996		1	0.996
		1			1
FTSE100 and SMI			CAC40 and SMI		
CCC	VARMA-GARCH	PS-GARCH	CCC	VARMA-GARCH	PS-GARCH
1	0.984	0.995	1	0.998	0.996
	1	0.992		1	0.996
		1			1

Table 4: Basel Accord Penalty Zones

Zone	Number of Violations	Increase in $k$
Green	0 to 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00
Note: The number of violations is given for 250 business days.		

Table 5: Mean Daily Capital Charge and AD of Violations

Model	Number of Violations	Mean Daily Capital Charge	AD of Violations	
			Maximum	Mean
CCC	7	9.009	2.125	0.498
VARMA-GARCH	6	9.760	1.974	0.454
PS-GARCH	7	9.180	1.902	0.442
Notes:				
(1) The daily capital charge is given as the negative of the higher of the previous day's VaR or the average VaR over the last 60 business days times $(3+k)$ , where $k$ is the penalty.				

Figure 1a: S&P500 Volatility

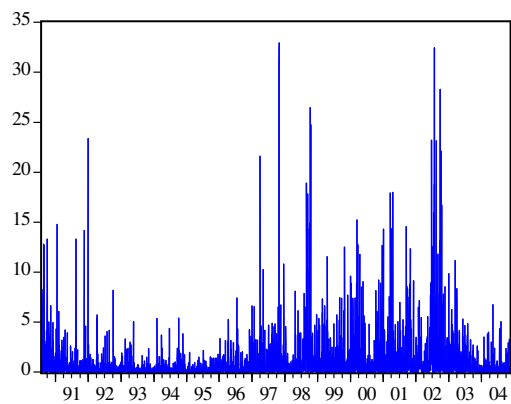


Figure 1b: FTSE100 Volatility

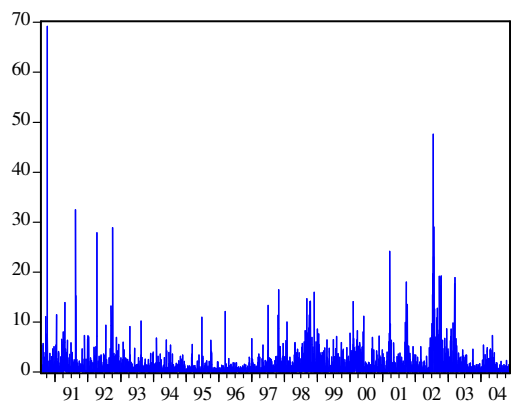


Figure 1c: CAC40 Volatility

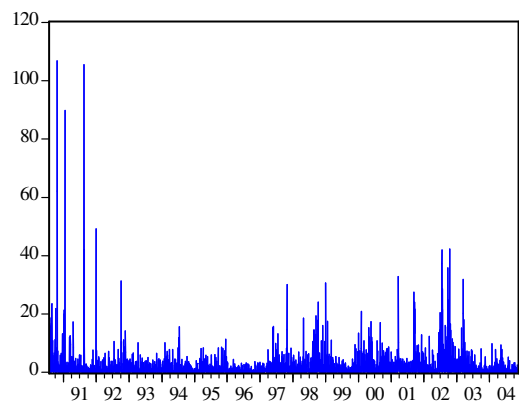


Figure 1d: SMI Volatility

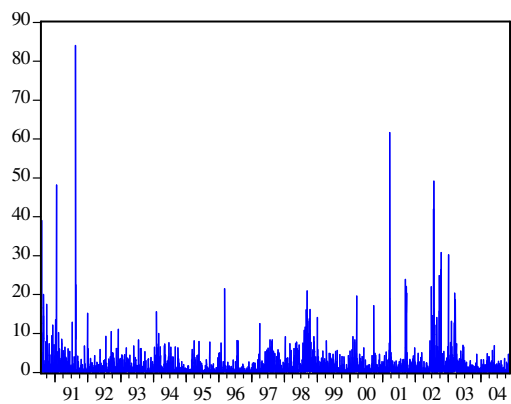




Figure 2: Portfolio Volatility Forecast

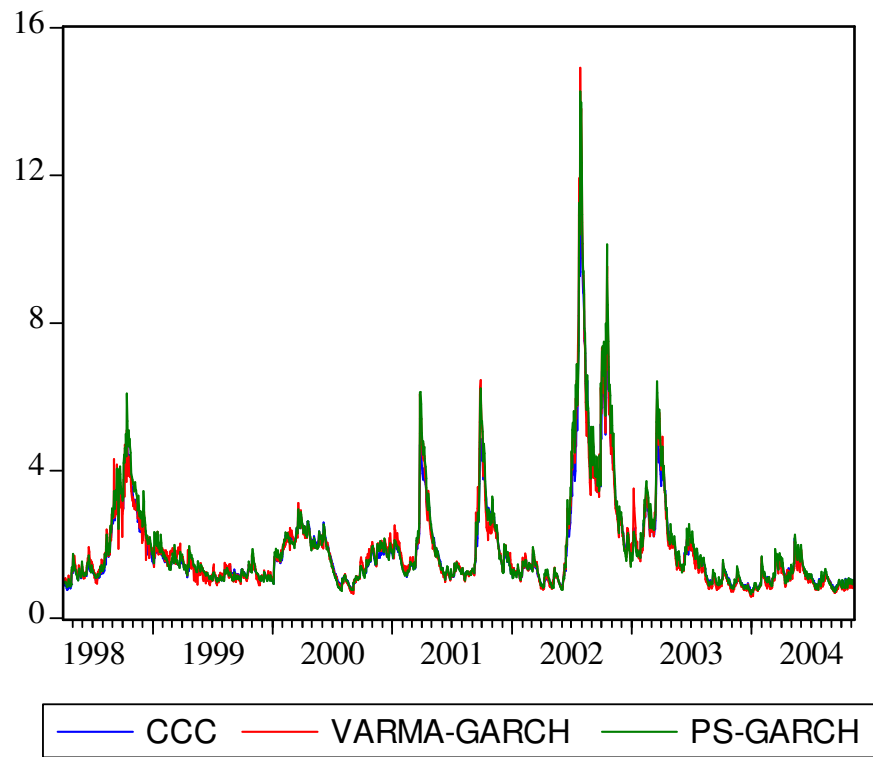


Figure 3: Rolling Conditional Correlation Forecasts Between S&P500 and FT SE100

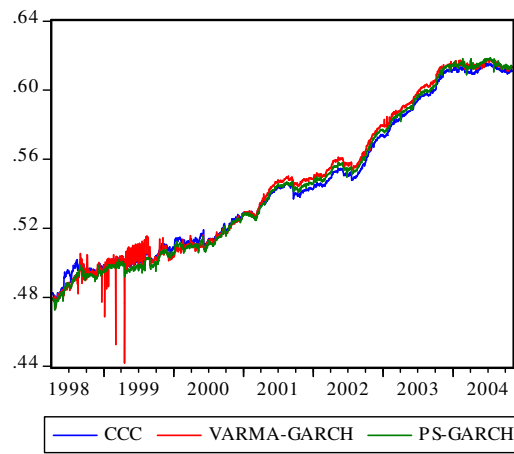


Figure 4: Rolling Conditional Correlation Forecasts Between S&P500 and CAC40

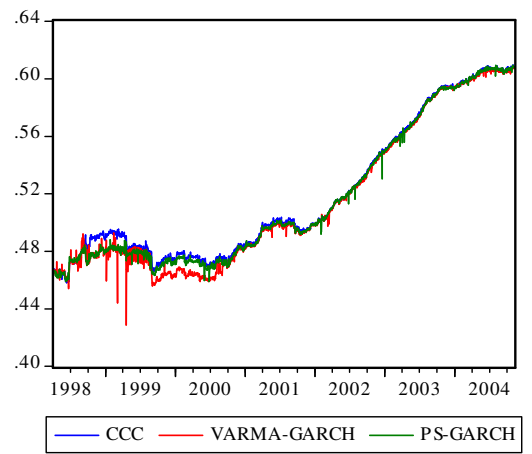


Figure 5: Rolling Conditional Correlation Forecasts Between S&P500 and SMI

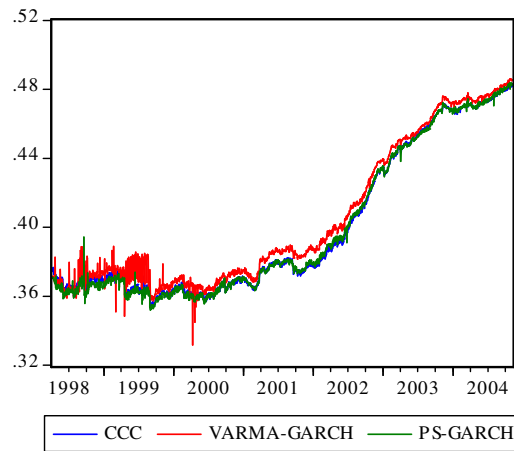


Figure 6: Rolling Conditional Correlation Forecasts Between FT SE100 and CAC40

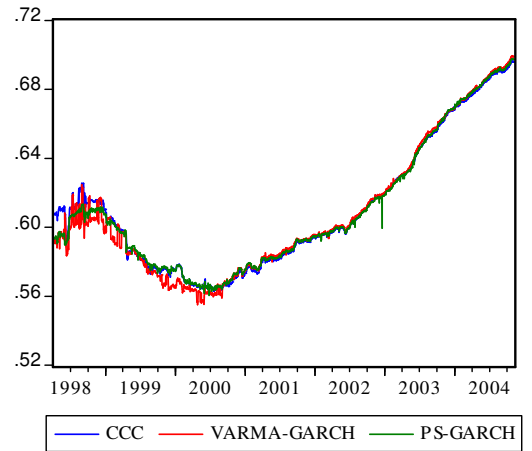


Figure 7: Rolling Conditional Correlation Forecasts Between FT SE100 and SMI

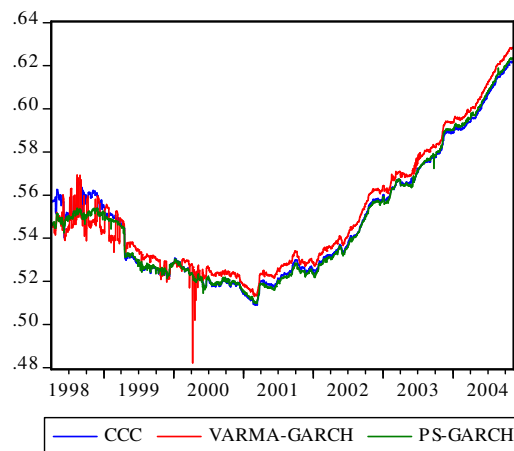


Figure 8: Rolling Conditional Correlation CAC40 and SMI

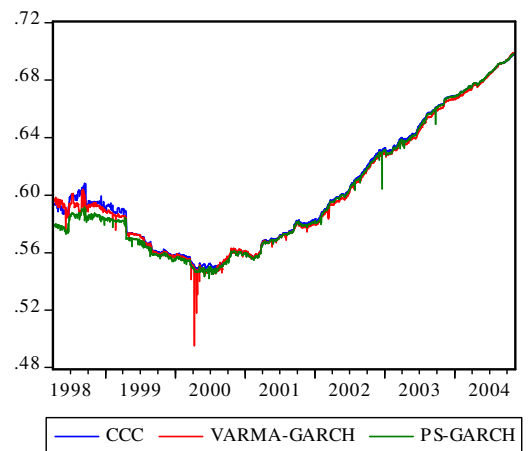


Figure 9: Realized Returns and CCC VaR Forecasts

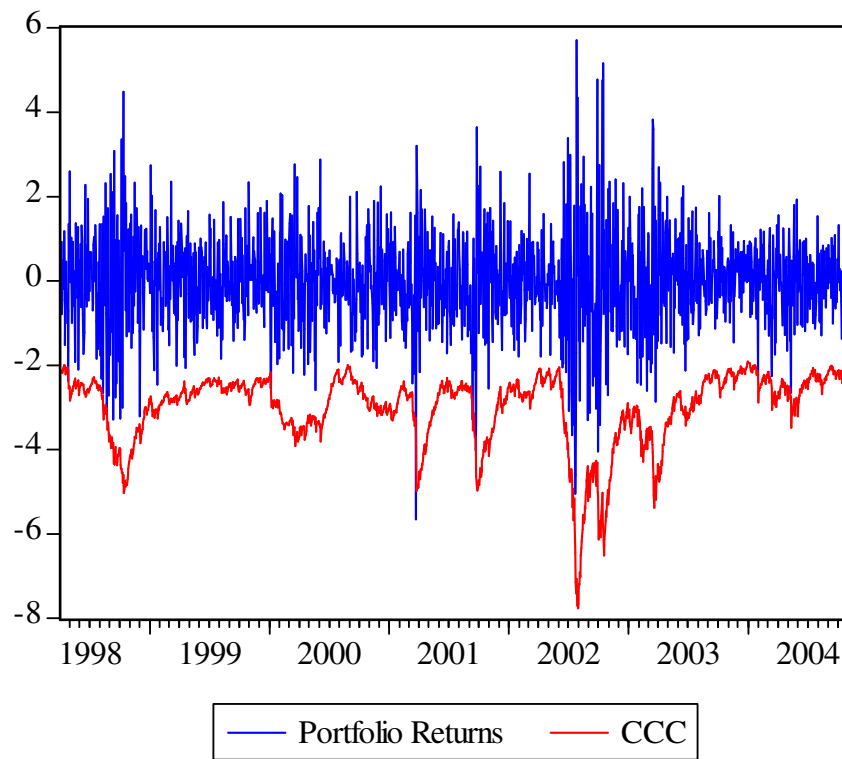


Figure 10: Realized Returns and VARMA-GARCH VaR Forecasts

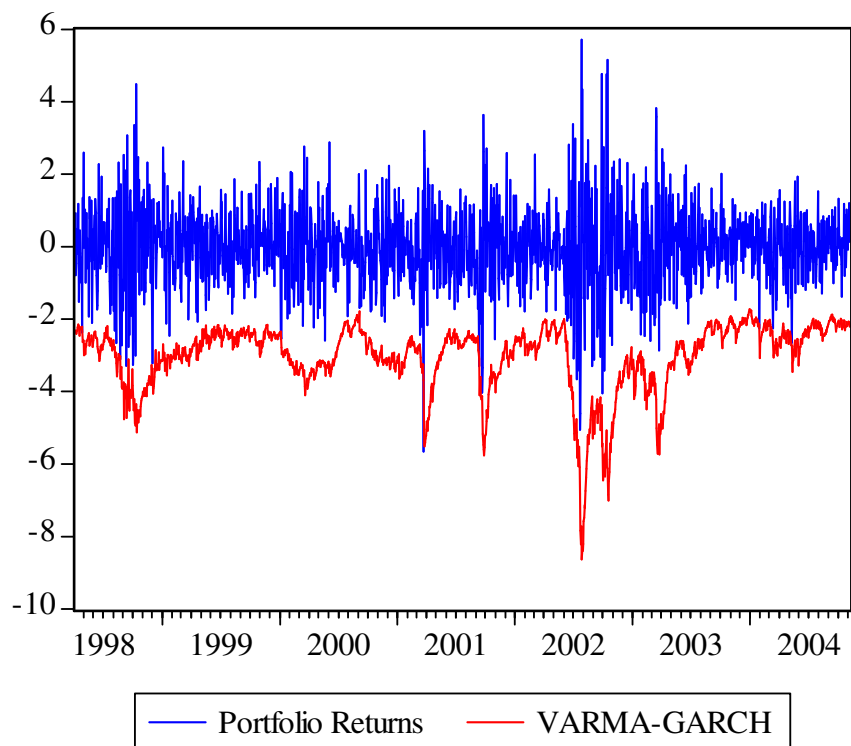


Figure 11: Realized Returns and PS-GARCH VaR Forecasts

