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International Convergence and Local Divergence

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Abstract

This paper presents an East-West endogenous-growth model that reproduces recent stylized facts applicable to the trade liberalization process of many developing countries: convergence with the rest of the world, higher internal divergence, increasing spatial concentration of economic activity and higher growth rates. We claim that the ongoing reduction of manufacturing trade costs may generate a net inflow of global demand towards the industrialized cores of developing countries. This will induce a reallocation of labor from traditional to modern sectors. In turn, such a sectoral shift may enlarge the catch-up (imitation) potential of developing countries and raise global growth rates, due to Grossman and Helpman's complementarity between imitative and innovative activities. Although advanced economies may become relatively worse off, the effect on growth rates may allow them to gain in absolute terms.

1 Introduction

China's gradual liberalization over the last decades is leading to a rapid process of catchup with more advanced economies. However, internal divergence has been rising, with the coastal areas benefitting much more than the more inland provinces. This experience has not been limited to China. Though growth in Mexico has been somewhat more disappointing, its catchup with the rest of the world has been paralleled by increasing divergence between the more advanced and the less advanced states of the country.

This paper explores the impact of a developing country's higher trade openness on convergence, not just with the rest of the world, but also within the liberalizing country. In addition to addressing these questions of relative development, it also analyzes its effect on global long-run growth. This is important: although the rest of the world may become relatively worse off, in absolute terms it may end up gaining due to the impact of higher trade openness on growth rates.

Our modelling tool is an East-West framework with an exogenous division within the eastern (and poorest) country. The East consists of an industrialized Core — which can potentially host both manufactures and a research sector devoted to imitating western patents — and a Periphery doomed to host just primary sectors under perfect competition. We assume that international-trade barriers for our homogeneous (primary) good do not decay at the same pace as those of manufactures, as if biased technological change was affecting differently the transaction costs of both sectors.[1] Since the western aggregate income is larger, an increase in manufacturing trade openness induces a net inflow of demand for eastern varieties, which raises the relative wage of the Core with respect to the West. Simultaneously, the relative wage of the Core with respect to the Periphery also rises, since primary goods remain barely as attractive to foreign consumers as before. Then, these widening income differentials within the East give rise to Periphery-Core migrations, which also enhances peripheral wages and favors East-West convergence. However, wages in the Core do not necessarily decay with migration, since some of the immigrants will become researchers, enlarging the eastern imitation potential and the fraction of world manufactures produced in the Core, which channels an even higher world demand towards the latter location.

As for its effects on growth, the agglomeration of labor in the Core turns out to be beneficial for global growth rates. In our framework imitation and innovation are complementary activities, which implies that a higher eastern catch-up potential spurs innovation in the West. The last effect holds because stronger imitation will reduce western wages and subsequently increase the value of a patent, raising the natural incentives to innovate. Taking all this into account, any restriction to Periphery-Core migration proves to be harmful in terms of steady-state growth, but not necessarily in terms of regional cohesion, since a higher catch-up potential in the Core may boost internal divergence patterns in the East.

The participation of China in the world trade and investment systems involves not only crucial consequences for the internal disparities within that country, but also for the international relocation of significant labor-intensive industries, which often shift from more developed towards less developed countries. For example, as illustrated by Woo (2003), "in mid-2003, the electronic and electrical firms in Penang, Malaysia, employed 17 percent fewer workers than in 2000". Meanwhile, Mexico's economic liberalization and trade integration in NAFTA has also been related to (internal) regional divergence and the threat of a "giant sucking sound" posed for some segments of the US economy. Well known empirical work has already estimated a significant effect of international

[1] For an empirical study that confirms this tendency, linked to the recent breakthrough of telecommunications, see Rauch (1999).

trade openness on higher growth and increasing regional inequality for these countries (see e.g. Wei (1993), Rodriguez-Pose and Sanchez-Reaza (2002)).

However, the study of the connections between trade openness, growth and regional inequality in developing countries had remained at a largely statistical level up to very recent times (see, e.g., Jian, Sachs and Warner (1996); Ying (1999); Kanbur and Zhang (2001); Fujita and Hu (2001); Huang, Kuo and Kao (2003)). Just a few papers have tried to introduce some specific economic modelling into the debate on the sources of inequality. We will briefly examine the explanations proposed Feenstra and Hanson (1997), Giannetti (2002) and Hu (2002). Moreover, since our framework is derived from the Grossman and Helpman (1991) model, our main innovation, compared to them, consists of explicitly incorporating trade costs in the model. We also portray a dual economy within the East, which allows us to modify the steady-state growth rate as trade shocks affect differently our two eastern regions and induce migratory flows towards the areas where imitation takes place.

Focusing on the case of Mexico, Feenstra and Hanson (1997) link rising wage inequality to the foreign capital inflows that followed NAFTA, positively correlated with the demand for skilled labor. Our model does not rely on FDI as the usual suspect behind regional divergence, since our main driving force is a fall in manufacturing trade costs, which shifts world demand towards the small economies once they are sufficiently opened to international trade. On the other hand, Giannetti (2002) develops an East-West endogenous-growth model inspired by similar EU stylized facts (international convergence accompanied by divergence within countries). However, gradual increases in trade openness are not the main driving force of the mechanism. Instead, strong regional disparities originate from international knowledge-spillovers, which determine regional comparative advantage and subsequent productive specialization. Finally, Hu (2002) is also an economic geography model, inspired by the case of China, but it does not model explicitly the Western economy and neither does it consider growth effects. The only difference in his model between the Coastal and the Interior region is a differential access to the Western market, without further institutional distinctions. The presence of vertical linkages and rural-urban migration also lead to agglomeration in the Coast as international trade costs fall. Nevertheless, our imitation-potential mechanism is absent from his model, which leads him to support the traditional view on the pro-convergence effects of interregional migration.

It is important to note that our results crucially depend on the Periphery being *exogenously* a rural economy, radically differentiated from the rest of the East concerning productive capabilities. However, we argue that – at least in the case of China – such a geo-economic structure is not endogenously derived from the typical interplay of centripetal and centrifugal forces, as described by a Core-Periphery model in a market economy. Instead, they come from the deliberate decisions made by a body of political authorities, who face a trade-off between their own objectives and those of the common population. In other words, we are going to argue in favor of the *political-economy origins* of the economic backwardness of Central and Western China, as opposed to alternative economic-geography motivations, like those modeled, for example, by Hu (2002). Therefore, this paper intends to describe the mechanism by which current regional disparities are aggravated, but renounces to study the underlying rationale behind the Core's specialization in manufactures and the peripheral specialization in the provision of energy, minerals, food or cheap labor.

There is substantial evidence favoring a political-economy explanation for China's Core-Periphery situation. Branstetter and Feenstra (1999) view the political process in China as trading off the social benefits of increased trade and foreign direct investment, against the losses incurred by state-owned enterprises due to such liberalization. One of the most solid conclusions they reached during some discussions with firm managers was that, to some degree, most foreign-invested enterprises compete with state-owned firms. The second conclusion is that "the Chinese government, both national and local, is acutely aware of that competition, and has taken steps to impede the ability of foreign firms to compete in the Chinese market". Accordingly, multinational executives have found related restrictions on their operations, e.g. export requirements, localization requirements, restrictions on domestic market access, requirements for technology transfer, ...Therefore, foreign-invested firms have mostly located along the coastal area "*following a line of least resistance*" (Gipouloux (1998)), i.e. FDI increases where there are fewer state-owned companies involved in industrial organization. In this respect, we reproduce a significant paragraph from his text:

"With a jolt, the opening up of the country and the dynamics of economic reform re-activated these divisions between coastal China and inland China, but they are *simply the traces of very ancient geo-economic dividing lines, still visible after having been blurred for three decades* (1949-1979), from the Communists' takeover to the beginning of the reforms. The 14 coastal cities that were opened in 1984 correspond essentially to the chain of ports opened under diplomatic and military pressure after the Opium Wars."[2]

That is, there is something much deeper than a simple "home-market effect" keeping those divisions in place.[3]

The rest of the paper is organized as follows. Section 2 derives the properties of a generic steady state, which initially shows a given distribution of populations in West, Core and Periphery.

[2] There have been sincere attempts from the Chinese authorities to switch from an uneven-national-priority strategy to a nation-wide implementation of FDI promotion. However, as Chunlai (1997) reports, "not only has the process of diffusion from the coastal region to the inland areas been slow, but also the outflow of skilled workers, technical personnel and capital from the inland areas to the coastal regions has been increasing. Perhaps, more important is that the coastal region has been getting more freedom in economic decision making from the central government than the inland regions".

[3] For a discussion on the highly distorted system of inland China's industrial relations, see e.g. Young (2000).

Section 3 contains the comparative-statics exercise (in the level of international trade openness) that reproduces our stylized facts, while allowing for interregional migration within the East. Section 4 concludes.

2 The model without migration

2.1 Environment

2.1.1 Overview

In our framework, Periphery-Core migration will be responsible for the scale effects that yield higher global growth rates. Our steady state will be characterized by the intersection of two curves: one of them describes the relative wage of the Core with respect to the Periphery for a given distribution of eastern population between both locations; the second one, also known as the "migration function", describes the amount of population willing to live in the Periphery for a given relative wage. Now we will derive the first of both curves, which implies solving the whole dynamic model regardless of the migration decision, which will be considered in section 3.

2.1.2 Endowments

As in Grossman and Helpman (1991), we consider 2 countries - East and West. One important novelty is the existence of three regions, i.e. we also include a Periphery within the East. The population of both countries is exogenously given (being L_s for the East and L_n for the West), since we do not allow for international migration. Nevertheless, there can be migrations within the East, which means that eastern people can move from Periphery to Core (and vice versa) in response to economic-opportunity variables; i.e. $L_s = L_a + L_c$, where L_a (the peripheral population) is an endogenous variable.

The availability of factors of production is different across regions, since every location has distinctive institutional features. There are three main productive factors: labor, researchers and financial capital. Labor can be employed in agriculture (in the Periphery) or in manufacturing (in the Core or the West), and researchers are exclusively located in the last two locations. Researchers in the West are used to conceive new varieties (startups), whereas researchers in the Core can only replicate the existing ones to produce them in the East at lower cost. Moreover, there is perfect occupational mobility between local manufacturing workers and researchers, in the sense that both have the same local earnings and are therefore indifferent between both occupations. The distribution of the population in the West and the Core between researchers and manufacturing labor will be endogenously derived in the model.

A household (or individual) from location k owns a measure β_{nk} of western firms and β_{ck} of eastern firms. The source of this financial capital were the previous gross savings of the household, which were used to finance the new manufacturing startups producing in the Core ($\dot{\beta}_{ck}$) or the West ($\dot{\beta}_{nk}$). When allocating their savings to startups from different locations, consumers must take into account that firms from the West will be imitated from the Core and drawn out of the market with some probability.

2.1.3 Preferences

Any representative household (or individual) k, living in that location k, maximizes (in every period t) an intertemporal utility function W_t^k such as

$$W_t^k = \int_t^{\infty} e^{-\rho(s-t)} \log \left[U_s \left(X_s^i, A_s \right) \right] \, ds \tag{1}$$

where W_t^k reflects the discounted utility flow that household k expects to obtain from period t onwards by acquiring manufactures (grouped into the composite X) and the homogeneous agricultural good (A). On the other hand, the particular form of U_s reveals the relative weight assigned to food and manufactures in the following way:

$$U_s = X_s^{\mu} A_s^{1-\mu}, \text{ where } 0 < \mu < 1 \tag{2}$$

The composite of manufactures X_s is a Dixit-Stiglitz subutility function over the aggregate measure of varieties invented up to period s,

$$X_s = \left[\int_0^{n(s)} x_j \left(s \right)^{\alpha} dj \right]^{\frac{1}{\alpha}} \tag{3}$$

where $0 < \alpha < 1$ is a positive measure of the substitutability between manufactures and $x_j\left(s\right)$ quantifies the household demand for variety j at time s, $\forall s \geq t$. These preferences imply that the individual appreciates the expansion of manufacturing diversity, since utility will grow as expenditure is more thinly divided among a growing number of varieties.

2.1.4 Technologies

In the global economy there is a continuum of industrial varieties with measure n, and $n = n_n + n_c$ (the addition of the measures from the West and the Core). This degree of product variety expands over time due to innovation. Moreover, an increase in the local measure of manufactures enlarges the stock of public knowledge and reduces future R&D costs. Grossman and Helpman's local stocks of knowledge are equal to n in the West - since all patents were originally made up there - and to n_c in the core. This implies that there are no international knowledge-spillovers.

The production function for every particular manufacture (and for the homogeneous primary good) is identical and very simple: 1 unit of labor generates 1 unit of final output. Labor is the only factor in the production of the primary good, whereas prior to the production of any manufacture it is necessary to incur a fixed cost (to invent or imitate the corresponding patent), which is financed by means of gross savings. By free entry in the innovative (and imitative) activity, such a fixed cost is at least equal to the market value of the patent. This value decreases with the local stock of public knowledge in this way:

$$v_c \leq \frac{a_m w_c}{n_c}, \text{ with equality when } \dot{n}_c > 0 \tag{4}$$

$$v_n \leq \frac{a w_n}{n}, \text{ with equality when } \dot{n} > 0 \tag{5}$$

where v_c and v_n denote the values of eastern and western patents, respectively, whereas $\frac{a_m}{n_c}$ and $\frac{a}{n}$ stand for the number of researchers needed to imitate a western patent in the Core and to create a new variety in the West. Our variables w_a, w_c and w_n denote the nominal wage in the Periphery, the Core and the West, respectively. Later we will establish some necessary and sufficient parameter restrictions so that imitation and innovation coexist, which implies that

$$w_n = \frac{n v_n}{a}; w_c = \frac{n_c v_c}{a_m} \tag{6}$$

Eastern researchers need to incur the previous fixed cost in order to replicate a western patent, while western researchers do it to invent one from scratch. On the other hand, we assume that our primary good is traded costlessly, whereas our parameter $\tau \geq 1$ introduces the classical iceberg-notion of international trade costs for manufactures: it is necessary to buy τ units of that good abroad to consume 1 unit at home. That is, we introduce manufacturing trade costs between East and West, but we assume away internal trade costs within the East.[4]

2.2 Static optimization

Productive firms must decide which prices to quote in every period to maximize profits. On the other hand, free entry into the innovative (imitative) activity guarantees that the expected stream of profits for the startup is equal to the actual cost of innovation (imitation).

[4]Since those internal trade costs do not change, without loss of generality we can make them equal to zero.

Consumers in any location not only decide how much to save, which equity to buy and which commodities to consume, but also choose their job (if they are not in the Periphery, they become either manufacturing workers or researchers) and their location of residence. The job decision is not problematic, since they will receive the local wage no matter whether they do research or not. On the contrary, as is usual in economic geography, we assume that expectations are adaptive when an eastern household chooses whether to migrate or not, i.e. they do not expect other households to move at the same time.

The function W_s^k is intertemporally maximized with respect to its ultimate arguments ($x_j(s), \forall j$, $\forall s \geq t$; $A(s)$ $\forall s \geq t$) at every period t, taking as given the expected temporal paths $v_n(s)$, $v_c(s)$, $n(s)$, $p_j(s)$ $\forall j$ and $p_a(s)$, $\forall s \geq t$. As Grossman and Helpman do, this problem can be decomposed into 2 parts:

- The static allocation of a given per-household expenditure E_s^k among the primary good and all kind of manufactures, which gives rise to a demand function for each of these commodities.

- The choice of an optimal path for E_s^k, given the possibility of saving and investing in equity of eastern and western firms.

We will proceed now to describe the first of both parts.

Let's denote by E the aggregate world expenditure and by γ the proportion of E spent by people from the West, which is an endogenous variable. Considering that demand for any variety comes from both western and eastern consumers who face different c.i.f. prices, we can derive the aggregate demand for any western (x_n) and eastern manufacture (x_c), taking into account (2), (3) and our previous definition of γ as follows:

$$x_n = \mu \cdot p_n^{-\in} \cdot \left[\frac{\gamma E}{n_n p_n^{1-\in} + \delta\ n_c\ p_c^{1-\in}} + \frac{(1-\gamma)\ \delta\ E}{\delta\ n_n p_n^{1-\in} +\ n_c\ p_c^{1-\in}} \right] \tag{7}$$

$$x_c = \mu \cdot p_c^{-\in} \cdot \left[\frac{\gamma\ \delta\ E}{n_n p_n^{1-\in} + \delta\ n_c\ p_c^{1-\in}} + \frac{(1-\gamma)\ E}{\delta\ n_n p_n^{1-\in} +\ n_c\ p_c^{1-\in}} \right] \tag{8}$$

where $\in = \frac{1}{1-\alpha}$. In expressions (7) and (8), as in Martin and Ottaviano (1999), $\delta = \tau^{1-\in}$ $(0 \leq \delta \leq 1)$ is a measure of trade openness in the global economy with respect to manufactures.

Concerning firms, they maximize profits at any period s taking into account a demand of the type (7) or (8) and the simple production function described above. As a result, both utility and profit maximization from expressions (3), (7) and (8) result in a common optimal price for all industrial firms in location k, which is a constant mark-up over marginal costs:

$$p_k = \frac{w_k}{\alpha}, \text{ for } k = \text{ West, Core.} \tag{9}$$

Then, from (9), per-period operating profits for any manufacturing firm in location k are

$$\pi_k = \left(\frac{1-\alpha}{\alpha}\right) w_k x_k \text{ for } k = \text{West, Core} \tag{10}$$

On the other hand, we assume that the wage differential between West and Core is high enough for eastern imitators to quote the unconstrained optimal mark-up. Therefore, this wide-gap assumption will only be satisfied if the original manufacturer can not undercut the eastern firm without incurring losses, i.e. iff[5]

$$\frac{w_c}{\alpha}\tau \leq w_n \tag{11}$$

Given that the primary sector is characterized by perfect competition and free entry, the agricultural price is equal to the peripheral wage and per-firm operating profits are zero. We assume that international transaction costs for primary products remain unaltered. So, without loss of generality, we state that these costs are just nil. Taking all this into account,

$$p_a = w_a = \frac{(1-\mu)\,E}{L_a} \tag{12}$$

2.3 Dynamic optimization

Now we have to face the intertemporal allocation of expenditure and savings, not only to distribute consumption along the time horizon, but also to finance new startups in the West and the Core. In order to allocate expenditure and savings over time, any household k must choose (in every period s) a variation in its portfolio composition, buying or selling equity from eastern and western firms. During that process the household needs to keep in mind that (in every period s) a fraction $m = \frac{\dot{n}_c}{n_n}$ of the western measure of varieties is copied by eastern imitators, which implies that the previous owners of those firms will lose their equity.

Let π_n and π_c denote the current operating profits of any western and eastern industrial firm, respectively. At every period s, a representative household from location k owns a measure $\beta_{nk}(s)$ of western firms and $\beta_{ck}(s)$ of eastern firms. Moreover, f_{nk} stands for the proportion of gross savings devoted to buying western equity. We will explore the properties of an interior equilibrium in which new startups from both countries are financed (i.e. $0 < f_{nk} < 1$).

Our control variables are E_k (household's expenditure) and $f_{nk}(s)$, whereas the state variables are $\beta_{nk}(s)$ and $\beta_{ck}(s)$. Then, the present-value Hamiltonian faced by any household in location k

[5] This assumption is useful to rule out strategic behavior in the pricing decisions of western and eastern firms.

at time t for the period s is the following:

$$H_k(s) = e^{-\rho(s-t)}.\log E_k(s) + \Phi_{nk}(s)\left[\frac{(w_k + \beta_{nk}\pi_n + \beta_{ck}\pi_c - E_k) f_{nk}(s)}{v_n} - m\beta_{nk}\right] + \quad (13)$$

$$+\Phi_{ck}(s)\left[\frac{(w_k + \beta_{nk}\pi_n + \beta_{ck}\pi_c - E_k)(1 - f_{nk}(s))}{v_c}\right] \quad (14)$$

The first-order condition for an interior solution for $f_{nk}(s)$ is the following:

$$\frac{\Phi_{nk}(s)}{v_n(s)} = \frac{\Phi_{ck}(s)}{v_c(s)},\ \forall s \quad (15)$$

The first-order condition with respect to $E_k(s)$ yields, due to equation (15), that

$$e^{-\rho(s-t)}\frac{1}{E_k(s)} = \frac{\Phi_{nk}(s)}{v_n(s)} = \frac{\Phi_{ck}(s)}{v_c(s)},\ \forall s \quad (16)$$

And therefore, by differentiating and using the first-order conditions with respect to the state variables,

$$\frac{\dot{E}}{E} = \frac{\dot{E}_c}{E_c} = \frac{\dot{E}_n}{E_n} = \frac{\dot{E}_a}{E_a} = \frac{\pi_n}{v_n} - m - \rho + \frac{\dot{v}_n}{v_n} = \frac{\pi_c}{v_c} - \rho + \frac{\dot{v}_c}{v_c} \quad (17)$$

The last expression shows how, in equilibrium, the profitability of western and eastern manufacturing firms must satisfy an arbitrage condition period by period.

2.4 Description of dynamic equilibrium without migration

2.4.1 System of differential equations

Now, by grouping terms, we can define $A = \frac{E}{nv_n}$ and $B = \frac{E}{n_c v_c}$. To characterize a dynamical system in A, B and $c = \frac{n}{n_c}$, we need to know first the dynamic behavior of the measures of manufacturing varieties, n_c and n. We will follow the evolution of the aggregate measure of manufactures in the core and the global economy $(\frac{\dot{n}_c}{n_c}, \frac{\dot{n}}{n})$ by looking at the labor-market-clearing conditions. These equilibrium conditions in the core and the north can be specified considering the available production function and the technology in the imitation and innovation processes:

$$L_c = a_m\frac{\dot{n}_c}{n_c} + n_c x_c \quad (18)$$

$$L_n = a\frac{\dot{n}}{n} + n_n x_n \quad (19)$$

On the other hand, the system describes the dynamics of A, B and c, but the separate evolutions of E, v_c and v_n can not be disentangled. As a consequence, Grossman and Helpman have one degree of freedom to normalize

$$E(t) = 1,\ \forall t \quad (20)$$

which implies (by equation (4)) that

$$A = \frac{1}{aw_n};\ B = \frac{1}{a_m w_c} \tag{21}$$

Instead of A and B, we will be interested in the evolution of the local nominal wages w_n and w_c. Therefore, using (7), (8), (10), (17), (18), (19) and (21), we are ready to set up the complete system of differential equations in w_n, w_c and c (when trade openness is almost perfect, i.e. when $\delta \to 1^-$) as follows:

$$\begin{aligned} \frac{\dot{w}_n}{w_n} &= \left[\frac{-(1-\alpha)}{a_m}\left(\frac{w_n^{\in-1}}{w_n^{\in-1}+(c-1)w_c^{\in-1}}\right)+\rho\right]+\left[\frac{L_n}{a}-\frac{\alpha(c-1)}{aw_n}\left(\frac{w_c^{\in-1}}{w_n^{\in-1}+(c-1)w_c^{\in-1}}\right)\right] \\ \frac{\dot{w}_c}{w_c} &= \rho+\left[\frac{L_c}{a_m}-\frac{1}{a_m}\left(\frac{w_n^{\in-1}}{w_n^{\in-1}+(c-1)w_c^{\in-1}}\right)\right] \\ \frac{\dot{c}}{c} &= \left[\frac{L_n}{a}-\frac{\alpha(c-1)}{aw_n}\left(\frac{w_c^{\in-1}}{w_n^{\in-1}+(c-1)w_c^{\in-1}}\right)\right]-\left[\frac{L_c}{a_m}-\frac{\alpha}{a_m}\left(\frac{w_n^{\in-1}}{w_n^{\in-1}+(c-1)w_c^{\in-1}}\right)\right] \end{aligned} \tag{22}$$

We will try to provide some intuition for the previous system of differential equations. Let's begin focusing on the first equation: the first term in square brackets contains the increase in nominal wages due to expenditure. Expenditure will raise more nominal wages the higher is the discount rate (ρ). But we also have another negative term ($\frac{-(1-\alpha)}{a_m}\left(\frac{w_n^{\in-1}}{w_n^{\in-1}+(c-1)w_c^{\in-1}}\right)$) next to the discount rate, and we can see that its absolute value is decreasing in $c = \frac{n}{n_c}$. That is, expenditure will make western wages grow more the higher is c, i.e. the lower is the imitation-potential of the Core. This happens because when the imitation potential is very low (c is very high), the expected life of a western patent is high and the profits offered by western firms in a given period are consequently low. This encourages people to spend (instead of saving and investing in western startups), which tends to increase nominal wages.

Let's have a look now at the second term (in square brackets) of the first equation. Its interpretation is much more straightforward: western wages will increase more the higher is innovation ($\frac{\dot{n}}{n}$), since higher innovation entails more demand for labor in the West. And innovation will be faster the more researchers (and the less manufacturing workers) you can find in the West. Since a higher imitation potential (a lower c) curtails the western demand for manufacturing workers, innovation (and wages) in the West will tend to go up the lower is c.

Therefore, in the first equation we can see that any variation in the eastern imitation potential (i.e. in c) has two opposite effects on western wages. On the one hand, as the expected life of a patent is shortened by more imitation, people receive higher annual profits and therefore save more (and spend less), reducing the growth rate of western wages. On the other hand, more imitation makes western workers shift to research (rather than manufacturing), which raises innovation and spurs future demand for labor in the West, raising the growth rate of nominal wages there.

The other two equations are easier to interpret. The second just tells us that wages in the Core will grow more the higher is ($\frac{\dot{n}_c}{n_c}$), since demand for labor will increase there. The third equation obviously reflects that c is increasing in the speed of innovation relative to the speed of imitation.

2.4.2 Innovation and imitation in steady state

If we prove that there are some values $c^*, w_n{}^*$ and $w_c{}^*$ for which $\dot{w}_n = \dot{w}_c = \dot{c} = 0$, this will imply that there exists a steady state for our system of differential equations established in (22). From the second differential equation in (22), in our candidate to steady state

$$\frac{1}{a_m}\left(\frac{w_n^{\in-1}}{w_n^{\in-1} + (c-1)w_c^{\in-1}}\right) = \alpha\left(\frac{L_c}{a_m} + \rho\right) \tag{23}$$

and from the third equation, (18), (19) and (23) we get that

$$g = \frac{\dot{n}}{n} = \frac{\dot{n}_c}{n_c} = (1-\alpha)\frac{L_c}{a_m} - \alpha\rho > 0 \tag{24}$$

We can observe that our innovation growth rate is exclusively determined by the monopoly power, the discount rate and the imitation capacity of the Core. It may look rather odd that the global growth rate does not depend on the innovative conditions in the West. In fact, this extreme result depends on the absence of international (West-East) knowledge spillovers. Once they are allowed in Grossman and Helpman (1991)'s model, it can be shown that both countries play a role in the determination of the steady-state growth rate: what matters is that such a rate is always increasing in the imitation capacity of the Core.

Therefore, from (4) and (24),

$$\frac{\dot{v}_c}{v_c} = \frac{\dot{v}_n}{v} = -g \tag{25}$$

This implies that the value of every firm shrinks in steady state at a constant rate. In other words, financial capital depreciates at the rate of innovation, and it is necessary to save to make up for that depreciation period by period. Now, from equations (20), (25) and also the arbitrage condition (17), we are ready to obtain reduced-form equations for the profits of any western and eastern industrial firm:

$$\pi_n = (\rho + m + g)\,v_n;\ \ \pi_c = (\rho + g)\,v_c \tag{26}$$

It is useful, as Grossman and Helpman do, to express c as a function of m and g, where $m = \frac{\dot{n}_c}{n_n}$ is our imitation rate. Since $m = g\frac{1}{(c-1)}$, we can solve now for c:

$$c = \frac{m+g}{m} \tag{27}$$

As a consequence, from (4), (10), (19), (26) and (27), we can restate the arbitrage condition corresponding to western manufactures as follows:

$$\frac{\pi_n}{v_n} = \left(\frac{1-\alpha}{\alpha}\right)\left[\frac{L_n}{a} - g\right]\left(\frac{m+g}{g}\right) = \rho + m + g \tag{28}$$

By combining (24) and (28), we can already derive a formal expression for the steady-state imitation rate m:

$$m = \left\{ \begin{array}{ll} 0, & \text{if } \frac{L_n}{a} \geq \frac{L_c}{a_m} \\ \frac{(1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]\left((1-\alpha)\frac{L_c}{a_m} - \alpha\rho\right)}{\alpha\rho - (1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]}, & \text{if } \frac{L_c}{a_m} \geq \frac{L_n}{a} \geq \frac{L_c}{a_m} - \frac{\alpha\rho}{1-\alpha} \\ \infty, & \text{if } \frac{L_n}{a} \leq \frac{L_c}{a_m} - \frac{\alpha\rho}{1-\alpha} \end{array} \right\} \tag{29}$$

As could be expected, m rises with the imitation potential of the Core relative to the western innovation capacity: $\left(\frac{L_c}{a_m} - \frac{L_n}{a}\right)$. We can already establish a first set of parameter restrictions so that the global economy exhibits a positive innovation rate and a positive measure of manufactures operate in both countries. That is, we want that $1 < c < \infty$, which requires $0 < m < \infty$ and $0 < g < \infty$. As we prove in the Appendix, this initial condition can be simply summarized as follows:

$$0 < \left[\frac{L_c}{a_m} - \frac{L_n}{a}\right] < \frac{\alpha\rho}{1-\alpha} \tag{30}$$

2.4.3 Absolute and relative wages in steady state

Now we will see how steady-state relative wages change in response to a fall in trade costs. But there are still several endogenous variables to be determined that are crucial for our comparative statics. Two of them are the relative wage of the Core with respect to the West ($\omega = \frac{w_c}{w_n}$) and γ. From equations (7), (8), (9), (18), (19) and (20), we can get an idea of the determinants of ω as follows:

$$\frac{x_n}{x_c} = \frac{L_n - ag}{L_c - a_m g}\frac{m}{g} = \omega^{\in} C(\delta, L_c, \omega) \tag{31}$$

$$\text{where } C(\delta, L_c, \omega) = \left[\frac{\frac{\gamma}{(g/m)\omega^{\in-1}+\delta} + \frac{(1-\gamma)\delta}{\delta(g/m)\omega^{\in-1}+1}}{\frac{\gamma\delta}{(g/m)\omega^{\in-1}+\delta} + \frac{(1-\gamma)}{\delta(g/m)\omega^{\in-1}+1}}\right] \tag{32}$$

We can see from the left-hand side of (31) that only the supply-side fundamentals - i.e. industrial workforces in both countries and innovation and imitation long-term capacities - can modify the relative size of firms ($\frac{x_n}{x_c}$). That means that any variation in international trade openness (δ) will be exactly offset in the long run by a countervailing adjustment of ω.

Our term $C(\delta, L_c, \omega)$ is a direct measure of the home-market advantage of one of the countries to offer higher wages for similar supply-side fundamentals. The country with a higher demand capacity (i.e. the West if $\gamma > 1/2$) will be able to reward better the labor force, since less demand will be wasted paying transaction costs there. Before we explore the relative-wage consequences of a rise in δ, we need to express γ in terms of the parameters for a steady-state situation. Next lemma will be of considerable help.

Lemma 1:

In any steady state without net migratory flows, any household's expenditure is identical to that household's income period by period. Therefore, the steady-state aggregate western and eastern incomes are equal to γ and $1-\gamma$, respectively, and there are no net savings.

Proof. See Appendix. ■

Subsequently, let's derive some formal expressions of western and eastern aggregate income. An implication of the last lemma is that a household's gross savings in steady state just cover the depreciation of previously-owned capital. Therefore, a representative household from location k will have an income

$$y_k = E_k = w_k + \rho\eta_{ck}a_m w_c + \rho\eta_{nk}a\left(\frac{g}{m+g}\right)w_n$$

where η_{ck} and η_{nk} denote the fraction of eastern and western firms, respectively, owned by that household. From (12), (20) and our definition of γ it is possible to come out with a neat expression of this variable as a fraction between zero and one:

$$\gamma = \frac{1}{1+\frac{\left(\frac{1-\mu}{w_n}\right)+\ \omega L_c\ +\rho\left[(1-\eta_{nn}L_n)a\left(\frac{g}{m+g}\right)+(1-\eta_{cn}L_n)a_m\omega\right]}{L_n\left[1+\rho\left(\eta_{nn}a\left(\frac{g}{m+g}\right)+\eta_{cn}a_m\omega\right)\right]}} \tag{33}$$

In the denominator of (33), w_n is an endogenous variable that has not been fully specified yet in terms of the parameters. So, we need to obtain an expression for local absolute wages as well. Let's define first

$$Q = \frac{m}{g}\omega^{1-\epsilon} \tag{34}$$

Now, if we plug (7) into (19), divide numerator and denominator of the latter expression by $\left(\frac{w_n}{\alpha}\right)^{1-\epsilon}$ and rearrange, eventually we find that

$$w_n = \frac{\alpha\mu}{(L_n - ag)}\left[\frac{\gamma}{1+\delta Q} + \frac{(1-\gamma)\,\delta}{\delta + Q}\right] \tag{35}$$

Proceeding in a similar way, we can solve for w_c from (18) as follows:

$$w_c = \frac{\mu}{(L_c + a_m\rho)}\left[\frac{\gamma\delta}{1+\delta Q} + \frac{(1-\gamma)}{\delta + Q}\right]Q \tag{36}$$

In the next section we derive a necessary and sufficient condition for an increase in ω in response to a marginal rise in trade openness (δ).

2.4.4 Comparative Statics

Proposition 1:

Concerning the distribution of financial wealth, assume that

$$\eta_{nn}L_n \to 1^-;\ \eta_{cn}L_n \to 0^+; \eta_{cc} = \eta_{ca} = 1/L_s \tag{37}$$

where η_{kl} is equal to the proportion of aggregate wealth from location k owned by any household living in location l. In that case, when the imitation potential of the Core is sufficiently small, the relative wage of the Core with respect to the West (ω) rises in response to higher trade openness if - and only if - the initial degree of trade openness is high enough, i.e. $\lim_{L_c \to a_m \frac{L_n}{a}^+} \left(\frac{d\omega}{d\delta}\right) > 0$ iff $\delta^2 > \frac{1-\mu}{\mu}$

Proof. See the Appendix. ■

There are two opposite effects of a reduction of international transaction costs on the relative wage ω. The first one has to do with the difference in aggregate income between East and West: a wealthier West will be likely to raise its demand for every eastern manufacture beyond the increase in aggregate eastern demand for any western good. This would result in a rise of ω (and international convergence[6]) if there were no other active forces. Let's call this the relative-size effect.

But there is still another effect. Since most of the industrial varieties are initially produced in the West, toughness of competition increases much more for the smaller market in the East (a firm suddenly faces many more competitors there as δ falls), which tends to depress ω and generate divergence. The strength of this price-index effect decreases with the initial degree of trade openness (δ), since higher values of δ imply that local price indices are almost identical to start with (i.e. the international market is almost fully open from the beginning). This means that when the initial level of trade costs is already very low, the demand flow is relatively more important, and convergence prevails.[7]

Therefore, for $\frac{d\omega}{d\delta}$ to be positive we do not only need a large differential in the size of both countries, but also a high enough initial value of δ. Under the assumptions of Proposition 1, a very high relative-size effect has been guaranteed (since the imitation capacity of the Core is infinitesimal), which makes the initial level of trade openness the only determinant of the evolution of relative wages.[8] In this respect, this proposition may shed some light on the determinants of protectionist policies: they may be more likely to arise in small countries when the current level of trade openness is low enough.

But we would like to know what happens to relative incomes also out of this extreme situation, i.e.

[6] We will talk about convergence in this paper when there exists convergence in *nominal* income, instead of real income or indirect utility. The reason why we adopted such an arbitrary convention is that nominal convergence is usually the aspect detected by national accounts, given the difficulty to access good local price-indices.

[7] The distributional assumptions we make in Proposition 1 are just technical (simplifying) assumptions. We can prove that it would be possible to distribute all world (financial) wealth in a strictly egalitarian way and the main result would not be affected. Furthermore, by making the Core's initial imitation capacity infinitesimal we guarantee the relative-size effect, and make the price-index effect the only relevant force in the comparative statics.

[8] We have proved that the distributional assumptions could be relaxed while preserving our main result.

for any initial distribution of eastern population between Core and Periphery. Our next objective will be obtaining the function $\omega_c = \frac{w_c}{w_a} = f(L_a, \delta)$ that determines the labor-market-clearing relative wage in the East as a function of L_a and δ. The intersection of this curve with an exogenous migration function $\omega_c = h(L_a)$, which yields the amount of people willing to live in the Periphery as a function of the relative wage, will offer the final-steady-state values $(L^*{}_a(\delta), \omega_c{}^*(\delta))$.

3 The model with migration

In this model, the introduction of migratory movements is the only way to strengthen the catch-up potential of the Core and henceforth increase the steady-state growth rate. Why are innovation and imitation complementary in this model? The answer is twofold:

- Firstly, as the imitation potential rises, the demand for manufacturing labor in the West goes down and then a higher proportion of the western population is devoted to research.

- Secondly, as the imitation potential increases, the expected life of a western patent shortens, which forces western firms to offer higher profits (given the arbitrage condition) and encourages more saving and investment in new startups.

And how can we get Periphery-Core migrations in the first place? We claim that such a Periphery-Core migration will arise if the Core is initially favored by trade shocks. These results can be related, for example, to the Chinese experience: Solinger (1995) and Poncet (2006) document how - despite severe migration restrictions imposed by the government - the amount of "floating population" undertaking rural-urban migration could reach 150 million people, and they are driven mostly by economic motivations in the destination area. "Officials say that by 2020 about 60% of population will be living in cities or towns, which implies that more than 200 million new people will move from the countryside by then" (The Economist, page 29). For those people, migration obviously has a cost. But this paper tries to shed some light on the static and dynamic gains for those migrants, for the whole Chinese population and for the rest of the world. Let´s now face the foundations of the migration decision.

3.1 Migration

In this subsection we draw partially from Faini (1996) to obtain a microfoundation for the migration function ($\omega_c = h(L_a)$).

Since we assumed away internal trade costs within the East, the price indices in both Core and

Periphery will be identical. Therefore, a comparison of local real incomes reduces to a comparison of local nominal incomes. We will also assume that the utility derived from a given income in the Core is lower than that in the Periphery, which may be due to congestion effects or undesirable living conditions in an industrial location. That asymmetry will be summarized by the parameter θ ($\frac{1}{1+\frac{L_s}{\rho a_m}} \leq \theta \leq 1$).

We are going to assume some degree of heterogeneity in the eastern population with respect to their willingness to live in the Core (summarized by θ_i, where θ_i measures the willingness of individual i to live in the Core). That heterogeneity will show in a certain statistical distribution of parameter θ among the Chinese people: in particular, it will be assumed that θ follows a uniform distribution $U[\frac{1}{1+\frac{L_s}{\rho a_m}}, 1]$

Since we will not consider migration costs, the individual who is indifferent between living in the Periphery or in Core for a given ratio of incomes will be implicitly characterized by the expression

$$\bar{\theta}(w_c + \frac{1}{L_s}\rho a_m w_c) = w_a + \frac{1}{L_s}\rho a_m w_c$$

where $\frac{1}{L_s}\rho a_m w_c$ is the net financial income received by any Chinese individual, and $\bar{\theta}$ represents the willingness of the last individual to move to the Core at the current real wages. Rearranging, we can rewrite the previous expression as

$$\bar{\theta} = \frac{1 + \rho a_m \omega_c \frac{1}{L_s}}{\omega_c \left(1 + \rho a_m \omega_c \frac{1}{L_s}\right)}$$

And the amount of population living in the Core will be given, after some algebra, by

$$L_c = L_s P\left[\theta \geq \bar{\theta}\right] = \frac{1 - \frac{1+\rho a_m \frac{\omega_c}{L_s}}{\omega_c\left(1+\rho a_m \frac{\omega_c}{L_s}\right)}}{\frac{L_s}{L_s+\rho a_m}} = \left(1 - \frac{1}{\omega_c}\right) L_s$$

i.e. the previous expression can be rewritten as

$$\omega_c = h(L_a) = \frac{L_s}{L_a} \tag{38}$$

This is a decreasing and convex function in L_a, which shows the steady-state amount of eastern population willing to live in the Periphery for a given relative wage. In the next section we will spell out the intuition and details of our main results.

3.2 Description of dynamic equilibrium with migration

Since we want to reproduce some stylized facts, it is convenient for us to rule out any price-index effect threatening to abort East-West convergence. Therefore, trade costs should be initially low enough to turn demand flows into the main result of an incremental openness. Then, the relative-size effect will remain as the single driving force. Therefore, $\gamma > 1/2$ appears as a natural requirement that (together with $\delta \longrightarrow 1^-$) could be enough to achieve international convergence in per-capita income. But let's provide first a sufficient condition for $\gamma > 1/2$ in terms of the parameters.

Lemma 2 :

Given our distributional assumptions in Proposition 1, $\lim_{\delta \longrightarrow 1^-}(\gamma) > 1/2$ if $L_n > \hat{L}_n(L_c)$, where $\hat{L}_n(L_c)$ is a monotone increasing function.

Proof. See Appendix. ■

As we can see, it turns out that international differences in aggregate income amount to a difference in the size of populations. The larger is the size of L_n relative to L_c,the larger will be the innovative capacity of the West relative to the imitation potential of the East. This implies that a larger proportion of the global array of manufacturing varieties will be produced in the West, raising the western real wage relative to the eastern one.

The wide-gap assumption made explicit in (11) involves that $\lim_{\delta \longrightarrow 1^-}(\omega) < \alpha$, from which we can also derive the following lemma.

Lemma 3:

There exists a unique upper-bound $L_c^* \geq L_c$ such that the wide-gap assumption holds together with the coexistence of a positive measure of western and eastern manufactures; i.e. there exists a unique L_c^* such that (11) and (30) are simultaneously satisfied iff

$$a_m \frac{L_n}{a} < L_c \leq L_c^* < a_m \left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha} \right] \forall a, a_m$$

Proof. See Appendix. ■

That is, for the Core's producers to be able to quote the unconstrained mark-up over marginal cost, it is necessary that the Core's population - which determines its imitation potential - is small enough relative to the western one: otherwise, the nominal wage in the Core would be too high and the western firms would find it profitable to undercut.

Our notion of steady state is partially characterized by the following equality:

$$\omega_c = f(L_a, \delta) = h(L_a) \tag{39}$$

where $\omega_c = h(L_a)$ is our migration function.

Now we will endogenously determine the curve $\omega_c = f(L_a, \delta)$. From (12), (24) and (36) we can obtain that

$$\lim_{\delta\to 1^-} \omega_c = \lim_{\delta\to 1^-} f(L_a,\delta) = \lim_{\delta\to 1^-} \left[\frac{L_a}{(1-\mu)} \frac{\mu\left(\frac{Q(L_a,\delta)}{1+Q(L_a,\delta)}\right)}{(L_s - L_a + a_m\rho)} \right] \tag{40}$$

where

$$\lim_{\delta\to 1^-} Q(L_a,\delta) = \left[\frac{(1-\alpha)\left[\frac{L_s-L_a}{a_m} - \frac{L_n}{a}\right]}{\alpha\rho - (1-\alpha)\left[\frac{L_s-L_a}{a_m} - \frac{L_n}{a}\right]} \right]^{1-\alpha} \left[\frac{\alpha(L_s - L_a + a_m\rho)}{L_n - \frac{a(1-\alpha)}{a_m}(L_s - L_a) + a\alpha\rho} \right]^{\alpha} \tag{41}$$

Here we can appreciate the two basic effects of a declining peripheral labor force ($\downarrow L_a$) on ω_c:

- First, the numerator and denominator of (40) directly capture the straightforward *labor-supply effect*: if new immigrants come from Periphery to Core, ω_c will tend to decrease for a given value of Q.

- Secondly, the quotient $\frac{Q(L_a,\delta)}{1+Q(L_a,\delta)}$ is decreasing in L_a because it reflects the gain in *imitation potential* of the Core after an inflow of former peripheral workers. This force tends to increase the fraction of the total measure of manufactures produced in the Core, which channels world demand to this location and can potentially raise w_c.

The relative strength of these two effects varies along the relevant range of values of L_a : $\left[L_s - a_m\frac{L_n}{a},\ L_s - L_c^*\right]$. In fact, $Q(L_a,\delta)$ acts as a positive measure of the imitation potential in the core. Moreover, additional migration reinforces much more that potential the lower $Q(L_a,\delta)$ is. In other words, once you have copied a high proportion of western varieties, it is harder for you to raise your local wage by further imitating: you have to compete - every time more toughly - with more and more producers in your own location.

In fact, since by (40) $f(0,\delta) = f\left(L_s - a_m\frac{L_n}{a}, \delta\right) = 0 \ \ \forall\delta$ and our function f is continuous in L_a,we know for sure that $f(L_a,\delta)$ shows an inverted-U shape $\forall\delta$. That is, we can observe both an upward-sloping part of the curve - where the labor-supply effect is stronger - and a downward-sloping one, with a dominant imitation-potential effect[9](see Figure 1).

In Figures 1 and 2 the horizontal axis measures the amount of population in the Periphery (L_a), and the vertical axis represents the relative wage of the Core with respect to the Periphery (ω_c). We can observe in Figure 1 how - due to the coexistence of a labor-supply and an imitation-potential effect with opposite effects on ω_c - the curve $f(L_a,\delta)$ has both an upward-sloping and a downward-sloping region.

In order to draw our arrows of motion, we have assumed that agents form their expectations in

[9]Provided that the whole range of values of La satisfies the wide-gap assumption, i.e. if $\mathrm{L}_s - a_m\frac{L_n}{a} < L_c^*$.

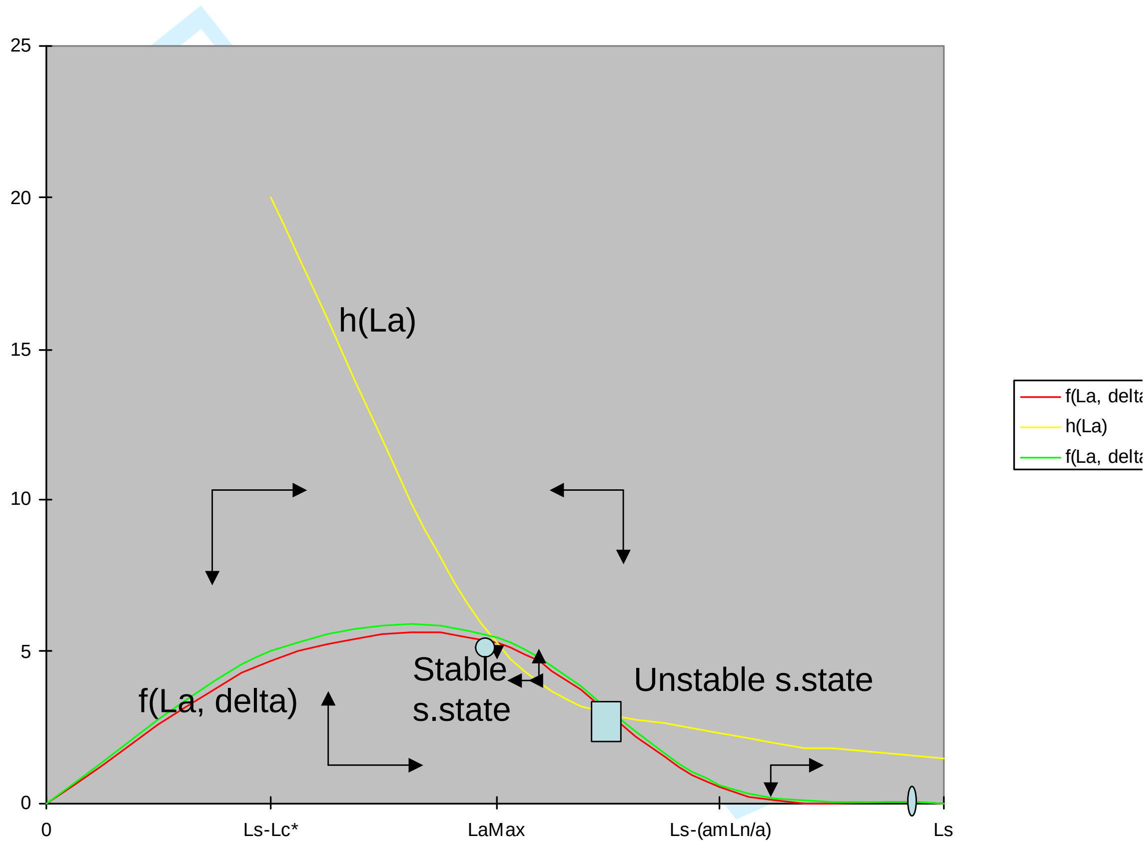

Figure 1: Effect of trade liberalization

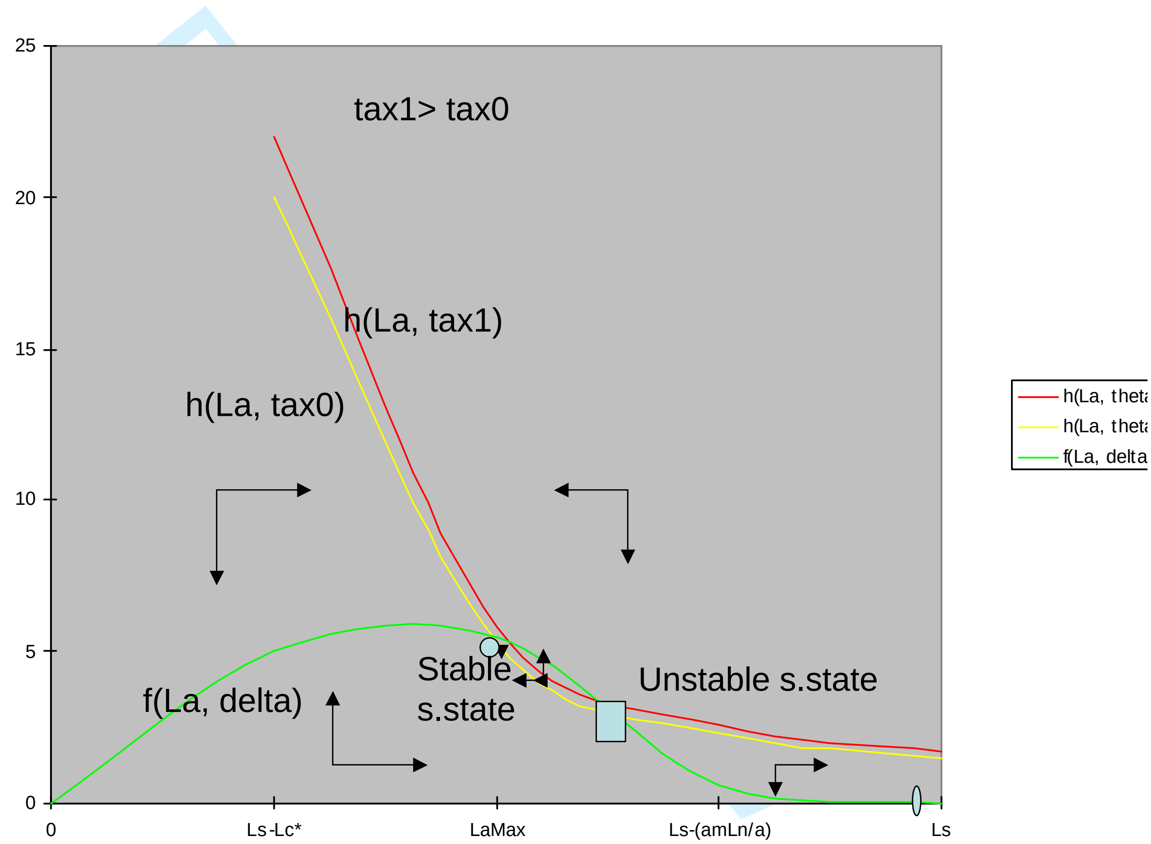

Figure 2: Effect of Core-Periphery redistribution

an *adaptive* way, as is usual in economic geography. This means that, when deciding how to allocate their financial capital, individuals take local populations as given and do not expect them to change; by the same token, when making their migratory choice, eastern individuals do not expect relative wages to vary at all (even when the economy is out of the steady state). As a result of this, we can see that the system has two stable steady states and an intermediate, saddle-path unstable one.

The first stable steady state concentrates all eastern population in the Periphery and there is no manufacturing activity at all within the East ($L_a = L_s$). It is necessary to have a critical mass of population (and researchers) in the Core to channel a sufficiently high share of world demand towards that location and raise the local wage, which will subsequently attract more population to repeat the cycle. If such a critical mass is achieved, the system will *evolve naturally* towards the second stable steady state, characterized by $\omega_c = f(L_a, \delta) = h(L_a)$ and also by $\frac{\partial h}{\partial L_a} < \frac{\partial f}{\partial L_a} < 0$.

In order to replicate our stylized facts, we will assume that the economy is initially situated in the second steady state (marked with a blue circle in Figure 1) and it receives a trade shock that will shift the $f(L_a, \delta)$-curve upwards, as represented by the transit from the red curve to the green curve in Figure 1. That is, the main characteristics of our relevant steady state are significant agglomeration effects on the Core's labor productivity and a considerable labor stickiness within the East.

3.3 Main results

As anticipated above, the main results we need to reproduce are international (East-West) convergence in per capita income, interregional (Core-Periphery) divergence within the East, higher concentration of labor in the Core and higher (global) growth rates. It may look counterintuitive the coexistence of potential Core-Periphery migrations and interregional divergence within the East. The reason why both phenomena coexist is that interregional divergence is obtained in terms of real income, but not in terms of utility. In the section about the derivation of migration functions, we assumed that people suffer from a congestion disutility in the Core (i.e. the marginal utility of a given income in the Core is lower than in the Periphery), and the parameter measuring congestion disutility follows a probability distribution. As more people move to the Core, the congestion disutility of the last mover becomes higher, and the ratio of Core/Periphery incomes is also higher than at the beginning.

Now we will obtain a sufficient condition for the ratio $R_{ca} = \frac{\text{per-capita income in the core}}{\text{per-capita income in periphery}}$ to increase in response to a marginal rise in δ.

Proposition 2:

Let $R_{ca} = \frac{Y_c/(L_s - L_a)}{Y_a/L_a}$ be the core-periphery relative per-capita income. If in the original steady state the following conditions are satisfied: a) $L_n > \hat{L}_n(L_s)$; b) $\frac{a_m L_n}{a} < L_c < L_c^*$; c) $\delta \to 1^-$; then :

$$\frac{dR_{ca}}{d\delta} > 0; \ \frac{d\omega_c}{d\delta} > 0; \frac{dg}{d\delta} > 0; \frac{dL_a}{d\delta} < 0$$

Proof. See Appendix. ■

With a sudden rise in δ, the dominance of the relative-size effect - when we are close to full openness - weakens the home-market advantage of the West. The subsequent rise in ω_c attracts a net migratory flow from Periphery to Core and increases our eastern imitation potential. Hence, the decrease in c caused by migrations channels more world demand towards eastern manufactures and exerts an upward pressure on the labor costs in the Core. This force countervails the labor-supply effect, which usually happens when industrial competition within the core is soft enough and eastern labor force is sufficiently sticky.

Given the significant agglomeration effects on labor productivity detected in the EU by Ciccone (2002) and in China by Au and Henderson (2006), accepting that $\frac{\partial f}{\partial L_a} < 0$ (i.e. that we are on the downward-sloping part of the function $f(L_a, \delta)$) does not look counterfactual. Neither does the extreme (interregional) stickiness of labor in many European and Asian countries (see Bentolila (1997); Fujita and Hu (2001)).

Let's try to face now the East-West convergence issue in a similar fashion.

Proposition 3:

If in our initial steady state $a_m \frac{L_n}{a} \leq L_c \leq L_c^*$, $L_n > \hat{L}_n(L_s)$ and $\delta \to 1^-$, then necessarily $\frac{dR_{ns}}{d\delta} < 0$, where R_{ns} is the relative per-capita income of the West with respect to the East.

Proof. See Appendix. ■

There are three forces involved in the comparative-statics evolution of relative East-West per-capita income, two of which exactly offset each other. These 2 opposite forces, whose joint effect is nil, can be described as follows:

- First, the net inflow of workers to the core enhances the innovation rate and, consequently, also the demand for labor in the West, which tends to raise w_n.

- At the same time, although the global economy innovates faster, a higher imitation potential raises the proportion of eastern manufactures. Hence, a lower proportion of total financial wealth owned by the West exactly makes up for the higher demand for researchers in that country. Therefore, the only effect capable of modifying γ comes from the aggregate demand for the manufactures produced in the West. This aggregate demand goes down in terms of our numeraire, since the western home-market advantage becomes weaker.

Corollary:

If in our initial steady state $a_m \frac{L_n}{a} \leq L_c \leq L_c^* < a_m \left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha} \right]$, $L_n > \hat{L}_n (L_s)$ and $\delta \to 1^-$, then, in our comparative-statics exercise

$$\frac{dR_{ca}}{d\delta} > 0;\ \frac{d\omega_c}{d\delta} > 0; \frac{dg}{d\delta} > 0; \frac{dL_a}{d\delta} < 0; \frac{d\gamma}{d\delta} < 0; \frac{d\omega_c}{d\delta} > 0 \text{ and } \frac{d\omega_a}{d\delta} > 0 \text{ (where } \omega_a = \frac{w_a}{w_n}) \ .$$

Proof. Straightforward from (12), (20) and the last 2 propositions. ■

It is remarkable that - in our framework - a decrease in international trade costs could be *potentially Pareto-improving*. This is true because both eastern locations unambiguously gain in terms of steady-state indirect utility; and although the western per-capita (nominal) income falls, that effect could be offset by the higher growth rate for a low enough discount rate (ρ).

Furthermore, Figure 2 shows how an intensification of a Core-Periphery income-redistribution policy within the East could reduce peripheral wages, wages in the Core (due to the foregone agglomeration effects) and the growth rate of the global economy[10] (for an analytical derivation of this result, see the Appendix). Nevertheless, structural changes in the Periphery - even if financed with transfers - could also enlarge the scale effects within the East and yield both convergence within China and higher global growth rates. However, this model does not lend itself to the study of public investment (there are no public goods) and structural change, so we can not assess quantitatively the relative virtue of promoting migration versus restructuring in the Periphery.[11]

4 Conclusions

We have studied an East-West endogenous growth model where exogenous institutional features play a major role: they determine the relative incidence of a biased shock in trade openness on two distinct eastern regions. Within our eastern country, we have considered a perfectly-competitive

[10] Therefore, the *Hukou* system may be having deleterious economic effects over China and even over the rest of the world, although its implementation could make sense from a political-economy point of view (see Solinger (1995)).

[11] De la Fuente (2004) creates a framework to study the optimal central-planner allocation of public investment among regions, for a given degree of income redistribution that can not be extended. When calibrated for the case of Spain, he finds that the allocation of public investment has probably been too redistributive. His model is essentially static and does not consider pecuniary externalities across locations or induced modifications in the local populations, as we do. If his model considered all these effects - according to our framework - we presume that the case for redistribution through public investment in Spain would be even weaker (we can not forget that this conclusion depends on the maintenance of a given interregional solidarity through income-redistribution programmes).

market structure for the Periphery together with some sources of agglomeration economies in the Core. As a result, we have reproduced our stylized facts, i.e. the coexistence of per-capita income convergence between countries and divergence within the same countries. The existence of scale effects generates a trade-off between Core-Periphery convergence and global steady-state growth. But not necessarily a trade-off between long-run growth rates and East-West convergence.

Our model has potentially interesting implications for the role of interregional transfers. In particular, we conclude that, no matter how generous interregional transfers are, if they do not help transform peripheral productive structures they can not prevent an asymmetric exposure to trade shocks. If transfers also refrained migratory flows, they could reduce the core-periphery gap, though only by lowering all easterners' labor income and the growth rate of the global economy.

On the other hand, if transfers were useful to industrialize the Periphery the scale-effects would be larger. In fact, this seems to be the recent choice of the Chinese authorities, aiming to reconcile higher growth and Core-Periphery convergence by means of the setup of new economic infrastructure in the latter location. This looks like an argument to advocate structural changes in the Periphery as opposed to direct transfers to household consumption. But, in order to elaborate on this, we need to do some welfare analysis requiring transitional dynamics and an explicit formulation of both migratory costs and structural-change costs, since we need a different framework to assess the relative virtue of promoting migration versus structural change in the Periphery. This is an interesting avenue for future research.

5 References

Au, Ch. and Henderson, V. (2006). *Are Chinese cities too small?.* Review of Economic Studies 73, 549-576.

Backus, D.; Kehoe, P. and Kehoe, T. (1992). *In search of scale effects in trade and growth.* Journal of Economic Theory.

Banerjee, A. (2006). *FDI in China and its economic impact.* World Review of Entrepreneurship, Management and Sust. Development.

Bentolila, S. (1997). *Sticky labor in Spanish regions.* European Economic Review 41, 591-598.

Branstetter, L. and Feenstra, R. (1999). *Trade and Foreign Direct Investment in China: a Political Economy Approach.* NBER Working Paper No. 7100.

Carlino, J.; Chatterjee, S. and Hunt, R. (2006). *Urban density and the rate of invention.* WP. Federal Reserve Bank of Philadelphia.

Chunlai, Ch. (1997). *Provincial Characteristics and Foreign Direct Investment Location Decision within China.* Working Paper, University of Adelaide.

Chunlai, Ch. (1997b). *Foreign direct investment and trade: an empirical investigation of the evidence from China.* Working Paper, University of Adelaide.

Ciccone, A. and Hall, R. (1996). *Productivity and the density of economic activity.* American Economic Review.

Ciccone, A. (2002). *Agglomeration effects in Europe.* European Economic Review.

De la Fuente, A. (2004). *Second-best redistribution through public investment: a characterization, and empirical test and an application to the case of Spain.* Regional Science and Urban Economics.

Esteban, J.M. (1994). *La desigualdad interregional en Europa y en Espana. En Crecimiento y convergencia regional en Espana y Europa.* Instituto de Analisis Economico-CSIC, Barcelona.

Faini, R. (1996). *Increasing returns, migrations and convergence.* Journal of Development Economics 49, 121-136.

Fujita, M. and Hu, D. (2001). *Regional disparity in China 1985-1994: the effects of globalization and economic liberalization.* The Annals of Regional Science.

Giannetti, M. (2002). *The effects of integration on regional disparities: convergence, divergence or both?.* European Economic Review.

Gipouloux, F. (1998). *Integration or Disintegration? The Spatial effects of Foreign Direct Investment in China.* China Perspectives 17.

Grossman, G. and Helpman, E. (1991). *Innovation and Growth in the Global Economy.* MIT Press.

Hu, D. (2002). *Trade, rural-urban migration and regional income disparity in developing countries: a spatial general-equilibrium model inspired by the case of China.* Regional Science and Urban Economics.

Huang, J.; Kuo, Ch; and Kao, A. (2003). *The Inequality of Regional Economic Development in China between 1991 and 2001.* Journal of Chinese Economic and Business Studies.

Jian, T.; Sachs, J. and Warner, A. (1996). *Trends in regional inequality in China.* NBER

Working Paper 5412.

Jones, Ch. (1999). *Growth: with or without scale effects?* American Economic Review.

Kanbur, R. and Zhang, X. (2001). *Fifty years of regional inequality in China: a journey through revolution, reform and openness.* Mimeo.

Martin, P. and Ottaviano, G. (1999). *Growing locations: industry location in a model of endogenous growth. European Economic Review* 43, 281-302.

Poncet, S. (2006). *Provincial migration dynamics in China: borders, costs and economic motivations.* Regional Science and Urban Economics 36, 385-398.

Rauch, J. (1999). *Networks versus markets in international trade.* Journal of International Economics 48, 7-35.

Solinger, D. (1995). *China's Urban Transients in the Transition from Socialism and the Collapse of the Communist "Urban Public Goods Regime".* Comparative Politics, January 1995.

Whalley, J. and Zhang, S. (2004). *Inequality in China and (Hukou) Labour Mobility Restrictions.* NBER WP. 10683.

Woo, W. T. (2003). *The economic impact of China's emergence as a major trading nation.* Mimeo, Columbia University.

6 Appendix

6.1 Steady-state fraction of manufacturing varieties in the Core

By (22), (26) and our definition of steady state,

$$\left[1+\frac{1-\alpha}{\alpha}\frac{c}{(c-1)}\right]\left[\frac{L_n}{a}-\left((1-\alpha)\frac{L_c}{a_m}-\alpha\rho\right)\right]-\left[(1-\alpha)\frac{L_c}{a_m}-\alpha\rho\right]\frac{1}{(c-1)}=\rho+\frac{L_n}{a} \tag{42}$$

Finally, solving for c in (42) we can get that

$$c^{*}=\frac{\alpha\rho}{(1-\alpha)\left[\frac{L_c}{a_m}-\frac{L_n}{a}\right]} \tag{43}$$

The trivial fact that $n \geq n_c$, i.e. $c \geq 1$, imposes our restriction (30) on the value of the parameters.

6.2 Income-redistribution policy between Core and Periphery

We are going to introduce a proportional income tax accompanied by a lump-sum rebate for the Chinese population. As will be shown, this form of Core-Periphery redistribution will reduce the willingness of Chinese population to live in the Core. Now we can characterize the willingness of the last individual to move to the Core $(\bar{\theta})$ as follows:

$$\bar{\theta}\left[w_c\left(1+\rho a_m \frac{1}{L_s}\right)(1-\Gamma)+G\right]=\left(w_a+\rho a_m w_c \frac{1}{L_s}\right)(1-\Gamma)+G \tag{44}$$

where Γ measures the proportional income tax and G the corresponding lump-sum rebate.

The balanced-budget condition that links the values of τ and G can be expressed as

$$L_s G=\Gamma\left[w_c \rho a_m+w_c L_c+w_a L_a\right]$$

Solving for G and replacing the value of G in (44), we can obtain an expression for $\bar{\theta}$ such as

$$\bar{\theta}=\frac{L_s(1-\Gamma)+(\rho a_m+\Gamma L_c)\,\omega_c}{\omega_c\left[L_s(1-\Gamma)+\rho a_m+\Gamma L_c\right]}$$

It is finally easy to check that

$$\lim_{\tau\to 0}\frac{d\,\bar{\theta}}{d\Gamma}>0 \text{ iff } \omega_c>1$$

i.e. within the relevant range of values for ω_c, higher taxation implies an upward shift of the curve $h(L_a)$ and a lower steady-state population in the Core.

6.3 Proof of Proposition 1

Proof. Let's rewrite the second part of expression (31) as follows:

$$C(\delta, L_c, \omega)=\frac{\gamma\left(m\omega^{1-\in}+\delta g\right)+(1-\gamma)\,\delta\left(\delta m\omega^{1-\in}+g\right)}{\gamma\delta\left(m\omega^{1-\in}+\delta g\right)+(1-\gamma)\left(\delta m\omega^{1-\in}+g\right)} \tag{45}$$

After a marginal increase in δ, the right-hand side of (31) has to remain constant, because nothing is altered in the left-hand side of the equality. Therefore,

$$\lim_{L_c\to a_m\frac{L_n}{a}^{+}}\frac{(dC/d\delta)}{C}=-\lim_{L_c\to a_m\frac{L_n}{a}^{+}}\frac{\in\frac{d\omega}{d\delta}}{\omega} \tag{46}$$

Then, if we take logs of (45) and compute the total derivative, we can get that

$$\frac{(dC/d\delta)}{C}=\frac{\left(\gamma\delta^2-(1-\gamma)\right)}{\delta\left[1-\gamma\left(1-\delta^2\right)\right]}-$$

$$-\frac{\frac{d\omega}{d\delta}\left[Q\,(\in-1)\left(\delta^2-\left(\gamma+(1-\gamma)\,\delta^2\right)\left(1-\gamma\left(1-\delta^2\right)\right)\right)+\frac{d\gamma}{d\omega}\delta\left(1-\delta^2\right)\right]}{\delta\left[1-\gamma\left(1-\delta^2\right)\right]} \tag{47}$$

From (46) and (47),

$$\frac{d\omega}{d\delta}=\frac{\omega\left(\gamma\delta^2-(1-\gamma)\right)}{\in\delta\left(1-\gamma\left(1-\delta^2\right)\right)+(\in-1)\,Q\left[\delta^2-\left(\gamma+(1-\gamma)\,\delta^2\right)\left(1-\gamma\left(1-\delta^2\right)\right)\right]+\frac{d\gamma}{d\omega}\delta\omega\left(1-\delta^2\right)} \tag{48}$$

In order to determine the sign of $\lim_{L_c\to a_m\frac{L_n}{a}^+}\frac{\frac{d\omega}{d\delta}}{\omega}$, it is useful to know the limit-value of ω when $L_c\to a_m\frac{L_n}{a}^+$. From (31),

$$\lim_{L_c\to a_m\frac{L_n}{a}^+}\frac{L_n-ag}{L_c-a_m g}\frac{m}{g}=\left[\lim_{L_c\to a_m\frac{L_n}{a}^+}\omega^{\in}\right]\left[\lim_{L_c\to a_m\frac{L_n}{a}^+}C(\delta,L_c,\omega)\right] \tag{49}$$

Our parameter restriction (30) guarantees that g >0 and then, from (29), (45) and (49), $0=\left[\lim_{L_c\to a_m\frac{L_n}{a}^+}\omega^{\in}\right]\left[\frac{\delta}{1-\gamma(1-\delta^2)}\right]$. As we can infer from (33), $0<\frac{\delta}{1-\gamma(1-\delta^2)}<\infty$ provided that $\delta>0$. Then, as a consequence,

$$\lim_{L_c\to a_m\frac{L_n}{a}^+}\omega=0^+ \tag{50}$$

Moreover, since we can easily check that $\lim_{L_c\to a_m\frac{L_n}{a}^+}\left(\frac{d\gamma}{d\omega}\right)$ is finite, from (**??**), (34) and (50) it is possible to conclude that $\lim_{L_c\to a_m\frac{L_n}{a}^+}\left(\frac{d\gamma}{d\omega}\right)\omega=\lim_{L_c\to a_m\frac{L_n}{a}^+}(Q)=0$, and therefore, by (48),

$$\lim_{L_c\to a_m\frac{L_n}{a}^+}\frac{\frac{d\omega}{d\delta}}{\omega}=\frac{\gamma\delta^2-(1-\gamma)}{\in\delta\left(1-\gamma\left(1-\delta^2\right)\right)} \tag{51}$$

Since the denominator of (51) is positive,

$$lim_{L_c\to a_m\frac{L_n}{a}^+}\left(\frac{d\omega}{d\delta}\right)>0 \text{ iff } \gamma\delta^2>(1-\gamma) \tag{52}$$

Next, from (33) and (35) we can obtain that

$$\lim_{L_c\to a_m\frac{L_n}{a}^+}\frac{(1-\mu)}{w_n}=\frac{(1-\mu)}{\mu}(L_n+a\rho) \tag{53}$$

Now, if we plug (53) into (33), we can restate condition (52) only in terms of the parameters:

$$lim_{L_c\to a_m\frac{L_n}{a}^+}\left(\frac{d\omega}{d\delta}\right)>0\ iff\ \eta_{nn}>\frac{1}{\mu\left(1+\delta^2\right)}\left[\frac{\left(1-\mu\left(1+\delta^2\right)\right)}{a\rho}+\frac{1}{L_n}\right] \tag{54}$$

Finally, taking into account our assumptions in (37),

$$lim_{L_c\to a_m\frac{L_n}{a}^+}\left(\frac{d\omega}{d\delta}\right)>0\quad iff\quad \delta^2>\frac{1-\mu}{\mu}$$

■

6.4 Proof of Proposition 2

Proof. From our definition of R_{ca}, our distributional assumptions (37) and Lemma 1 we can derive that in any steady state

$$R_{ca}=\frac{\omega_c\left[(L_s-L_a)+\rho a_m\left(1-\eta_{ca}L_a\right)\right]}{\left(1+\rho\eta_{ca}a_m\omega_c\right)(L_s-L_a)} \tag{55}$$

From (39), any marginal variation in δ must yield the following migratory reaction between steady states:

$$\lim_{\delta\to 1^-}\frac{dL_a}{d\delta}=\lim_{\delta\to 1^-}\left[\frac{\frac{\partial f}{\partial\delta}}{\left(\frac{\partial h}{\partial L_a}-\frac{\partial f}{\partial L_a}\right)}\right] \tag{56}$$

The assumptions of the proposition guarantee that the denominator in (56) is negative. As to the numerator, from (31) and (45) we can obtain that

$$\lim_{\delta\to 1^-}\frac{\partial f}{\partial\delta}=\frac{L_a}{(1-\mu)}\frac{\mu}{(L_s-L_a)}\left[\frac{(2\gamma-1)\,Q+\frac{\partial Q}{\partial\delta}}{(1+Q)^2}\right] \tag{57}$$

and

$$\lim_{\delta\to 1^-}\frac{\partial Q}{\partial\delta}=-\left(\frac{m}{g}\right)(\in -1)\left[\lim_{\delta\to 1^-}\omega^{-\in}\right]\left[\lim_{\delta\to 1^-}\frac{\partial\omega}{\partial\delta}\right] \tag{58}$$

Now, from (31) and (45) we can conclude that

$$\omega=C^{-\frac{1}{\in}}\left(\delta,L_a,\omega\right).\left[\lim_{\delta\to 1^-}\omega\right]\ \forall\delta,\text{ since }\lim_{\delta\to 1^-}C\left(\delta,L_a,\omega\right)=1 \tag{59}$$

After some computations, we can additionally get from Lemma 3 and (45) that

$$\lim_{\delta\to 1^-}\frac{\partial C}{\partial\delta}=1-2\gamma<0 \tag{60}$$

Finally, expressions (59) and (60) imply that

$$\lim_{\delta\to 1^-}\frac{\partial\omega}{\partial\delta}=\left[\lim_{\delta\to 1^-}\omega^{-\in}\right].\frac{(2\gamma-1)}{\in}>0 \tag{61}$$

If we now go backwards, plugging (61) into (58) and then (58) into (57), our final result after rearranging is that $\lim_{\delta\to 1^-}\frac{\partial f}{\partial\delta}=\frac{L_a}{(1-\mu)}\frac{\mu}{(L_s-L_a)}.\lim_{\delta\to 1^-}\left[\frac{\left(\frac{m}{g}\right)(2\gamma-1)\omega^{1-\in}}{(1+Q)^2\in}\right]>0$. This positive sign means, by (56), that $\frac{dL_a}{d\delta}<0$. And hence, from (20), $\frac{dg}{d\delta}>0$. Since

$$\frac{dR_{ca}}{d\delta}=\frac{\partial R_{ca}}{\partial\omega_c}\frac{\partial\omega_c}{\partial\delta}+\left[\frac{\partial R_{ca}}{\partial\omega_c}\frac{\partial\omega_c}{\partial L_a}+\frac{\partial R_{ca}}{\partial L_a}\right].\frac{dL_a}{d\delta} \tag{62}$$

we must obtain now the expressions for $\frac{\partial R_{ca}}{\partial\omega_c}$ and $\frac{\partial R_{ca}}{\partial L_a}$ to clarify unambiguously which is the sign of (62). Then, from (37) and (55),

$$\frac{\partial R_{ca}}{\partial\omega_c}=\left[1+\frac{\rho a_m\left(1-\eta_{ca}L_a\right)}{(L_s-L_a)}\right].\frac{1}{\left(1+\eta_{ca}a_m\rho\omega_c\right)^2} \tag{63}$$

$$\frac{\partial R_{ca}}{\partial L_a}=\frac{\rho a_m\left(1-\eta_{ca}L_a\right)}{(L_s-L_a)^2}.\frac{\omega_c}{\left(1+\eta_{ca}a_m\rho\omega_c\right)} \tag{64}$$

If we consider simultaneously (62) and (63), we can easily observe that

$$\lim_{\delta\to 1^-}\frac{dR_{ca}}{d\delta}>\lim_{\delta\to 1^-}\frac{dL_a}{d\delta}\left[\frac{\partial R_{ca}}{\partial\omega_c}\frac{\partial\omega_c}{\partial L_a}+\frac{\partial R_{ca}}{\partial L_a}\right]$$

which means that $\lim_{\delta\to 1^-}\frac{dR_{ca}}{d\delta}>0$ if $\frac{dh}{dL_a}<\lim_{\delta\to 1^-}\left(\frac{\partial f}{\partial L_a}\right)$. Finally, if we focus on the evolution of ω_c, its total derivative can be proved to be positive provided that $\gamma>1/2$ and $\frac{dh}{dL_a}<\left(\frac{\partial f}{\partial L_a}\right)$, since

$$\frac{d\omega_c}{d\delta}=\frac{\partial f}{\partial \delta}.\left[\frac{\frac{dh}{\partial L_a}}{\left(\frac{\partial h}{\partial L_a}-\frac{\partial f}{\partial L_a}\right)}\right] \tag{65}$$

■

6.5 Proof of Proposition 3

Proof. Since L_n and L_s are invariant in our model, from Lemma 1 we can infer that $\frac{dR_{ns}}{d\delta}<0$ iff $\frac{d\gamma}{d\delta}<0$.

The easiest way to compute $\frac{d\gamma}{d\delta}$ is by considering expressions (35) and (37). Let

$$D(L_c,L_n)=\left[L_n+\rho a\left(\frac{g}{m+g}\right)\right] \tag{66}$$

From (24), (29), (37) and (73), $\gamma=w_n\left[L_n+a\left[\rho-\frac{1}{(\in-1)}\left(\frac{L_c}{a_m}-\frac{L_n}{a}\right)\right]\right]$, and by taking logs and differentiating

$$\lim_{\delta\to 1^-}\frac{d\gamma}{d\delta}\frac{1}{\gamma}=\frac{-a_m\left(\frac{dL_a}{d\delta}\right)}{a_m\in(L_n-ag)}-\lim_{\delta\to 1^-}\frac{\left[\frac{dQ}{d\delta}+(2\gamma-1)\,Q\right]}{(1+Q)}+\frac{dD}{d\delta}\frac{1}{D} \tag{67}$$

It is easy to show that, precisely,

$$\frac{dD}{d\delta}\frac{1}{D}=\frac{a_m\left(\frac{dL_a}{d\delta}\right)}{a_m\in(L_n-ag)} \tag{68}$$

and therefore, by (58), (61), (67) and (68),

$$\lim_{\delta\to 1^-}\frac{d\gamma}{d\delta}\frac{1}{\gamma}=-\lim_{\delta\to 1^-}\frac{\left[\frac{\partial Q}{\partial L_a}\frac{dL_a}{d\delta}+\frac{(2\gamma-1)Q}{\in}\right]}{(1+Q)}<0 \tag{69}$$

Apart from the assumptions of this proposition, expressions (41) and (69) ensure that $\lim_{\delta\to 1^-}\frac{d\gamma}{d\delta}<0$. ■

6.6 Proof of Lemma 1

Proof. Let $\eta_{nk}=\frac{\beta_{nk}}{n_n}$ and $\eta_{ck}=\frac{\beta_{ck}}{n_c}$ be the proportion of eastern and western equity, respectively, owned by a representative household living in location k, where β_{nk} and β_{ck} are the absolute measures of western and eastern firms owned by that household. Then, the amount of gross savings for any household living in k can be expressed as follows:

$$(Gross\ Savings)_k=GS_k=w_k+\eta_{ck}n_c\pi_c+\eta_{nk}n_n\pi_n-E_k \tag{70}$$

We know that in our steady state $\frac{\dot{\eta}_{jk}}{\eta_{jk}} = \frac{\dot{\beta}_{jk}}{\beta_{jk}} - g = 0$, i.e. $\frac{\dot{\beta}_{jk}}{\beta_{jk}} = g$ $\forall j =$ West, core;$\forall k =$ West, core, periphery. Therefore,

$$\frac{\dot{\beta}_{nk}}{\beta_{nk}} = \frac{GS_k f_{nk}}{v_n \beta_{nk}} - m = \frac{\dot{\beta}_{ck}}{\beta_{ck}} = \frac{GS_k\left(1 - f_{nk}\right)}{v_c \beta_{ck}} = g \tag{71}$$

where f_{nk} is the proportion of total gross savings devoted to the purchase of western equity. Then, from (71),(4) and (28), we can easily solve for GS_k:

$$GS_k = (m+g)\eta_{nk} a \left(\frac{g}{m+g}\right) w_n + g\eta_{ck} a_m w_c \tag{72}$$

On the other hand, it is easy to see from (4) and (25) that the instantaneous variation in the value of previously-owned assets, considering also the effect of imitation, is the following:

$$\frac{\partial V_k}{\partial t} = -(m+g)\,\eta_{nk} a \left(\frac{g}{m+g}\right) w_n - g\eta_{ck} a_m w_c$$

where V_k is the value of previously-owned assets by a household in location k. Since, by (72) and the last equation, $(Net\ Savings)_k = GS_k + \frac{\partial V_k}{\partial t} = 0\ \forall t$ in any steady state, any household's wealth is kept constant along the balanced growth path, i.e.

$$y_k = E_k = w_k + \rho\eta_{ck} a_m w_c + \rho\eta_{nk} a \left(\frac{g}{m+g}\right) w_n \tag{73}$$

where y_k is household k's income, $\forall k =$ West, Core, Periphery in steady state. ■

6.7 Proof of Lemma 2

Proof. From (33) we can check that

$$\lim_{\delta\to 1^-} \gamma > 1/2 \text{ iff } (1-\mu) < \lim_{\delta\to 1^-} \left[w_n \left(L_n + a\rho \left(\frac{g}{m+g}\right)\right) - w_c\left(L_c + a_m\rho\right)\right] \tag{74}$$

As we can conclude after inspecting expressions (24), (29), (35) and (36), condition $\lim_{\delta\to 1^-}\gamma > 1/2$ can only be satisfied iff (74) holds. Now we just have to look for a sufficient condition that guarantees (74). From our definition of Q in expression (34), condition (74) can be restated as follows:

$$\frac{(1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]}{\alpha\rho - (1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]} \cdot \left[\frac{\alpha\left(L_c + a_m\rho\right)}{L_n - ag}\right]^{\epsilon - 1} < P^{\epsilon} \tag{75}$$

By the assumptions established in this lemma, necessarily $P^{\epsilon} > 0$. Let's now define the function

$$H(L_c, L_n) = \frac{(1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]}{\alpha\rho - (1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]} \cdot \left[\frac{\alpha\left(L_c + a_m\rho\right)}{L_n - ag}\right]^{\epsilon - 1} - P^{\epsilon} \tag{76}$$

It is easy to see that $\frac{\partial H}{\partial L_c} \geq 0$ and $\frac{\partial H}{\partial L_n} \leq 0 \; \forall L_c, L_n$. Therefore, a sufficient condition for (74) follows from any situation in which $H(L_c, L_n) < 0$. We want to search for a relation between the initial values of L_c and L_n that ensures that $H(L_s, L_n) < 0$ and hence that $\lim_{\delta \to 1^-} \gamma > 1/2$. For any initial value of L_c that satisfies (24) and (30), we can determine that, from (76),

$$H(L_c, \frac{aL_c}{a_m}) = -P^{\in} < 0 \text{ and } H(L_c, a\left[\frac{L_c}{a_m} - \frac{\alpha\rho}{1-\alpha}\right]) > 0 \tag{77}$$

Since the equality $H(L_c, L_n) = 0$ contains an implicit function $\hat{L}_n(L_c)$ for which $\frac{\partial L_n}{\partial L_s} = -\frac{\partial Q/\partial L_s}{\partial Q/\partial L_n} > 0$ $\forall L_c, L_n$, then $\hat{L}_n\,(L_c)$ is an increasing function in L_c. Since $H(L_s, L_n)$ is a monotone and continuous function in L_n, from (77) we can apply Bolzano's theorem to state that

$$\exists \text{ a unique function } \hat{L}_n(L_c) \text{ such that } H(L_c, \hat{L}_n(L_c)) = 0 \; \forall L_c \tag{78}$$

Finally, from the sign of the partial derivatives above, we can say with certainty that $\forall L_c$, if $L_n > \hat{L}_n(L_c)$ then $H(L_c, L_n) < 0$, which means that $Q < P$ and hence that $\lim_{\delta \to 1^-} \gamma > 1/2$. ■

6.8 Proof of Lemma 3.

Proof. From (11) we can express the wide-gap assumption when $\delta \to 1^-$ as

$$\frac{\left(L_n - \frac{(1-\alpha)L_c}{a_m} + \alpha\rho\right)}{\alpha\left(L_c + a_m\rho\right)} \cdot \left[\frac{(1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]}{\alpha\rho - (1-\alpha)\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right]}\right] \leq \alpha^{\in} \tag{79}$$

Rearranging and rewriting (79) with an equality, we get the following quadratic equation in L_c :

$$\left[\frac{L_c}{a_m} - \frac{L_n}{a}\right] = \frac{\frac{\alpha\rho}{(1-\alpha)}}{\left(1 + \frac{\left(L_n - a\frac{(1-\alpha)}{a_m} + a\alpha\rho\right)}{\alpha^{\frac{2-\alpha}{1-\alpha}}\left(L_c + a_m\rho\right)}\right)} \tag{80}$$

Since, from condition (30), $L_c > 0$ and $L_n > \frac{a}{a_m}L_c - \frac{a\alpha\rho}{(1-\alpha)}$, we can conclude that the denominator of the right-hand side of (80) is bigger than 1. This means that at least one root $L_{c1}^*$of (80) satisfies for sure the inequality $a_m\frac{L_n}{a} < L_{c1}^* < a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]$, because the right-hand side is positive and smaller than $\frac{\alpha\rho}{1-\alpha}$. Now we have to make sure that $L_{c1}^*$is a unique root within the interval $(a_m\frac{L_n}{a}, a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right])$.

If we formally restate (79) we can obtain the following inequality:

$$Z(L_c) = EL_c^2 + FL_c + G \leq 0 \quad (81)$$

where

$$E = \frac{a_m \alpha^{\frac{2-\alpha}{1-\alpha}} - a(1-\alpha)}{a_m^2} \quad (82)$$

$$F = \frac{L_n\left(\frac{2-\alpha}{1-\alpha} - \alpha^{\frac{2-\alpha}{1-\alpha}}\frac{a_m}{a}\right) + \alpha\rho\left[a + \alpha^{\frac{1}{1-\alpha}} a_m \left(2 - \frac{1}{1-\alpha}\right)\right]}{a_m}$$

$$G = -\left[\frac{L_n}{a}\left[\alpha\rho\left(a + \alpha^{\frac{1}{1-\alpha}} a_m\right) + L_n\right] + \alpha^{\frac{3-2\alpha}{1-\alpha}} . \frac{1}{1-\alpha} a_m \rho^2\right]$$

We can see that, in principle, the signs of E and F are undetermined but that of G is clearly negative, which implies that $Z(0) < 0$. Let's explore now the implications of the 2 possibilities concerning the sign of E:

-If $E > 0$ then, since $Z(0) < 0$, $Z(L_c)$ is necessarily a quadratic function with one positive and one negative root. Therefore, we know for sure that there is a unique L_{c1}^* such that $Z(L_{c1}^*) = 0$ and $a_m \frac{L_n}{a} < L_{c1}^* < a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]$. Given that this curve cuts the horizontal axis from below, conditions (81) and (30) will be satisfied.

-If $E < 0$, $Z(L_c)$ will be now a concave function with at least one positive root L_{c1}^*, but in principle it could have another one within our particular interval $\left[a_m \frac{L_n}{a}, a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]\right]$. In order to reject this latter possibility, it will be enough to show that $Z(a_m \frac{L_n}{a}) < 0$ and $Z(a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]) > 0$,which would imply that the other root is out of our interval.

It is possible to check that

$$Z\left(a_m \frac{L_n}{a}\right) = -\frac{(\in -1)^{\in+2}}{\in^{\in+1}} . \rho a_m . \left(\frac{L_n}{a} + \rho\right) < 0$$

$$Z(a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]) = \left(\frac{\in -1}{\in}\right)\rho L_n\left(\in^2 + \in -2\right) > 0$$

Again, since this curve intersects the horizontal axis from below, if $E < 0$ the wide-gap case is compatible with positive measures of manufactures in both countries iff $a_m \frac{L_n}{a} < L_c \leq L_c^* < a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]$.

To summarize, if $\delta \to 1^-$, $\forall a_m$ and a, $\forall \alpha \in (0,1)$, $\exists$ a unique L_c^* such that both (11) and (30) hold iff $a_m \frac{L_n}{a} < L_c \leq L_c^* < a_m\left[\frac{L_n}{a} + \frac{\alpha\rho}{1-\alpha}\right]$, where L_c^* is the smallest positive root of equation (80). ■