Political support for the private system to finance political parties[*]

Jenny De Freitas[†]

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Abstract

In a Downsian model of political competition we compare the equilibrium tax and redistribution level obtained from two systems to finance parties' political campaigns: the public and the private system. In the private system ideological voters make campaign contributions to increase the chances of winning of their preferred party. In the public system parties receive funds from the government. If voters are sufficiently ideological the private system induces high aggregate spending. Nevertheless, it may be supported by a majority of voters given the indirect effect contributions have on the equilibrium redistribution level and parties' probability of winning.

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1 Introduction

Campaign finance regulation has been the subject of public debate since recent reforms on U.S. (the Bipartisan Campaign Reform Act, 2002) and Canada (Bill C-24, 2003). Different arguments in favor and against more public funds in politics and the fairness of the system shed light on the complexity to drive an unambiguous answer on the optimal policy to finance political parties' campaigns.

Previous literature on this subject argue that the optimality of a policy, the cap on contributions, for example, depends strongly on the assumptions made on the motivation of interest groups and the informativeness of the campaign.

This paper investigates the political sustainability of the private system to finance political parties. While the public system favors centrist policies on both, the economic and non-economic issues. The private system allows for more extremist outcomes that may fit better some societies. We find that a majority of voters may favor the private system when asymmetry on the intensity of preferences toward the non-economic issue is large.

We assume interest groups give campaign contributions to increase the voting share of an alike candidate (e.g. Grossman and Helpman 1996, 2001). We take the approach of Grossman and Helpman (1996), Ortuño-Ortín and Schultz (2005) and Roemer (2006) under which campaign spending do not provide information to voters. It increases the probability of winning among uninformed or impressionable voters.

We assume parties held fixed a position toward the non-economic issue. Political parties choose a purely redistributive proportional income tax to maximize their probability of winning. Parties face a trade-off: On the one hand, they increase the probability of winning by choosing a more centrist policy on the economic issue, that attracts middle class voters (because by assumption voters in the middle class are swing voters); and, on the other hand, they want to please their partisans by moving toward a more extreme policy in order to increase campaign contributions that also increase their chances of winning the elections among uninformed voters. Note that our parties are Downsian, the only motivation to choose a policy different from the preferred tax of the median voter is to collect money from partisan voters or deter contributions to the rival party. We study what the expected policy in the economic and non-economic issues is in two different (and extreme) models to finance political campaign: the private and the public system. In the private system, voters can freely contribute to their most preferred party. In the public system, private donations to political parties are illegal and parties can only receive funds from the state in proportion to their voting share. We analyze in which circumstances (ideology intensity, effectiveness of campaign, number of informed voters) a private system would be supported by a majority of voters.

As in Glazer and Gradstein (2005) voters/citizens make campaign contributions to increase the chances of winning of their preferred party. In their paper parties maximize contributions, since the problem is unidimensional parties' policies need to diverge sufficiently enough to attract contributions. In our paper parties maximize the probability of winning. Since campaign spending increases the probability of winning parties trade-off extreme policies that attract higher contributions and moderate policies that please the median voter. Such a trade-off is not present in Glazer and Gradstein (2005).

In the public system both parties have equal chances of winning and the equilibrium tax rate is the most preferred tax rate of the median voter. In the private system the equilibrium income tax is closer to the preferred income tax of the less ideological voter and the preferred party of the most ideological voter has the highest probability. In our setting political campaign does not bring any information to voters. In the private system competition among ideological voters (contributors) may induce excessive spending in political campaign. Still, it may be preferred by a majority of voters given the indirect effect contributions have on the equilibrium income tax and parties' probability of winning.

We also compare the welfare implications of different policy reforms as a cap on contributions. An intermediate cap decreases aggregate contributions and benefits the group of voters not constrained by the cap. An strict cap, that constraints both ideological voters, makes the median voter better off.

We develop some numerical examples to understand how the different effects in favor and against the private system to finance political parties interact. The private system does not seem to have enough support from voters when ideological voters are equally or almost equally attached to political parties. The reason is that in such a case, where the society is homogenous, voters favor the system that brings then the highest economic utility, having ideological voters opposite economic preferences, a coalition of the median voter and at least one ideological voter will prefer the public system over the private one. When ideological preferences are asymmetric enough both ideological voters are pleased. The economic issue is closer to the preferred one by the less ideological voter. The expected non-economic issue (the party in office) is closer to most ideological voters preferred one. Both ideological voters then, support the private system to finance political parties. In more asymmetric (heterogenous) societies the private system has higher political support.

The paper is organized as follows: In section 2 we comment the related literature. Section 3 presents the model and the characteristics of a private system to finance political campaigns. The contribution stage is developed in section 4. The equilibrium income tax and the introduction of a cap on contributions are described in section 5. In section 6 we characterize the equilibrium in a purely public system to finance political parties. In section 7 we compare both systems. Section 8 concludes.

2   Literature Review

Previous literature on political campaign finance focuses on the welfare impact of policy reforms as a ban of contributions, the introduction of a ceiling either on contributions or on campaign spending. One of the central lessons that has emerged from this literature has been that both positive and normative conclusions depend strongly on assumptions made about the role of campaign advertisement, the rationality of voters and the motivation of contributors.

Formal models of elections with campaign contributions can be categorized according to two aspects. The first distinction concerns assumptions about the motivation of campaign contributions. An interest group may contribute money to influence the policy outcome, the influence motive; or in order to receive services or policy favors from the candidate, the service motive for contributions. Alternatively, an interest group may contribute to an alike candidate to increase her chances of winning. This is known in the literature as the electoral or position-induced motive. Magee (2001, 2002) has empirical evidence in favor of service-motivated contributions and the electoral motive for contributions. Ansolabehere et al. (2003) find that money to finance campaign spending comes mainly from individual donations, their study shows evidence that favors the electoral or position-induced motive for contributions.

The second distinction concerns assumptions about the effects of campaign spending on voter behavior and election outcome. The effect of contributions on the election outcome depends on whether we assume that voters are impressionable or rational. Impressionable voters, in general, do not have a specific policy position either in the economic or the ideological dimension. They are called impressionable because they vote with a higher probability for the party advertising the most. On the other hand, when campaigns are informative, they provide information to voters on the parties platforms. Rational voters use this information to vote for the party that gives them the highest utility.

When campaign advertisement is informative (e.g. Schultz 2007; Coate 2004a, 2004b; Vanberg 2004) voters are assumed to be rational. They update their beliefs about the policy position or type of a candidate as a function of advertisement received by parties (in general there are two parties).

When campaign advertisement is informative different assumption on the motivations of interest-group contributions have different welfare implications. If the motivation of interest-group contributions is position-induced (e.g. Coate 2004a) contribution limits benefit interest groups by decreasing competition among them. If instead we have a service-motive for campaign contributions (e.g. Coate 2004b) a cap on contributions is welfare enhancing. It will reduce policy favors without any effect on the quality of the selected

leader.

None of these works consider asymmetries in the access to funds by parties. Vanberg (2004) introduces such asymmetries by assuming that the interest group associated with party R is bigger. Party R is wealthier since the same per-capita contribution gives a higher total contribution level. The higher probability of winning of the advantaged party from the larger population covered by the advertisement outset the lower probability of winning among uninformed voters. The probability of winning is, then, independent on the access to funds by parties. So, contribution limits (such that party L is not constrained) favor members of the wealthier interest group since it decreases per-capita contributions levels.

Roemer (2006) considers a very different setting, parties are endogenously formed and political campaigns are not informative, they reach impressionable voters. Within the party, contributions are shared efficiently. When campaigns are not informative, asymmetries in the access to funds skew the policy outcome in favor of the financially stronger party, i.e. the pivotal voter is richer than the median income voter. Similar results are found in Ortuño-Ortín and Schultz (2005) with ideological parties (à la Wittman). The availability of private contributions allows the wealthier party, say party R, to choose a more extreme policy. The expected platform then is more to the right than if only public funding was allowed.

3 Privately financed political parties

Assume we have three groups of voters differentiated by their earnings ability w_j, indexed by $j = 1, 2, 3$ with $0 < w_1 < w_2 < w_3$. The proportion of voters in each group is the same and equals $\frac{1}{3}$. Income is linear in labor supply and takes the form $y_j = w_j l_j$. Therefore, per-capita income of this economy is $\mu = \frac{1}{3}(y_1 + y_2 + y_3)$. Within each group, there is a fixed proportion $1 - \rho \in (0, 1)$ of uninformed or impressionable voters. Parties can win these impressionable voters only by campaigning. Informed voters are also differentiated by their intensity of preferences toward parties (or the non-economic issue that characterizes them). In this dimension we identify two types of voters: ideological and indifferent or swing voters. For simplicity we assume that voters in group 1 and 3 are ideological. Voters have a bias α_j toward party L, with $\alpha_1 > 0$, $\alpha_2 = 0$ and $\alpha_3 < 0$. Ideological preferences are then, perfectly correlated with income. We often refer voters 1 and 3 as ideological voters.

Political competition takes place between two office motivated parties, L and R. They hold fixed their political position toward the non-economic issue. The pliable issue is a marginal income tax t_P that finances a lump-sum transfer r_P, with $P = L, R$. The non-economic issue can be interpreted as a political position that does not directly affect the tax revenue and the redistribution level. One example is the 'moral values' issue that played

an important role in the Bush campaign.

Ideological voters play two roles. They may be contributors to political parties in a first stage and voters in a second stage.

Political parties choose a policy platform (t_P, r_P), $P = L, R$, and spend contributions received from ideological voters on political campaign to attract uninformed or impressionable voters. Anticipating the effect of the announced platform on the level of contributions, political parties choose the political platform that maximizes the probability of winning given the policy chosen by their rival.

The game goes as follows: first, parties announce simultaneously the political platforms (t_P, r_P), $P = L, R$. We assume full commitment to the platform announced. Second, ideological voters give contributions (if any) to their preferred party. Contributions received by political parties finance political campaigns that influence the voting decision of impressionable voters. At the third stage all voters take part in the election. The winning candidate implements the policy announced. At the last stage of the game voters make consumption and labor decisions. The model is solved backwards.

3.1   The tax schedule

A proportional tax with marginal tax rate $t \in [0, 1]$ is collected to finance a lump-sum transfer or redistribution level r. The budget balanced condition gives us a redistribution function $r(t)$,

$$r(t) = t\mu(t) \tag{1}$$

Note that average income is endogenously determined and depends upon the labor decision of voters. If labor supply is decreasing in taxes (leisure is a normal good) the redistribution function is concave in t. The peak of the Laffer curve, the t that maximizes redistribution is $\bar{t} : \mu(t) + t\frac{\partial \mu}{\partial t} = 0$. From labor disincentives $\mu(.)$ is decreasing in t.

3.2   Voters

The economic utility is represented as:

$$u(x_j, l_j) = x_j - \frac{1}{2}l_j^2$$

Where x_j and l_j are consumption and labor supply of a voter in group j. Consumption equals after-tax income, $x_j = (1-t)w_jl_j + r$. Once the voting stage takes place the winning party implements the platform announced. Voters make consumption and labor decisions that maximize utility. The optimal labor supply is $l_j = (1-t)w_j$. Labor supply is strictly positive, for $t < 1$. The income level of a voter j is given by $y_j = (1-t)w_j^2$. The peak of

the Laffer curve is $\bar{t} = \frac{1}{2}$.

Informed voters

Informed voters are divided in ideological, group 1 and 3; and swing voters, voters in group 2. By ideological we mean that they have clear and relatively extreme views toward the non-economic issue that characterizes a party. The indirect utility of a voter in group j if party L wins the election is,

$$V_{j,L} = u_j(t_L) + \alpha_j \tag{2}$$

Where t_L is the income tax rate announced by political party L, and $u_j(t)$ is the indirect economic utility of voter j at the optimal labor supply. The lump-sum transfer r is determined from the budget balanced condition (1). The parameter α_j measures the relative position (or attachment) of a voter in group j toward the non-economic issue that characterizes party L. Net from contributions indirect utility for group j is $V_{j,L} - C_j$.

It can be easily showed that the preferred income tax of a voter in group j is t_j, for $j = 1, 2, 3$:

$$t_j = \max\left\{ 0, \frac{\overline{w}_2 - w_j^2}{2\overline{w}_2 - w_j^2} \right\} \quad j = 1, 2$$

$$\text{and} \quad t_3 = 0$$

Where $\overline{w}_2 = \frac{1}{3}\sum w^2$. It can be easily checked that $t_1 \in \left(0, \frac{1}{2}\right)$ and $t_2 < t_1$. The preferred tax of group 2, t_2, will be strictly positive as long as w_2 is smaller than $\sqrt{\overline{w}_2}$. Assume the median earnings ability, w_2, is smaller than the average earnings ability, $\overline{w} = \frac{1}{3}\sum w$; which guarantees $t_2 > 0$. This comes from the following ordering[1]: $\sqrt{\overline{w}_2} > \overline{w} > w_2$. From now on we characterize voters by their preferred income tax t_j and their ideology α_j. Another distinction between ideological and swing voters is that campaign contributions are given by ideological voters. In equilibrium voters in group 2 do not have interest in contributing.

Uninformed voters

We assume swing or impressionable voters are captured (influenced) by political advertisement. The effect of campaign advertisement of party L and R that spend C_L and C_R, respectively, increases the probability of winning of party L among uninformed voters by $g(C_L, C_R)$.

We next identify properties and conditions the function $g(.)$ satisfy.

1. Symmetry: Given two levels of campaign spending $C_L, C_R \geq 0$, $g(C_L, C_R) = -g(C_R, C_L)$.

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[1] For the variance of w to be positive we need $\overline{w} < \sqrt{\overline{w}_2}$.

2. g is twice continuously differentiable.

3. We assume $g_{11} < 0$ and $g_{22} > 0$; which implies $g_{11}g_{22} - (g_{12})^2 < 0$, where g_i is the derivative of g with respect to the $i-$th argument.

4. The marginal gain from one extra dollar in campaign spending by party L if party R does not make political campaign ($C_R = 0$) tends to infinity. By symmetry the same applies to party R, $\lim_{C_L \to 0} g_1(C_L, 0) = \infty$ and $\lim_{C_R \to 0} g_2(0, C_R) = -\infty$.

In some circumstances it would be better for both groups of ideological voters not to contribute to political parties. Condition 4 rules out this possibility in equilibrium. If $C_R = 0$ the probability of winning of party L would increase sharply if voters in group 1 contribute 1\$ to party L (analogous for group 3). Condition 4, then, guarantees us an equilibrium with strictly positive contributions. Condition 2 guarantees that the equilibrium with positive contributions is asymptotically stable. In section 3 we give three examples of functions satisfying these properties and conditions.

3.3   Parties

Parties announce a policy platform: an income tax and a redistribution level that maximizes their probability of winning. Parties full commit to the platform announced. Once in office parties they implement their preferred position toward the non-economic issue hold fixed during election campaigns. It is held fixed during the election campaigns. We develop the error distribution approach (Roemer, 2001) to model uncertainty. Whenever $V_{j,L} > V_{j,R}$ group j prefers t_L to t_R. Parties are confident about this up to a margin of error, ε. The error parties make in measuring the probability of winning is uniformly distributed in $[-\beta, \beta]$. Among the informed voters, the probability that group j will vote for party L is,

$$\Pr(\varepsilon \leq V_{j,L} - V_{j,R})$$

We assume for simplicity that α_1 and $\mid \alpha_3 \mid$ are sufficiently high. Indeed for $\alpha_1, \mid \alpha_3 \mid > \beta$ group 1 and 3 probability to vote for party L and R, respectively, equals one. The probability that group 2 votes for party L is,

$$\pi_2 = \begin{cases} \frac{1}{2} + \frac{1}{2\beta}(V_{2,L} - V_{2,R}) & \text{if} \quad -\beta \leq X \leq \beta \\ 1 & \text{if} \quad X < -\beta \\ 0 & \text{if} \quad X > \beta \end{cases}$$

where $X = V_{2,L} - V_{2,R}$.

The function π_2 represents party L's probability to win the support of informed voters in group 2. There is a proportion of ρ informed voters. Among uninformed voters, party L can also increase its probability of winning by campaigning.

The probability of winning of party L taking into account informed and uninformed voters is,[2]

$$\pi(t_L, t_R) = \frac{\rho}{3} \sum \pi_i + (1 - \rho)\left(\frac{1}{2} + g(C_L, C_R)\right)$$

Note that the announced income tax will induce a certain level of contributions from ideological groups. Parties, then, anticipate the effect of the income tax announced on contributions.

4 Contribution stage

Parties are the means to get implemented the most preferred position toward the non-economic issue of ideological voters. If party L wins the election it benefits all voters in group 1. In this sense party L generates a positive externality to voters in group 1 if she wins the election (analogous for group 3 and the party R). Following the literature on private contributions to public goods (e.g. Andreoni 1988, 1998; Bergstrom et al. 1986) we know that the most ideological voters will contribute to their most preferred party. When deciding on contributions voters already know the policy position undertaken by parties. In any case a group would not contribute to a party he would not vote for at the voting stage. As long as $V_{1,L} - V_{1,R} > 0$ group 1 whenever it contributes it does it to party L, while group 3 will contribute to party R, if it contributes. Indeed ideological voters are advocates of the non-economic issue associated with a party. Campaigning increases uninformed voters support toward such non-economic issue. Voters in group 2 are not ideological, if they contribute to a party is because they perceive that by contributing they can increase the probability of winning of the party that brings then the highest economic utility. Though, in equilibrium voters in group 2 do not contribute. As we will see later, when platforms converge group 2 is indifferent among the two parties.[3]

The expected utility of a voter in group 1 takes the following form,

$$EV_1(t_L, t_R) = \pi V_{1,L} + (1 - \pi)V_{1,R} - C_1$$

Ideological voters choose a contribution level $C_1 \geq 0$ that maximizes their expected utility.

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[2]In equilibrium, for α_1 and $|\alpha_3|$ sufficiently high, informed voters in group 1 votes for party L while informed voters in 3 votes for party R. This will be proved later, when determining the platforms chosen by parties.

[3]The difference in utilities, $V_{2,L} - V_{2,R}$, need to be sufficiently high for group 2 be willing to contribute. Indeed it equals zero if platforms convergence, which is the case, see section 5.

Whenever α_1 is sufficiently large, group 1 will be willing to contribute. If there is a continuum of identical voters in each group an additional *moral* principle assumption should be made to escape the free-riding problem. See, for instance, Roemer (2006), which assumes voters follow a Kantian principle: Where all group members, if equal, pay the same, an individual deviation imply then, a deviation for all members of the group. A deviation from the contribution level for one single voter would imply a deviation of the same size by all of the voters in that group. This guarantees strictly positive levels of contributions.

Group 1 chooses C_1 that equals party L's campaign spending, by maximizing $EV_1(t_L, t_R)$ taking C_R as given. From here on we refer to C_1, C_L or C_3, C_R indistinctly although they are different concepts. The first order condition:[4]

$$\frac{\partial EV_1}{\partial C_L} = \frac{\partial \pi}{\partial C_L}(V_{1,L} - V_{1,R}) - 1 \leq 0 \qquad (C_L = 0 \text{ if inequality})$$

For the particular function of probability of winning,[5]

$$(1 - \rho) g_1(C_L, C_R)(V_{1,L} - V_{1,R}) - 1 = 0 \tag{3}$$

$$C_L = m\left(\frac{1}{(1-\rho)(V_{1,L} - V_{1,R})}, C_R\right)$$

where $m = g_1^{-1}$, since m is decreasing in $\left(\frac{1}{(1-\rho)(V_{1,L} - V_{1,R})}\right)$, contributions to party L increase with utility difference $V_{1,L} - V_{1,R}$, which is increasing in the ideological preferences of group 1, α_1. The higher the proportion of informed voters, ρ, the lower is the contribution level. Because campaign spending will be less effective, there will be a small increase in the probability of winning for a party spending an extra dollar in campaigning.

Analogous for group 3.

$$\frac{\partial EV_3}{\partial C_R} = \frac{\partial \pi}{\partial C_R}(V_{3,L} - V_{3,R}) - 1 \leq 0 \quad (C_R = 0 \text{ if inequality})$$

For the particular function of probability of winning,

$$(1 - \rho) g_2(C_L, C_R)(V_{3,L} - V_{3,R}) - 1 = 0 \tag{4}$$

$$C_R = n\left(\frac{1}{(1-\rho)(V_{3,R} - V_{3,L})}, C_L\right)$$

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[4]Second order conditions are satisfied provided that group 1 votes for party $L : V_{1,L} - V_{1,R} > 0$. Otherwise, group 1 would not choose to contribute. Note that the platform was already announced and parties fully commit to the platform announced. The same logic applies when determining contributions by group 3, this time $V_{3,L} - V_{3,R} < 0$.

[5]Note that $C_L = 0$ when $C_R = 0$ is not a best response for group 1 if condition 4 on the function $g(C_L, C_R)$ holds.

where $n = -g_2^{-1}$. Some boundaries on contributions should be introduced to guarantee an *affordable* contribution level. We assume that $C_j \le x_j(t_e)$, $j = 1, 3$.

Lemma 1 *If properties 1-4 of the function $g(C_L, C_R)$ are satisfied a unique equilibrium with positive contributions exists.*

Proof. *In the appendix.* ■

Contribution levels depend on parties tax schedules. If party L please their partisans (voters from group 1), this has two effects on contributions: it increases contributions from group 1, because it increases the economic advantage from having party L in power $(V_{1,L} - V_{1,R})$. Since taxes are purely redistributive group 3 will be paying higher taxes under the party L policy, then, contributions from group 3 to party R may also increase. Each party is aware of the effect of its announced platform on contributions given to itself and its rival party. Notice that if party L chooses a policy platform on the pliable issue (taxes) favoring group 1, it is solely because this allows party L to increase contributions from that group. Party L, then, may benefit group 2 or even group 3 if by doing so she increases the political support from informed voters in group 2 or decreases contributions to party R more than contributions to her decrease.

5   Choice of platform

Political parties choose their platforms knowing that campaign contributions will be affected by their policy choices. Party L maximizes the probability of winning, π, while party R maximizes $1 - \pi$. The first order conditions for the party L and R problem are respectively,

$$\frac{\rho}{6\beta} \frac{\partial V_{2L}}{\partial t_L} + (1 - \rho) \left(g_1(C_L, C_R) \frac{\partial C_L}{\partial t_L} + g_2(C_L, C_R) \frac{\partial C_R}{\partial t_L} \right) = 0 \qquad (5)$$

$$\frac{\rho}{6\beta} \frac{\partial V_{2R}}{\partial t_R} - (1 - \rho) \left(g_1(C_L, C_R) \frac{\partial C_L}{\partial t_R} + g_2(C_L, C_R) \frac{\partial C_R}{\partial t_R} \right) = 0 \qquad (6)$$

Group 2 has no *a priory* bias toward any of the parties. By assuming $\alpha_2 = 0$ we avoid consideration of policy choices when there is an advantaged candidate (e.g. Aragonès and Palfrey 2005; Sahuguet and Persico 2006). The conditions to have convergence of platforms is the symmetry on contribution schedules: $\left(\frac{\partial C_P}{\partial t_L} + \frac{\partial C_P}{\partial t_R} \right) |_{t_L = t_R = t} = 0$, $P = L, R$. If the marginal cost of an additional contribution is independent of the platform announced by parties from symmetry of $g(.)$ symmetry on contributions hold (see appendix A).

Party L faces a trade-off between maximizing the probability of winning favoring informed voters in group 2 and maximizing total contributions from group 1 without increasing too much contribution competition (indirect effect on C_R).

As a consequence the equilibrium income tax is not necessarily equal to the median voter preferred income tax, t_2. Next proposition summarizes the condition under which the equilibrium income tax is lower (higher) than the median voter preferred income tax.

Proposition 1 *1) Suppose the second order conditions are satisfied. The equilibrium tax schedule is $t_L = t_R = t_e$, that solves (5). The equilibrium contribution levels solve $C_L = m\left(\frac{1}{\alpha_1(1-\rho)}, C_R\right)$ and $C_R = n\left(\frac{1}{|\alpha_3|(1-\rho)}, C_L\right)$.*
2) The equilibrium income tax is higher or equal than t_2 whenever,

$$\gamma\left(\frac{|\alpha_3|}{\alpha_1}\right) \geq \left(\frac{w_3^2 - w_2^2}{w_2^2 - w_1^2}\right), (equal\ to\ t_2\ if\ equality) \tag{7}$$

Where $\gamma = g_1\left(g_1 g_{22} - g_2 g_{12}\right)/g_2\left(g_1 g_{12} - g_2 g_{11}\right)$. Otherwise it will be lower than t_2.
In the symmetric case: $\alpha_1 = |\alpha_3| = \alpha$, the equilibrium income tax will be smaller than the median preferred income tax as long as $w_2 < \overline{w}$, where $\overline{w}$ is the mean earnings ability.
Proof. *In the appendix* ∎

The left hand side of (7) is the relative intensity of party preferences, weighted by the parameters of the advertisement function. The higher is $|\alpha_3|$ with respect to α_1 the easier condition (7) will be satisfied and the closer will be the tax rate to t_1. This is a swing voter effect: the tax schedule favors the less ideological voters. For $|\alpha_3|$ high, the gain in contributions from group 3 to party R when t_R decreases is small because $(V_{3,R} - V_{3,L})$ is already very high. Party R will rather favor the median voter in order to increase the probability of winning. If α_1 is relatively small party L could increase campaign spending by increasing t_L from t_2 (without increasing too much contributions to its rival). When t_L is above t_2 a further increase in t_L increases contributions to party L but decreases the utility of the median voter. The effect of the advertisement function on the equilibrium income tax is harder to analyze. It is interesting to note, though, that in many examples γ equals one.[6] For these cases (7) becomes:

$$\frac{|\alpha_3|}{\alpha_1} \geq \left(\frac{w_3^2 - w_2^2}{w_2^2 - w_1^2}\right)$$

The right hand side of (7) is the relative earnings differential with respect to voter 2. The equilibrium income tax moves in a direction that benefits the voter with earnings further away from w_2. This is because the closer w_1, say, to w_2, the closer will be t_1 to t_2. Contributions from group 1 will be already high if party L chooses $t_L = t_2$. Instead party L would find more profitable to choose a lower income tax rate than t_2 in order to

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[6]This is the case for the following advertisement functions: $g(C_L, C_R) = \frac{(C_L)^k - (C_R)^k}{2(C_L)^k + (C_R)^k}$, for any $0 < k \leq 1$.

decrease group 3 contributions to party R. Then, at the symmetric case, the equilibrium income tax benefits more group 3, not because is richer but because the income distribution is right-skewed.

Applying the implicit function theorem to (3) and (4) from properties and conditions $1-3$ of $g(.)$ the effect of an increase in t_L on C_L and C_R is, respectively,

$$\frac{\partial C_L}{\partial t_L} = \frac{1}{\alpha_1 \alpha_3 \left(g_{11} g_{22} - (g_{12})^2\right)} \left(-g_1 g_{22} \alpha_3 \frac{\partial V_{1L}}{\partial t_L} + g_2 g_{12} \alpha_1 \frac{\partial V_{3L}}{\partial t_L}\right) \text{ and} \qquad (8)$$

$$\frac{\partial C_R}{\partial t_L} = \frac{1}{\alpha_1 \alpha_3 \left(g_{11} g_{22} - (g_{12})^2\right)} \left(g_1 g_{12} \alpha_3 \frac{\partial V_{1L}}{\partial t_L} - g_2 g_{11} \alpha_1 \frac{\partial V_{3L}}{\partial t_L}\right) \qquad (9)$$

We replace (8) and (9) in the first order condition for a maximum of party L. For $g_2 \left(g_1 g_{12} - g_2 g_{11}\right) \neq 0$, rearranging terms, we have:

$$\frac{\rho}{6\beta} \frac{\partial V_{2L}}{\partial t_L} + \frac{g_2 \left(g_1 g_{12} - g_2 g_{11}\right)(1-\rho)}{\alpha_1 \alpha_3 \left(g_{11} g_{22} - (g_{12})^2\right)} \left(\gamma \left|\alpha_3\right| \frac{\partial V_{1L}}{\partial t_L} + \alpha_1 \frac{\partial V_{3L}}{\partial t_L}\right) = 0 \qquad (10)$$

In the case where $g_{12} = 0$ the above expression becomes:

$$\frac{\rho}{6\beta} \frac{\partial V_{2L}}{\partial t_L} - (1-\rho) \left(\frac{(g_1)^2}{\alpha_1 g_{11}} \frac{\partial V_{1L}}{\partial t_L} + \frac{(g_2)^2}{\alpha_3 g_{22}} \frac{\partial V_{3L}}{\partial t_L}\right) = 0 \qquad (11)$$

From (10) and (11), the equilibrium income tax in the private system will be closer to t_2 the higher are α_1 and $\left|\alpha_3\right|$. Note that at the equilibrium income tax we maximize a weighted sum of voters utility. The weights on voters' utility are inversely related to their attachment toward parties, α_j with $j = 1, 3$.

We next show some examples. The equilibrium income tax and the levels of contributions shares some properties. In general the level of contributions to party L (party R) increases with α_1 ($\left|\alpha_3\right|$).and both, C_L and C_R decreases with the proportion of informed voters, ρ, since competition in contributions falls as the proportion of votes that can be bought by campaign advertisement decreases. Nevertheless, the proportion of informed voters do not affect the parties's probability winning. The probability of winning of party L increases with $\alpha_1 / \left|\alpha_3\right|$.

Consider the symmetric case, $\alpha_1 = \left|\alpha_3\right| = \alpha$. From Proposition 1 we know that $t_e < t_2$ whenever $w_2 < \overline{w}$ at the symmetric case, the equilibrium income tax will be lower the higher is the uncertainty toward the vote of the median voter, β, as a marginal increase in campaign spending would be more profitable than a marginal increase in voters' 2 utility. The political power of ideological voters decreases with α, then, t_e approaches the preferred tax rate of the median voter as α increases. It also approaches t_2 as ρ increases, the marginal benefit

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of pleasing the median informed voter increases with the proportion of informed voters.

Example 1 *Assume* $g\left(C_L, C_R\right) = \frac{C_L - C_R}{2(C_L + C_R)}$, *that satisfies conditions* $1-4$ *(if* $C_R = 0$*, the probability of winning among uninformed voters goes to* 1 *when* C_L *is slightly higher than zero). Contribution levels:* $C_L = \frac{(1-\rho)\alpha_1^2|\alpha_3|}{(\alpha_1+|\alpha_3|)^2}$, $C_R = \frac{(1-\rho)\alpha_1\alpha_3^2}{(\alpha_1+|\alpha_3|)^2}$. *The equilibrium income tax in the private system,*

$$t_e = \max\left\{0, \frac{2\beta\left(A\left(w_3^2 - w_1^2\right) + 3\left(\overline{w}_2 - \frac{2}{3}w_2^2\right)\right) + \rho\left(\alpha_1 + |\alpha_3|\right)\left(\overline{w}_2 - w_2^2\right)}{2\beta\left(A\left(w_3^2 - w_1^2\right) - 3\left(\overline{w}_2 - w_2^2\right)\right) + \rho\left(\alpha_1 + |\alpha_3|\right)\left(2\overline{w}_2 - w_2^2\right)}\right\}$$

where $A = \frac{\alpha_1 - |\alpha_3|}{\alpha_1 + |\alpha_3|}$. *In the symmetric case where* $\alpha_1 = |\alpha_3| = \alpha$, *for* $\beta \leq \frac{2}{3}\alpha\rho$, *the equilibrium income tax is given by:* $t_e = \frac{\left(\frac{2}{3}\alpha\rho - \beta\right)\left(\overline{w}_2 - w_2^2\right)}{\beta\left(\overline{w}_2 + w_2^2\right) + \frac{2}{3}\alpha\rho\left(2\overline{w}_2 - w_2^2\right)}$. *It equals zero otherwise.*

Example 2 *Assume* $g\left(C_L, C_R\right) = \frac{(C_L)^k - (C_R)^k}{2\left((C_L)^k + (C_R)^k\right)}$, *with* $0 < k \leq 1$, *that satisfies conditions* $1-4$. *In the symmetric case where* $\alpha_1 = |\alpha_3| = \alpha$, *contribution levels are:* $C_L = C_R = \frac{1}{4}k\alpha\left(1 - \rho\right)$. *The equilibrium income tax in the private system,*

$$t_e = \frac{\left(\frac{2}{3}\alpha\rho - k\beta\right)\left(\overline{w}_2 - w_2^2\right)}{k\beta\left(\overline{w}_2 + w_2^2\right) + \frac{2}{3}\alpha\rho\left(2\overline{w}_2 - w_2^2\right)}$$

for $\beta \leq \frac{2}{3k}\alpha\rho$, *and equals zero otherwise. Note that* t_e *decreases with* k, *which is a measure of the effectiveness of campaign spending. The higher is* k, *the higher the competition in contributions, the closer the tax rate will be to zero, the preferred tax rate of group* 3, *this is a consequence of the assumption on the skewness of the distribution, the equilibrium income tax is closer to the preferred tax of the voter with earnings ability further away from* t_2.

Example 3 *Assume* $g\left(C_L, C_R\right) = \ln C_L - \ln C_R$, *that satisfies conditions* $1-4$ *for* $\alpha_1, |\alpha_3|$ *sufficiently close. In the symmetric case where* $\alpha_1 = |\alpha_3| = \alpha$, *contribution levels are:* $C_L = C_R = \alpha\left(1 - \rho\right)$. *The equilibrium income tax in the private system,*

$$t_e = \frac{\left(\frac{1}{6}\alpha\rho - \beta\right)\left(\overline{w}_2 - w_2^2\right)}{6\beta\left(\overline{w}_2 + w_2^2\right) + \alpha\rho\left(2\overline{w}_2 - w_2^2\right)}$$

for $\beta \leq \frac{1}{6}\alpha\rho$, *and equals zero otherwise. For this advertisement function, where only absolute difference in campaign spending matters, it is easier to have a zero tax rate.*

Note that the private system may generate excessive spending, specially for $\alpha_1 = |\alpha_3|$, under which $\pi = 1/2$. In this case parties would be equally well with no campaign spending, but, the indirect effect through taxes makes the difference. An ideological voter may prefer an equilibrium with positive contributions if it induces a higher economic or ideological utility that compensates for the cost of the contributions.

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5.1 Cap on contributions

The introduction of a cap on contributions can have different effects whenever it is stringent. It decreases competition between lobbies by giving an upper bound to expenditure in contributions. Ideological voters save money but the introduction of a cap on contributions changes the equilibrium income tax announced by both parties.

Suppose $\overline{C}$ is the cap on contributions. If $\overline{C} > C_R, C_L$, the contribution cap is not stringent, then we still have the same results as in proposition 1. We next show that an intermediate cap on contributions that constraints party $R : C_L < \overline{C} < C_R$, for example, implies a higher equilibrium tax than in the absence of a cap. Consider an equilibrium income tax $t_e < t_2$ and $C_L < \overline{C} < C_R$. If both parties were choosing the same platform t_e contributions to party L and party R would be C_L and $\overline{C}$, respectively. Party L has incentives to move toward a higher tax rate $t' > t_e$. By doing so she increases contributions from group 1 and the utility of the swing voter, still $t' < t_2$, without increasing contributions to the rival party since C_R can not exceed $\overline{C}$. But party R can decrease contributions to party L to the original level C_L by replicating party L platform. Party L will find profitable to increase t_L until it reaches t_2, then a further increase in t_L implies a trade-off between increasing party L contributions and decreasing voter 2 utility. Indeed the equilibrium income tax satisfies,

$$\frac{\rho}{6\beta}\frac{\partial V_{2L}}{\partial t_L} + \frac{g_1(1-\rho)}{\alpha_1\alpha_3 g_{11}}\left(|\alpha_3|\frac{\partial V_{1L}}{\partial t_L}\right) = 0 \tag{12}$$

From (12) it can be easily shown that an intermediate cap on contributions level benefits voters in group 1 The equilibrium income tax will be higher than t_2 (or closer to t_1) and the probability of winning of party L increases by decreasing relative contribution $\frac{C_R}{C_L}$. If instead $C_R < \overline{C} < C_L$ the equilibrium income tax will be smaller than t_e and the relative advantage of group 1 will decrease. The intermediate cap on contributions, then, benefits the party that is not constrained by the cap and their partisans. This is in contrast with Vanberg (2004), where the cap on contributions decreased competition between lobbies allowing the funding advantaged group to save some money without any effect on the platform announced by parties. Here, the intermediate cap on contributions lowers the political power of the interest group contributing the most moving the equilibrium tax toward the preferred income tax of the opposite group.

To summarize our findings,

Proposition 2 *A very strict cap on contributions benefits group 2. While an intermediate level that only constraints the party with highest spending benefits the group that contributes to the opposite party.*

Proof. *In the appendix.* ∎

If the cap on contribution is sufficiently stringent it will decrease the margin of victory but, the private system with a cap on contribution do not necessarily benefit the median voter.

6 Public System

In the public system taxes are collected to finance the lump-sum transfer r and a fixed total cost of campaigning S. The new budget balance condition becomes,

$$r(t) = t\mu(t) - S$$

As in many European countries, parties obtain contributions in proportion to their voting shares. Following Ortuño-Ortín and Schultz (2005) we assume that today contributions depend upon the expected voting share from the election. Under rational expectations the expected voting share equals the effective voting share. We have a similar result to Ortuño-Ortín and Schultz; but here, since our parties are Downsian, the equilibrium income tax corresponding to the public system fully converges to the swing voter's preferred income tax.

Proposition 3 *The equilibrium income tax corresponding to the public system is the median preferred income tax* t_2. *The probability of party* L *of winning the election is* $\pi = 1/2$. **Proof.** *In the appendix.* ∎

Parties face no trade-off. If they want to maximize the probability of winning there is no point in attracting loyal voters (partisans) since they can not contribute. In order to maximize contributions, parties have to increase the probability of winning. The equilibrium income tax rate is then the median, w_2, preferred income tax rate. The revenue collected with such a tax has to be used to finance redistribution and political campaign with a fixed (exogenous) total cost of S.

7 Political support for the private system

From the results obtained in the previous sections we can assert that the comparison between the private and the public system to finance political campaign is not obvious. Depending on the distribution of earnings ability and ideological preferences the equilibrium income tax may be higher or lower than the preferred income tax of the median voter. Since voters have economic and non-economic preferences ($\alpha's$) when comparing different systems, we have to take into account these preferences.

When comparing each voter's welfare from each system the relevant information from the private system is the equilibrium income tax, t_e, the probability of winning of party L and the cost of campaigns. We know that in the public system the equilibrium income tax is t_2. The probability of winning of party L $\frac{1}{2}$. We assume the political campaign cost is exogenous and equals S, which is the minimum expenditure needed to finance political parties campaigns. The public system does not waste resources, since S is small, and benefits the median voter.

Assume $g(C_L, C_R) = \frac{C_L - C_R}{C_L + C_R}$ from Example 1. We restrict α_1, α_3 and ρ to guarantee affordable contribution levels, such that $x_j > C_j$. Let $V_j(t_2)$, be the expected indirect utility of voter j in the public system and $V_j(t_e)$ the expected utility of j in the private system with $j = 1, 2, 3$. Define $S_j^* = V_j(t_2) - V_j(t_e)$,

$$S_j^* = u_j(t_2) - u_j(t_e) + \left(\frac{1}{2} - \pi\right)\alpha_j + \frac{(1-\rho)\alpha_j^2 |\alpha_{-j}|}{(\alpha_1 + |\alpha_3|)^2}; \quad j = 1, 2, 3$$

If $S_j^* \geq S$, then voter j prefers the public to the private system. He prefers the private system otherwise. The political support for the private system is then, decreasing in S_j^*. Note that S_j^* is increasing in the level of contributions. As the intensity of preferences toward the non-economic issue increases we have two direct effects: On the one hand the cost of political campaign increases in the private system, which increases support for the public system. The probability of winning of the party associated with the voter with relatively higher α increases, which increases support by such a voter for the private system. We also have an indirect effect through taxes, as α_j increases the favored voter is further away from w_j, this increases j support for the public system. How the support for the private system is determined as a function of $\alpha's$ is not straightforward.

In our particular example from convergence of platforms $\pi = \frac{2}{3}\rho + (1-\rho)\left(\frac{1}{2} + \frac{\alpha_1 - \alpha_3}{\alpha_1 + |\alpha_3|}\right)$, S_1^* can be rewritten as,

$$S_1^* = u_1(t_2) - u_1(t_e) + \frac{\alpha_1}{(\alpha_1 + |\alpha_3|)}\left((1-\rho)\left((|\alpha_3| - \alpha_1) + \frac{\alpha_1 |\alpha_3|}{(|\alpha_3| + \alpha_1)}\right) - \frac{1}{6}\rho(|\alpha_3| + \alpha_1)\right)$$

Group 1 (similar for group 3) trade-offs economic and ideological benefits, net of cost, that in general (at least in the asymmetric case) are not aligned.

In the symmetric case $\pi = \frac{1}{2}$. Then $S_j^* = u_j(t_2) - u_j(t_e) + \frac{(1-\rho)\alpha}{4}$. Intensity of preferences toward parties increase S_j^* by increasing competition among ideological voters that foster campaign contributions. Indirectly, the equilibrium income tax is closer to the median preferred income tax as α increases, since $\frac{1-\rho}{4\alpha}$ is the (endogenously determined) weight of ideological voters utility on the parties objective. Since $u_j(t_2) - u_j(t_e)$ tends to zero as α increases, we expect less support toward the private system for α sufficiently large.

To capture the different trade-offs, we develop a numerical example to shed light on which effects dominates.

7.1 Numerical example

The parameter values we assume $\left(\beta = \frac{1}{4}, w_j, \ j = 1, 2, 3\right)$ and variables values $(\overline{w}_2, t_2)$ that we use through the example are the following:

Table I.a: *Parameters*					
$\mathbf{w}_1$	$\mathbf{w}_2$	$\mathbf{w}_3$	$\overline{\mathbf{w}}_2$	$\mathbf{t}_2$	$\mathbf{r}_2$
1	2	3	4.67	0.125	0.51

Table I.b: *Parameters*					
$\mathbf{w}_1$	$\mathbf{w}_2$	$\mathbf{w}_3$	$\overline{\mathbf{w}}_2$	$\mathbf{t}_2$	$\mathbf{r}_2$
1	1.5	3.5	5.17	0.36	1.19

Note that mean earnings ability equals 2 in both examples, but the distribution function on Table 1.b is more right skewed, in other words inequality in earnings is higher since median earnings ability, w_2, is lower than average.

We next summarize the main results from observation of Tables II and III below.

For any ρ if $\alpha_1 = |\alpha_3| = \alpha$ the public system is preferred by the median voter and one of the ideological voters. This holds as long as $|\alpha_1 + \alpha_3|$ is sufficiently small (note that perfect symmetry implies $|\alpha_1 + \alpha_3| = 0$).

If $\alpha_1 > |\alpha_3|$ the economic equilibrium will favor voters in group 3 (see section 4) and the probability of winning of party L will be higher than $\frac{1}{2}$. Being group 3 in those cases less ideological, they will prefer the private system over the public system for any S. Analogous for $\alpha_1 < |\alpha_3|$, in which case group 1 will benefit for a larger redistribution level in the private system. When is group 1 going to support the private system even with $\alpha_1 > |\alpha_3|$? She is going to support the private system as long as the non-economic benefit (higher probability of winning of party L) outweighs the economic loss from less redistribution. This is the case when $|\alpha_1 + \alpha_3|$ is sufficiently large, i.e. when intensity of preferences toward parties are asymmetric enough.

The effect of the size of informed voters: A decreased number of informed voters makes campaign contributions more effective. This is why the probability of winning of a given party increases faster as the voters' group α_j associated to such party increases. In general, the favored voter, Σ, gets further away from the median voter as the proportion of uninformed voters increases.[7]

The effect of wage dispersion: Comparing tables a and b we observe that taxes and redistribution are higher in type b tables where the median voter is poorer ($w_2 = 1.5$).

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[7]In type a tables, when $\rho = \frac{1}{2}$, the favored voter position do not decrease when $\alpha_1 > |\alpha_3|$ and do not increase when $|\alpha_3| > \alpha_1$ with respect to the position held at $\rho = \frac{3}{4}$, exceptions are found in type b tables.

Even though redistribution is higher, the favored voter is in general relatively richer than the corresponding median voter.[8]

Consider $\sum S_i^*$ as a proxy of welfare of the public system with respect to the private system, when preferences toward parties are asymmetric enough, higher welfare in the public system is associated with a higher proportion of informed voters. When preferences toward parties are symmetric higher welfare in the public system is associated with a lower proportion of informed voters. From the tables below we can observe that welfare in the public system is larger when inequality in earnings ability is low, but this seems to be true when the asymmetry of $\alpha's$ is small enough.[9]

Table II.a: *Results* $(\rho = 3/4)$							
$\alpha_1, |\alpha_3|$	*3,.3*	*.3,3*	*3,3*	*.6,3*	*3,2*	*.3,.3*	*.3,.7*
π^{PR}	0.704	0.295	0.5	0.333	0.55	0.5	0.4
Σ	>2.16	1.773	2.062	1.845	2.125	>2.16	1.897
t_e	0	0.246	0.082	0.213	0.032	0	0.186
r_e	0	0.866	0.351	0.782	0.172	0	0.707
S_1^*	-0.158	-0.189	0.308	-0.078	0.311	0.412	-0.099
S_2^*	0.042	0.039	0.005	0.021	0.023	0.042	0.010
S_3^*	-0.477	-0.019	-0.0005	-0.010	-0.187	-0.525	0.234
$\sum S_i^*$	-0.593	-0.169	0.313	-0.067	0.148	-0.072	0.145

Table II.b: *Results* $(\rho = 3/4)$							
$\alpha_1, |\alpha_3|$	*3,.3*	*.3,3*	*3,3*	*.6,3*	*3,2*	*.3,.3*	*.3,.7*
π^{PR}	0.704	0.295	0.5	0.333	0.55	0.5	0.4
Σ	>2.27	1.471	1.829	1.573	1.955	>2.27	1.915
t_e	0	0.367	0.261	0.343	0.207	0	0.225
r_e	0	1.201	0.996	1.164	0.847	0	0.901
S_1^*	0.344	0.062	0.314	0.137	0.264	0.915	0.240
S_2^*	0.526	0.0002	0.040	0.001	0.096	0.526	0.075
S_3^*	-2.363	-0.509	-0.461	-0.512	-0.789	-2.41	-0.919
$\sum S_i^*$	-1.493	-0.446	-0.107	-0.374	-0.428	-0.971	-0.604

---

[8]In absolute terms, for a given ρ, compare $(\Sigma - m)_b - (\Sigma - m)_a$, the difference between the favored voter earnings ability and the median voter earnings ability in type a and b tables. This difference is always positive, which means that the favored voter at $w_2 = 1.5$ is relatively richer with respect to the median voter than the favored voter at $w_2 = 2$.

[9]Computation for a broader range of parameters are available from author upon request.

Table III.a: *Results* $(\rho = 1/2)$							
$\alpha_1, \|\alpha_3\|$	*3,.3*	*.3,3*	*3,3*	*.6,3*	*3,2*	*.3,.3*	*.3,.7*
π^{PR}	0.909	0.091	0.5	0.167	0.6	0.5	0.3
Σ	>2.16	1.708	2.082	1.801	2.16025	>2.16	1.884
t_e	0	0.273	0.067	0.233	0.007	0	0.193
r_e	0	0.925	0.290	0.836	0.034	0	0.727
S_1^*	-0.710	-0.161	0.542	0.005	0.453	0.431	-0.068
S_2^*	0.042	0.058	0.009	0.032	0.042	0.042	0.012
S_3^*	-0.409	-0.454	0.120	-0.313	-0.104	-0.507	0.232
$\sum S_i^*$	-1.078	-0.557	0.672	-0.277	0.391	-0.034	0.177

Table III.b: *Results* $(\rho = 1/2)$							
$\alpha_1, \|\alpha_3\|$	*3,.3*	*.3,3*	*3,3*	*.6,3*	*3,2*	*.3,.3*	*.3,.7*
π^{PR}	0.909	0.091	0.5	0.167	0.6	0.5	0.3
Σ	>2.27	1.463	1.926	1.592	2.077	>2.27	1.960
t_e	0	0.369	0.220	0.337	0.142	0	0.204
r_e	0	1.203	0.887	1.155	0.629	0	0.838
S_1^*	-0.207	0.129	0.580	0.263	0.459	0.933	0.332
S_2^*	0.526	0.0003	0.080	0.002	0.194	0.526	0.100
S_3^*	-2.296	-1.049	-0.543	-0.941	-1.005	-2.394	-1.093
$\sum S_i^*$	-1.977	-0.920	0.116	-0.676	-0.353	-0.934	-0.662

From the previous numerical example we observed that it is possible to have a majority of voters in favor of the private system to finance political parties if their ideological preferences toward parties are asymmetric enough. In more homogenous societies, the public system dominates.

8 Conclusion

We study what would be the political support for the purely private system to finance political campaigns. It is worth noting that in our setting, the private system generates excessive campaign spending if voters are strongly ideological and a non-median equilibrium redistribution level. Still, we can find a majority of voters in favor of the private system because of the existing trade-off between ideological and economic preferences. In other words, as a consequence of voters' diversity on the intensity of preferences toward parties, we can find two (ideological) voters that prefer the private system and only one group favoring the public system. When ideological preferences are asymmetric enough all ideological

voters support the private system to finance political parties. In the private system the voter with the highest intensity of preferences obtains a higher probability of winning for her preferred party than in the public system and the equilibrium income tax is closer to the less ideological voter.

We also find different economic implications that differ from the ones existing in the traditional literature. In the symmetric case, if the median voter is poorer than the mean ($w_2 < \overline{w}$), it is true that the equilibrium tax rate in the private system is lower than the preferred tax rate of the median voter (in line with Roemer, 2006 and Ortuño-Ortín and Schultz, 2005). But, when we consider the asymmetric case the equilibrium income tax may be higher or lower than the median voter preferred income tax. Income inequality increases redistribution in absolute terms but the favored voter in the private system is relatively richer than the median voter when inequality in earnings ability is large.

We analyze the political support and the economic effects of a private system to finance political parties' campaign. In order to capture the individual incentives to make contributions that we find empirically, we assume that voters are contributors. A service-motive for political contributions may imply different conclusions. An imperfect correlation between types, i.e. some voters in group one and three contribute to party R and party L, respectively, will enrich our previous analysis. The introduction of service-motivated contributions and an imperfect correlation between types is left for further research.

Appendix

Appendix A: **Proof of Symmetry on Contributions**
If $t_L = t_R$, $\frac{\partial C_P}{\partial t_L} = -\frac{\partial C_P}{\partial t_R}$ with $P = L, R$.
Contribution levels satisfy equations (3) and (4) simultaneously. Define,

$$
\begin{aligned}
G_1 &= (1 - \rho) g_1 (V_{1,L} - V_{1,R}) - 1 \\
G_2 &= (1 - \rho) g_2 (V_{3,L} - V_{3,R}) - 1
\end{aligned}
$$

$\frac{\partial C_L}{\partial t_L}$ and $\frac{\partial C_R}{\partial t_L}$ $\left(\text{and } \frac{\partial C_L}{\partial t_R}, \frac{\partial C_R}{\partial t_R}\right)$ are implicitly determined at $G_1 = 0$, $G_2 = 0$.
Applying the implicit function theorem, the first derivative of C_L with respect to t_L is

$$
\frac{\partial C_L}{\partial t_L} = \frac{1}{D} \left(-g_1 g_{22} (V_{3L} - V_{3R}) \frac{\partial V_{1L}}{\partial t_L} + g_2 g_{12} (V_{1L} - V_{1R}) \frac{\partial V_{3L}}{\partial t_L} \right) \tag{13}
$$

where $D = (V_{1L} - V_{1R})(V_{3L} - V_{3R}) \left(g_{11} g_{22} - (g_{12})^2 \right)$

The first derivative of C_L with respect to t_R is

$$\frac{\partial C_L}{\partial t_R} = \frac{1}{D}\left(g_1 g_{22}\left(V_{3L}-V_{3R}\right)\frac{\partial V_{1R}}{\partial t_R} - g_2 g_{12}\left(V_{1L}-V_{1R}\right)\frac{\partial V_{3R}}{\partial t_R}\right) \qquad (14)$$

Clearly $\left(\frac{\partial C_L}{\partial t_L}+\frac{\partial C_L}{\partial t_R}\right)|_{t_L=t_R=t}=0$. Analogous for C_R : $\left(\frac{\partial C_R}{\partial t_L}+\frac{\partial C_R}{\partial t_R}\right)|_{t_L=t_R=t}=0$. As long as V_j is the same whatever the party implementing the policy (same prices w_j) and the marginal cost of contributions is constant, symmetry on contributions hold.

<u>Appendix B</u>: **Proof of propositions.**

Proof of Lemma 1:

From (3), the f.o.c. of group 1's problem, the slope of the best-response function $C_L\left(C_R\right)$ is, $\frac{\partial C_L}{\partial C_R}=-\frac{g_{12}}{g_{11}}$.

From (4), the f.o.c. of group 3's problem we have that, $\frac{\partial C_R}{\partial C_L}=-\frac{g_{21}}{g_{22}}$.

Note that from Condition 3, $sgn\left(\frac{\partial C_L}{\partial C_R}\right)=-sgn\left(\frac{\partial C_R}{\partial C_L}\right)$. Since $C_R\left(C_L\left(s\right)\right)$ is monotonic decreasing in s with $C_R\left(C_L\left(0\right)\right)>0$ and $C_R\left(C_L\left(\overline{s}\right)\right)\leq\overline{s}$ where $\overline{s}=\max\{x_{3,L},x_{3,R}\}$. Both $C_L\left(C_R\right)$ and $C_R\left(C_L\right)$, then, cross once at $C_R,C_L>0$. From condition 2 the equilibrium is asymptotically stable. If condition 4 is satisfied the $(0,0)$ equilibrium is precluded. There exist, then, a unique equilibrium with positive contributions. ∎

Proof of Proposition 1:

1) Platform convergence comes from the symmetry on contributions: $\frac{\partial C_P}{\partial t_L}=-\frac{\partial C_P}{\partial t_R}$, $P=L,R$, proved above (Appendix A). Suppose that at the equilibrium income tax second order conditions are satisfied (we perform a numerical example at section 6 for which there are satisfied). Then t_e that solves (5) when $t_L=t_R=t_e$ maximizes π.

2) From (5) evaluated at $t_L=t_2$, the equilibrium income tax will be higher than t_2 if

$$(1-\rho)\left(g_1\frac{\partial C_L}{\partial t_L}+g_2\frac{\partial C_R}{\partial t_L}\right)|_{t_L=t_R=t_2}>0 \qquad (15)$$

Applying the implicit function theorem to (3) and (4) when platforms converge we obtain equations (8) and (9). Substituting those expressions in (15), the condition to have $t_e>t_2$ is the following

$$\frac{g_2\left(g_1 g_{12}-g_2 g_{11}\right)(1-\rho)}{\alpha_1\alpha_3\left(g_{11}g_{22}-\left(g_{12}\right)^2\right)}\left(\gamma\left|\alpha_3\right|\frac{\partial V_{1L}}{\partial t_L}+\alpha_1\frac{\partial V_{3L}}{\partial t_L}\right)|_{t_e=t_2}>0 \qquad (16)$$

From Condition 3, $\alpha_1\alpha_3\left(g_{11}g_{22}-g_{12}^2\right)>0$ (remember that $\alpha_3<0$), for $g_2\left(g_1 g_{12}-g_2 g_{11}\right)>0$, which is satisfied for all $g\left(.\right)$ in examples 1-3; (16) becomes: $\left(\gamma\left|\alpha_3\right|\frac{\partial V_{1L}}{\partial t_L}+\alpha_1\frac{\partial V_{3L}}{\partial t_L}\right)|_{t_e=t_2}>$

0. Substituting $t_2 = \frac{\overline{w}_2 - (w_2)^2}{2\overline{w}_2 - (w_2)^2}$ at $\frac{\partial V_{iL}}{\partial t_L} = -(1 - t_2)(w_i)^2 + (1 - 2t_2)\overline{w}_2$, for $i = 1, 3$, in the above equation and rearranging terms the condition to have $t_e > t_2$ is

$$\Longleftrightarrow \gamma |\alpha_3| \left(-\frac{\overline{w}_2}{2\overline{w}_2 - w_2}(w_1)^2 + \frac{(w_2)^2}{2\overline{w}_2 - w_2}\overline{w}_2 \right) +$$

$$\alpha_1 \left(-\frac{\overline{w}_2}{2\overline{w}_2 - w_2}(w_3)^2 + \frac{(w_2)^2}{2\overline{w}_2 - w_2}\overline{w}_2 \right) > 0$$

$$\Longleftrightarrow \gamma |\alpha_3| (w_2)^2 + \alpha_1 (w_2)^2 - \gamma |\alpha_3| (w_1)^2 - \alpha_1 (w_3)^2 > 0$$

It can be easily shown that the above condition leads to (7) in Proposition 1.

When $\alpha_1 = |\alpha_3| = \alpha$ from the contribution stage and symmetry on contributions, both contribution levels are equal: $C_L = C_R = C$. From symmetry on contributions and differentiability of g, the following equalities are satisfied: $g_{22}(C_L, C_R) = -g_{11}(C_R, C_L)$ and $-g_{12}(C_R, C_L) = g_{21}(C_L, C_R) = g_{12}(C_L, C_R)$. Then, at the symmetric case, $-g_{12}(C, C) = g_{12}(C, C)$, which implies $g_{12}(C, C) = 0$. Substituting the last equality yields $\gamma = -\frac{g_{22}}{g_{11}} = 1$, (7) turns into

$$1 = \frac{|\alpha_3|}{\alpha_1} \geq \left(\frac{w_3^2 - w_2^2}{w_2^2 - w_1^2} \right)$$

At the symmetric case the condition for $t_e \geq t_2$ is simply $(w_3^2 - w_2^2) \leq (w_2^2 - w_1^2)$. This condition can not be satisfied for $(w_2)^2 < \overline{w}_2$. Indeed, by substituting $\overline{w}_2 = \frac{1}{3}(w_1^2 + w_2^2 + w_3^2)$, $(w_2)^2 < \overline{w}_2$ implies that $(w_3^2 - w_2^2) > (w_2^2 - w_1^2)$. From the variance of $w : \overline{w}_2 - (\overline{w})^2 > 0$, we have $\overline{w}_2 > (\overline{w})^2$, then, the following ordering is satisfied when $w_2 < \overline{w}$: $\overline{w}_2 > (\overline{w})^2 > (w_2)^2$. The equilibrium income tax is then, lower than t_2 when $w_2 < \overline{w}$ and $\alpha_1 = |\alpha_3|$.

Proof of Proposition 2:

Consider a strict cap on contribution such that $C_L, C_R > \overline{C}$. Such a cap on contribution decreases competition among ideological voters. It benefits group 2 since the objective of parties is just to maximize the utility of the swing voter (group 2). Indeed suppose both parties announce t_2, then the campaign spending by both parties equals $\overline{C}$. A deviation from t_2 by one party decreases the political support of the swing voter to this party and from the cap on contributions C_P, $P = L, R$ can not increase above $\overline{C}$. Then,. Such a deviation is not profitable.

Consider an intermediate cap on contribution, $\overline{C}$, such that $C_L < \overline{C} < C_R$. Consider $t_R = t_e$ if party L benefits their partisans $(t_L > t_R)$ she increases the probability of winning among uninformed voters that compensate the possibly loss in median voter support, note that $\frac{\partial C_R}{\partial t_L}\big|_{(t_e, t_e)} = 0$,

$$\frac{\rho}{6\beta}\frac{\partial V_{2,L}}{\partial t_L} + (1 - \rho) g_1 \frac{\partial C_L}{\partial t_L} > 0 \tag{17}$$

Party R will find profitable to replicate party L platform, platforms converge in equilibrium. When $(t_L, t_R) = (t'_e, t'_e)$ the expression (17) equals zero a further increase (or decrease) in t_L will decrease party L's probability of winning. Analogous for party R.

Proof of Proposition 3:

The probability of winning is

$$\pi = \frac{1}{2} + \frac{1}{2\beta} g \left(\pi^e S, (1 - \pi^e) S \right)$$

At $\pi^e = 0$, $\pi \geq 0$ and at $\pi^e = 1$, $\pi \leq 1$. The rational expectations probability of winning satisfies $g(.)$ is increasing in π. A fixed point π satisfying the rational expectations equilibrium exist $\pi^e = 1/2$. It will be a stable fixed point as long as $\pi > 0$ at $\pi^e = 0$. The objective function of party L at $\pi^e = 1/2$,

$$\pi = \frac{\rho}{3} + \frac{\rho}{3} \left(\frac{1}{2} + \frac{1}{2\beta} (V_{2,L} - V_{2,R}) \right) + (1 - \rho) \left(\frac{1}{2} + g \left(\frac{1}{2} S, \frac{1}{2} S \right) \right)$$

Clearly both parties choose the preferred tax rate of the median voter.

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