Partial coordination and mergers among quantity-setting firms

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Abstract

We analyze horizontal mergers in a collusive environment by using an infinitely repeated game where (i) a subset of collusive firms is exogenously given and (ii) partially collusive arrangements are allowed for. We show that, in our model, there is no clear relation between the existence of mergers and full collusion at equilibrium. However, we demonstrate that the presence of mergers generally leads to a price increase. Also, we show that cartel firms have less incentives to merge than firms in a Cournot oligopoly, and that collusion increases fringe firms' incentives to merge.
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1. Introduction

Horizontal mergers are usually seen as anti-competitive. In addition to the fact that merging two firms into one reduces the number of firms in an oligopoly, and can thus bring non-collusive equilibria closer to monopoly values, it is well known that under fairly general conditions, mergers foster collusion. More specifically, the minimum discount factor above which collusion is sustainable decreases with each merger (see for example Osborne, 1976; Vives, 1999).[1] In such models, it is assumed that collusion is absent before firms have the

choice to merge. The purpose of this paper is to study whether mergers increase incentives to collude when some collusion was already present.

We develop a multi-period oligopoly model with homogeneous, quantity-setting firms, an exogenous subset of which are assumed to collude, while the remaining (fringe) firms choose their output levels non-cooperatively.[2] We use subgame perfect Nash equilibria – henceforth, SPNE – as solution concept. It is well known that our repeated game setting exhibits multiple SPNE collusive agreements. Therefore, to select among those equilibria, following Verboven (1997) we adopt the particular criterion of (i) restricting strategies to grim "trigger strategies", and (ii) choosing the cartel's profit

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[1] An exception is Davidson and Deneckere (1984): they obtain the opposite result by assuming that firms are not allowed to redistribute output after a merger.

[2] The assumption of a cartel involving a subset of firms is based on the fact that some of the best known examples of cartels involve only a part of the industry. Some significant cases are the citric acid, the carbonless paper or the North Atlantic shipping industries.

maximizing allocation for each possible value of the discount factor, which means that cartel firms coordinate their output even when the joint profit maximization agreement does not correspond to a SPNE of the repeated game.

Our main result is that, in a collusive market, the existence of mergers without synergies leads to a price increase. This insight extends the results by Farrell and Shapiro (1990) and Spector (2003) to the case where firms can partially coordinate their output before they are allowed to merge. Also, our model enables us to show that the minimum discount factor required for the joint profit maximization agreement to be sustainable may either increase or decrease as a result of a merger. This leads us to believe that very little can be said about the relation between the effect of mergers on the above mentioned threshold and the competitive effects of mergers. Thus, the interpretation that mergers foster collusion by decreasing this minimum discount factor, as traditionally argued from an anti-trust viewpoint, can be misleading.

Using our model, we also study the incentives of firms to merge, and we show that (i) cartel firms have less incentives to merge than firms in the Cournot oligopoly, and (ii) collusion increases fringe firms' incentives to merge. These results extend those obtained by Salant et al. (1983) – henceforth, SSR – to the case of pre-merger collusion.

The rest of the paper is structured as follows. In Section 2, we present the model. In Section 3, we analyze the effect of mergers on price and firms' incentives to merge. Section 4 tests the robustness of our results using an optimal punishment – the "stick-and-carrot strategies" proposed by Abreu (1986, 1988) – and establishes that the main results continue to hold when all firms collude. We conclude in Section 5. All proofs are grouped together in the Appendix.

2. The model

We consider an industry with N firms. Each firm produces a quantity of a homogeneous product at a constant marginal cost $c > 0$. The industry inverse demand is given by the piecewise linear function:

$$p(Q) = \max(0, a - Q),$$

where Q is the industry output, p is the output price, and $a > 0$ with $a > c$. We assume that $K \in [2, N)$ firms, indexed by $k = 1,..., K$ – henceforth, cartel firms – behave cooperatively so as to maximize their joint profits. The remaining $(N-K)$ firms constitute the fringe and choose their output in a non-cooperative way.

We assume that only one cartel is formed, and we take K as exogenously fixed.[3] The assumption of an exogenously given subset of firms colluding is based on the fact that cartels often involve an agreement between firms which can easily coordinate with each other (e.g. because they are based in the same country or have a common corporate culture). The fringe consists of foreign firms or new entrants that could not coordinate their behavior with the cartel firms even if they wish so.[4]

We assume that firms compete repeatedly over an infinite horizon with complete information (i.e. each of the firms either fringe or cartel observes the whole history of actions) and discount the future using a discount factor $\delta \in (0, 1)$. Following the cartel and fringe literature, we assume that in each period the cartel behaves as a Stackelberg leader with respect to the fringe.[5] Time is discrete and dates are denoted by $t = 1, 2,....$ In this framework, a pure strategy for firm k is an infinite sequence of functions $\{S_k^t\}_{t=1}^{\infty}$ with $S_k^t : \sum^{t-1} \to \mathcal{Q}$ where $\sum^{t-1}$ is the set of all possible histories of actions (output choices) of all cartel firms up to $t-1$, with typical element σ_j^τ, $j = 1,..., K$, $\tau = 1,..., t-1$, and $\mathcal{Q}$ is the set of output choices available to each cartel firm. Following Friedman (1971), we restrict our attention to the case where each cartel firm is only allowed to follow grim trigger strategies. In words, these strategies are such that cartel firms adhere to the collusive agreement until there is a defection, in which case they revert forever to the static N-firm Cournot equilibrium.[6] Let q and q_n denote the output corresponding respectively to collusion and Cournot non-cooperative behavior. Since we restrict attention to trigger strategies, $\{S_k^t\}_{t=1}^{\infty}$ can be specified as follows. At $t=1$, $S_k^1 = q$, while at $t=2, 3,...$

$$S_k^t(\sigma_j^\tau) = \begin{cases} q & \text{if } \sigma_j^\tau = q \text{ for all } j = 1,...,K \text{ and } \tau = 1,...,t-1 \\ q_n & \text{otherwise} \end{cases}.$$

$$(1)$$

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[3] Using the concept proposed by d'Aspremont et al. (1983) for cartel stability, it is well known that if we endogenize cartel formation only cartels containing just over half the firms in the industry are stable (see for instance Donsimoni et al., 1986; Shaffer, 1995).

[4] As an example, three North-American and five European firms in the citric acid industry were fined for fixing prices and allocating sales in the worldwide market. Their joint market share was around 60%. The rest of the producers included a variety of minor companies based in Eastern Europe, Russia and China (see Levenstein et al., 2003).

[5] The seminal papers in this literature are Selten (1973) and d'Aspremont et al. (1983) in a static model, and Martin (1993) in a dynamic setting.

[6] We note that the punishment consists of cartel firms losing the strategic advantage of the leadership. This is based on the fact that, in a symmetric Cournot model, an endogenous sequence of play between a cartel and a Cournot fringe will assign a leader's role to the cartel and a follower's role to the fringe (see Shaffer, 1995).

Regarding fringe firms, their optimal response consists of maximizing their current period's payoff, in such a way that if each cartel firm produces q, then the output produced by each fringe firm, that we denote by q_f, is

$$q_f = \max\left\{0, \left(\frac{a - c - Kq}{N - K + 1}\right)\right\}.$$

The profit function of a cartel firm is given by

$$\Pi^c(N, K, q) = (a - c - Kq - (N - K)q_f)q$$

and that of a fringe firm by

$$\Pi^f(N, K, q) = (a - c - Kq - (N - K)q_f)q_f.$$

As shown by Friedman (1971), cartel firms producing q in each period can be sustained as a SPNE of the repeated game with the strategy profile (1) if and only if for given values of N, K and δ, the following condition is satisfied

$$\frac{\Pi^c(N, K, q)}{1 - \delta} \geq \Pi^d(N, K, q) + \frac{\delta \Pi(N)}{1 - \delta} \qquad (2)$$

where $\Pi^d(N, K, q)$ denotes the profits attained by an optimal deviation from a collusive output q, and $\Pi(N)$ denotes the Cournot equilibrium profits. Multiplicity of equilibria is obtained since condition (2) is satisfied for different collusive outputs. To select among such equilibria, we follow Verboven (1997) and, thereby, we choose the profit maximizing allocation for the cartel (i.e. the allocation that solves the problem: $\max_q \Pi^c(N, K, q)$ subject to (2)). Then, if δ exceeds a certain critical level, (2) is not a binding constraint, and the distribution of output in the cartel is the symmetric distribution of the output of a unique Stackelberg leader (namely, $q = \frac{a-c}{2K}$). If δ is below that critical level, (2) is a binding constraint and the distribution of output is the solution to the equality constraint in (2). Thus, in this case the equilibrium quantities (q, q_f) depend on δ. Let us denote the above mentioned critical level of the discount factor by $\bar{\delta}(N, K)$. It can be verified that

$$\bar{\delta}(N, K) = \frac{(N + 1)^2}{(N + 1)^2 + 4K(N + 1 - K)}.$$

It is worthwhile noting that our model exhibits equilibria where coordination on distinct output levels exists.

Definition 1. For each $K \leq N$ collusion is partial (respectively, full) whenever $\delta \in (0, \bar{\delta}(N, K))$ (respectively, $\delta \geq \bar{\delta}(N, K)$).

One may wonder how q varies with δ when collusion is partial. The sign of $\frac{\partial q}{\partial \delta}$ depends on the number of firms

in the cartel: when the cartel is small $\left(K < \frac{N+1}{2}\right)$ cartel firms' output increases with δ. Conversely, when the cartel is large $\left(K > \frac{N+1}{2}\right)$ cartel firms' output decreases with δ. Intuitively, in the first case cartel firms mimic the Stackelberg leader and, therefore, wish to increase their market share. In the second case, since the cartel controls most of the market, cartel firms cut production to increase price.

Lemma 1. *When collusion is partial, price decreases (respectively, increases) with δ if $K < \frac{N+1}{2}$ (respectively, if $K > \frac{N+1}{2}$).*

Furthermore, it can be easily verified that, for each $\delta > 0$, price under partial collusion is above (respectively, below) the Cournot price if $K > \frac{(N+1)}{2}$ (respectively, if $K < \frac{N+1}{2}$).[7]

3. Partial coordination and mergers

In this section, we analyze the effect of horizontal mergers on the extent to which firms compete. Also, we study the incentives of firms to merge.

3.1. The effect of mergers on price

It is widely believed by theorists and policy makers that horizontal mergers increase the propensity of firms to collude. The existing literature concentrates on analyzing the effect of mergers on the minimum discount factor required to sustain full collusion. For instance, in the repeated symmetric linear Cournot game the monopoly outcome can be sustained as a SPNE using trigger strategies if $\delta \geq \frac{(N + 1)^2}{(N + 1)^2 + 4N} = \delta(N)$, where N is the number of firms. Given that $\frac{\partial \delta(N)}{\partial N} > 0$, it is harder to collude with more firms, and therefore mergers foster collusion. Our model also permits us to study the effect of mergers on $\bar{\delta}(N, K)$. Assume that the timing of the game is as follows: first, there is the cartel formation; second, mergers among cartel or fringe firms occur; third, firms produce.[8] Assume also that the marginal

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[7] One can also check that profits of each cartel firm are enhanced by cartelization relative to symmetric Cournot oligopoly. Regarding fringe firms, profits are enhanced by cartelization if $K > \frac{N+1}{2}$.

[8] Throughout the paper, we will consider only mergers among cartel firms or among fringe firms. If we consider mergers between a cartel and a fringe firm, it makes sense to follow the reasoning of Huck et al. (2001): "If a leader merges with a follower in a market with quantity competition the new firm will stay a leader because the old firm can still use the old commitment technology of the former leader to commit itself on high output". However, in our model, since a fringe firm earns higher profits than a cartel firm when $K > \frac{N+1}{2}$, it is not clear which should be the status of the new entity.

costs of the firms remain unchanged after the merger. Then, once $M+1$ cartel firms have merged there will be $K-M$ cartel firms and $N-(K-M)$ fringe firms. Similarly, once $M+1$ fringe firms have merged there will be K cartel firms and $N-K-M$ fringe firms. Surprisingly, the result is partially ambiguous: it is easy to show that mergers may either increase or decrease the minimum discount factor above which full collusion is sustainable. More specifically, the merger of $M+1$ cartel firms helps full collusion ($\overline{\delta}(N-M, K-M)<\overline{\delta}(N, K)$) if $K>\frac{N+1}{2}$ and it hinders full collusion if $K<\frac{N+1}{2}$. On the other hand, the merger of $M+1$ fringe firms helps full collusion ($\overline{\delta}(N-M, K)<\overline{\delta}(N, K)$) if $K<\frac{N+1}{2}$ and it hinders full collusion if $K>\frac{N+1}{2}$.

From the analysis above one may conclude that, in our model, it is not clear that the existence of horizontal mergers favours a collusive behavior at equilibrium. However, in the present case if firms fail to sustain full collusion they can sustain partial collusion. As a consequence, it is more informative to use our model to study the effect of mergers on a variable that reflects more precisely the degree of collusion in the industry. We choose the price for that purpose. We obtain that the existence of mergers generally leads to a price increase. This result is presented in the following two propositions.

Proposition 2. *The merger of $M+1$ cartel firms does not decrease price.*

The price increase is always strict except for values of the discount factor such that full collusion is sustained before and after the merger. In this case, the aggregate output of cartel firms is the joint Stackelberg outcome and remains unchanged after the merger. Therefore, fringe firms do not change their output either in response to the merger and price does not increase. Interestingly enough, price increases when full collusion was sustainable before the merger but it is not sustainable after the merger. The intuition is as follows. It can be easily shown that $\delta\in(\overline{\delta}(N, K), \overline{\delta}(N-M, K-M))$ occurs only when the cartel is relatively large before the merger $\left(K>\frac{N+1}{2}\right)$ but relatively small after the merger $\left(K-M<\frac{N-M+1}{2}\right)$, which means that the merger involves a large fraction of cartel firms. In this case, the reduction in the aggregate output of the merging firms is larger than the increase in the aggregate output of the non-merging firms.[9] As a consequence, price increases even though full collusion cannot be sustained after the merger.

Proposition 3. *The merger of $M+1$ fringe firms strictly increases price.*

Price strictly increases even when full collusion is sustained before and after the merger. In this case, the aggregate output of cartel firms does not change after the merger. However, fringe firms contract their aggregate output when they merge and this raises price. We observe also that price increases when full collusion was sustainable before the merger but it is not sustainable after the merger. In this case, it can be verified that $\delta\in(\overline{\delta}(N, K), \overline{\delta}(N-M, K))$ is only true if $K>\frac{N+1}{2}$. Intuitively, when the cartel is relatively large each fringe firm produces more than each cartel firm, in such a way that a reduction in the number of fringe firms reduces the industry output although the aggregate output of the non-merging firms is expanded. Consequently, even though full collusion cannot be sustained after the merger, price increases.

We can now compare our results with those obtained by Farrell and Shapiro (1990) and Spector (2003): mergers without synergies raise price not only in the Cournot oligopoly but also in a collusion model. Furthermore, our results provide the interpretation that, when the anti-trust analysis of horizontal mergers is focused on firms' ability to sustain full collusion, it is not always clear whether mergers foster collusion or not. This is so because with partial collusion, a merger may either increase or decrease $\overline{\delta}(N, K)$. Yet, as shown by Propositions 2 and 3, mergers generally lead to a price increase.[10]

3.2. The incentives to merge

It is well known that the price increase does not guarantee by itself the profitability of a merger. For example, in a Cournot setting (as in SSR) although mergers increase price, they are (generally) not profitable because non-merging firms react to the merger by expanding their output.[11] Therefore, given that the present

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[9] The output expansion of non-merging firms is a well known result by SSR for the case of pre-merger Cournot competition. We note that it also holds in our model when collusion is partial.

[10] Compte et al. (2002) show that in a setting where firms have asymmetric capacities the effect of mergers is also ambiguous whenever it involves the largest firm. In particular, mergers increasing market share asymmetry may hinder collusion. However, in their paper, restricting attention to the sustainability of full collusion is validated by the fact that full collusion is sustainable whenever some collusion is sustainable. By contrast, in our paper if $\delta<\overline{\delta}(N, K)$ collusion is only partial.

[11] SSR show that the minimum profitable merger under Cournot oligopoly involves at least 80% of the firms in the industry. This paradoxical result is valid only in Cournot environments, and generally fails to hold in differentiated Bertrand models. This is an issue not raised here and left for future research. It would also be interesting to see an extension to a wider range of demand functions (see Cheung, 1992; Faulí-Oller, 1997).

model encompasses the Cournot case if $\delta=0$, we can check whether the results by SSR are sensitive to the assumption of pre-merger Cournot competition.

Given the nature of each firm's profits in our model, it is easy to see that we can write the profits of each firm as a function of δ. We denote the incentives to merge for $M+1$ cartel firms and $M+1$ fringe firms respectively by

$$\Pi^c(N-M, K-M, \delta) - (M+1)\Pi^c(N, K, \delta) \qquad (3)$$

$$\Pi^f(N-M, K, \delta) - (M+1)\Pi^f(N, K, \delta). \qquad (4)$$

We denote the (unique) root in M such that (3) is equal to zero by M_c^*. Equivalently, the (unique) root in M such that (4) is equal to zero is denoted by M_f^*. For given values of N, K and δ, M_c^* and M_f^* represent respectively the minimum number of cartel and fringe firms that are involved in the minimum profitable merger. Thus, $\frac{M_c^*}{K}$ and $\frac{M_f^*}{N-K}$ denote respectively the fraction of cartel and fringe firms required for a merger to be profitable.

Proposition 4. *When collusion is partial, the fraction of cartel firms required for a merger to be profitable is larger than the fraction of firms required in the symmetric Cournot oligopoly.*

Cartel firms have less incentives to merge than firms in the Cournot oligopoly. Intuitively, in absence of synergies derived from a merger, if firms are already sustaining a collusive agreement a merger loses attractiveness as an anti-competitive device.

As for fringe firms' incentive to merge, we obtain:

Proposition 5. *When collusion is partial, the fraction of fringe firms required for a merger to be profitable is larger than under full collusion.*

The intuition behind Proposition 5 is as follows. As mentioned earlier, unprofitability of horizontal mergers with quantity competition comes from the fact that non-merging firms react to the merger by increasing their output. In our model, when collusion is partial, cartel firms expand their output in response to the merger of fringe firms. However, under full collusion the aggregate output of cartel firms is the output of a unique Stackelberg leader and remains unchanged after the merger of fringe firms.

4. Extensions

To test the robustness of our results it is natural to consider a set of strategies that are less grim than the trigger strategies. We consider here the two-phase output path (with a stick-and-carrot pattern) presented by Abreu (1986, 1988). Unfortunately, the optimality of the simple penal code of Abreu occurs in a model where firms *simultaneously* choose their output. Therefore, we cannot use such a penal code in the model presented in Section 2. Instead, we choose the approach consisting of restricting attention to the limit case $K=N$, and considering an industry of $N\geq 2$ collusive firms, indexed by $i=1,...,N$.

As in Section 2, the strategy space consists of a sequence of decisions rules, describing each player's action as a function of the past history of the play. Then, a pure strategy for firm i is an infinite sequence of functions $\{S_i^t\}_{t=1}^{\infty}$ with $S_i^t: \sum^{t-1} \to \mathcal{Q}$ where $\sum^{t-1}$ is the set of all possible histories of actions of all firms up to $t-1$, with typical element σ_j^τ, $j=1,...,N$, $\tau=1,...,t-1$, and $\mathcal{Q}$ is the set of output choices available to each firm. Following Abreu (1986, 1988), we restrict our attention to the case where each firm is only allowed to follow a stick-and-carrot strategy. In other words, we assume that if a deviation from the collusive agreement occurs, then all firms expand their output for one period so as to drive price below cost and return to the most collusive sustainable output in the remaining periods, provided that every player went along with the first phase of the punishment. Let q and q^p denote the output produced by each firm in a collusive and in a punishment phase respectively. $\{S_i^t\}_{t=1}^{\infty}$ can be specified as follows. At $t=1$, $S_i^1=q$, while at $t=2, 3,...$

$$S_i^t(\sigma_j^\tau) = \begin{cases} q & \text{if } \sigma_j^\tau = q \text{ for all } j = 1,...,N \text{ and } \tau = 1,...,t-1 \\ q & \text{if } \sigma_j^\tau = q^p \text{ for all } j = 1,...,N \text{ and } \tau = t-1 \\ q^p & \text{otherwise.} \end{cases}$$

$$(5)$$

Under the conditions specified in Abreu (1986), all firms producing q in each period can be sustained as a SPNE of the repeated game with the strategy profile (5). As in Section 2, a multiplicity of equilibria is obtained. Then, for given values of N and δ, we choose the profit maximizing allocation for the cartel. Straightforward calculations show that if $N>3 + 2\sqrt{2} \simeq 5.83$ the cutoff of the discount factor is $\overline{\delta}(N) = \frac{(N-1)^2}{(N+1)^2}$. Let us concentrate on this case.

Proposition 6. *If $K=N$, the merger of $M+1$ firms does not decrease price.*

The price increase is strict except if $\delta \geq \overline{\delta}(N)$, in which case firms sell at the monopoly price before and after the merger.[12]

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[12] We note that in this case the minimum discount factor required to sustain full collusion decreases with each merger $\left(\frac{\partial \overline{\delta}(N)}{\partial N} > 0\right)$.

Regarding the incentives to merge, if we denote the profit of a firm by $\Pi^c(N, \delta)$, then the incentive for $M+1$ firms to merge is given by: $\Pi^c(N-M, \delta)-(M+1)\Pi^c(N, \delta)$.

Proposition 7. *If $K=N$ and $0<\delta<\bar{\delta}(N)$, the fraction of firms required for a merger to be profitable is larger than in the symmetric Cournot oligopoly.*

Propositions 6 and 7 parallel Propositions 2 and 4 for the $K=N$ case. Thus, we have studied a more severe punishment and established that, when all firms collude, the results of Section 3 continue to hold.

5. Concluding comments

We have developed a theoretical framework to study the effects of horizontal mergers in a collusive environment, a problem that, to the best of our knowledge, has not been extensively considered. We show that although the critical discount factor above which joint profit maximization is sustained may increase (due to a merger), the effect of a merger on price is unambiguous; price increases. Our results provide an interpretation contrary to the traditional anti-trust view that mergers only foster collusion if they decrease the critical discount factor above which full collusion can be sustained. Also, the fact that we assume that firms set quantities, enables us to compare our results with those obtained by Farrell and Shapiro (1990) and Spector (2003). Mergers without synergies raise price not only in the Cournot oligopoly but also when some collusion was already present before the merger.

We also analyze the incentives to merge. We prove that the results by SSR are sensitive to the assumption of pre-merger Cournot competition. We demonstrate that cartel firms have less incentives to merge than firms under Cournot competition, and that collusion increases fringe firms' incentives to merge.

The framework we have worked with is, admittedly, a particular one. To analyze real-world cases of mergers, firms' capacities, cost asymmetries or synergies should also be considered. We believe that those are subjects for future research.

Appendix A

Proof of Lemma 1. Price is given by the following expression:

$$p(N,K,\delta) = \begin{cases} \dfrac{(1+N)^2(a+cN)-(1-2K+N)(a(1+2K+N)+c(N+N^2-2K(2+N)))\delta}{(1+N)^3-(1+N)(1-2K+N)^2\delta} & \text{if } <\bar{\delta}(N,K) \\ \dfrac{a+c(1+2(N-K))}{2(N-K+1)} & \text{if } \geq \bar{\delta}(N,K). \end{cases} \tag{6}$$

If $\delta<\bar{\delta}(N, K)$, then $\dfrac{\partial p(N,K,\delta)}{\partial \delta} = \dfrac{4(a-c)K(2K-1-N)(1+N)}{((1+N)^2-(1-2K+N)^2\delta)^2}\left.\begin{array}{l} > 0 \text{ if } K > \dfrac{N+1}{2} \\ < 0 \text{ if } K < \dfrac{N+1}{2} \end{array}\right.$. $\square$

Proof of Proposition 2. From (6) it is easy to see that $p(N, K, \delta)$ is continuous in $\bar{\delta}(N, K)$. We denote the pre and post-merger cutoff respectively by $\bar{\delta}^1$ and $\bar{\delta}^2$. First, we prove that the following is true:

$$p(N-M, K-M, \delta) > p(N, K, \delta) \text{ if } \delta<\min\{\bar{\delta}^1, \bar{\delta}^2\}. \tag{7}$$

Note that $p(N-M, K-M, 0)>p(N, K, 0)$. On the other hand, $\exists$ only one $\delta\in(0, 1)$ such that $p(N-M, K-M, \delta)=p(N, K, \delta)$. We denote it by δ^0. We can check that if $\bar{\delta}^1<\bar{\delta}^2$, then $\delta^0>\bar{\delta}^1$, but if $\bar{\delta}^1>\bar{\delta}^2$, we have $\delta^0>\bar{\delta}^2$. If $\bar{\delta}^1=\bar{\delta}^2$, then $\delta^0=\bar{\delta}^2=\bar{\delta}^1$. Therefore, (7) is true. If $\delta<\min\{\bar{\delta}^1, \bar{\delta}^2\}$, (7) ensures that a merger (strictly) increases price. If $\delta\geq\max\{\bar{\delta}^1,$

$\overline{\delta}^2\}$ pre and post-merger price are the same, as $\frac{a+c(1+2(N-K))}{2(N-K+1)} = \frac{a+c(1+2(N-M-(K-M)))}{2(N-M-(K-M)+1)}$. For the remaining values of δ, we have two different relevant cases to consider: the first one is If $M > (N+1)\frac{2K-N-1}{K}$ and $\overline{\delta}^1 < \overline{\delta}^2$. The second case is if $M < (N+1)\frac{2K-N-1}{K}$ and $\overline{\delta}^1 > \overline{\delta}^2$. We know that $p(N, K, \overline{\delta}^1) = p(N-M, K-M, \overline{\delta}^2) = \frac{a+c(1+2(N-K))}{2(N-K+1)}$. In the first case, it is enough to check that $p(N-M, K-M, \delta)$ is strictly decreasing with δ. This is true if $M > 2K - N - 1$, and this holds since $M > (N+1)\frac{2K-N-1}{K}$. In the second case it is enough to check that $p(N, K, \delta)$ is strictly increasing with δ, and from Lemma 1 this is true if $2K - N - 1 > 0$, and this holds because otherwise $M < (N+1)\frac{2K-N-1}{K}$ could never hold. $\quad\square$

Proof of Proposition 3. From (6) it is easy to check that the following holds:

$$\frac{\partial p(N, K, \delta)}{\partial N} < 0. \tag{8}$$

We denote the pre-merger and post-merger cutoff by $\overline{\delta}^1$ and $\overline{\delta}^2$ respectively. If $\delta < \min\{\overline{\delta}^1, \overline{\delta}^2\}$, (8) is enough to prove that price strictly increases. If $\delta \geq \max\{\overline{\delta}^1, \overline{\delta}^2\}$, we can see that since $p(N, K, \overline{\delta}^1) = \frac{a+c(1+2(N-K))}{2(N-K+1)} < p(N-M, K-M, \overline{\delta}^2) = \frac{a+c(1+2(N-M-K))}{2(N-M-K+1)}$, price strictly increases. For the remaining situations, we have two relevant cases: the first is if $M > (N+1)\frac{N+1-2K}{N-K+1}$, and then we have that $\overline{\delta}^1 < \overline{\delta}^2$. The second case is if $M < (N+1)\frac{N+1-2K}{N-K+1}$, we have then that $\overline{\delta}^1 > \overline{\delta}^2$. In the first case, since $p(N-M, K, \overline{\delta}^1) > p(N, K, \overline{\delta}^1)$, it is enough to check that $p(N-M, K, \delta)$ is increasing with δ, which is true if $M > N - 2K + 1$, and this holds since $M > (N+1)\frac{N+1-2K}{N-K+1}$. In the second case, since $p(N-M, K, \overline{\delta}^2) > p(N, K, \overline{\delta}^2)$ (remember (8)), from Lemma 1 $p(N, K, \delta)$ is decreasing with δ if $N+1 > 2K$, and this holds because otherwise $M < (N+1)\frac{N+1-2K}{N-K+1}$ could never be true. $\quad\square$

Proof of Proposition 4. Cartel firms have incentives to merge when (3) is equal or greater than zero. From SSR we know that $\frac{\partial(\Pi^c(N-M,K,0)-(M+1)\Pi^c(N,K,0))}{\partial M}\Big|_{M=0} < 0$. At the same time, since $\frac{\partial(\Pi^c(N-M,K,\delta)-(M+1)\Pi^c(N,K,\delta))}{\partial \delta}\Big|_{M=0} = 0$ we have that the incentive constraint equal to zero has only one root in M, denoted by M_c^*, (for all $M+1 \in (1, K)$). Basically, what we have to do is proving that $\Pi^c(N-M, K-M, 0) - (M+1)\Pi^c(N, K, 0) > \Pi^c(N-M, K-M, \delta) - (M+1)\Pi^c(N, K, \delta)$. If $\delta < \overline{\delta}(N, K)$, then $\Pi^c(N, K, \delta) = \frac{(a-c)^2(1-N(2+N)(-1+\delta)-\delta+4K^2\delta((1+N)^2+(1-2K+N)(3-2K+3N)\delta))}{((1+N)^3-(1+N)(1-2K+N)^2\delta)^2}$.

Since $\Pi^c(N, K, \delta) - \Pi^c(N, K, 0) = \frac{4(a-c)^2(1-2K+N)^2(1-\delta)\delta}{((1+N)^2-(1-2K+N)^2\delta)^2} > 0$, and $\Pi^c(N, K, \delta)$ is a continuous and monotonic function with respect to δ, it is enough to prove that $\frac{\partial(\Pi^c(N-M,K-M,\delta)-(M+1)\Pi^c(N,K,\delta))}{\partial \delta}\Big|_{\delta=0} = 4(a-c)^2\left(\frac{4(K-M)}{(1-M+N)^3} - \frac{4K^2(1+M)}{(1+N)^4} + \frac{4K(1+M)}{(1+N)^3} + \frac{1+M}{(1+N)^2} + \frac{4(K-M)^2}{(1-M+N)^4} + \frac{1}{(1-M+N)^2}\right) > 0$. It is tedious but straightforward to show that the last expression is positive for $K \leq N$, $M+1 \leq K$ and $\delta < 1$. $\quad\square$

Proof of Proposition 5. Fringe firms have incentives to merge when (4) is equal or greater than zero. Following the reasoning of the last proof, we have to prove that $\Pi^f(N-M, K, \overline{\delta}(N-M, K)) - (M+1)\Pi^f(N, K, \overline{\delta}(N, K)) > \Pi^f(N-M, K, \delta) - (M+1)\Pi^f(N, K, \delta)$.

If $\delta < \overline{\delta}(N, K)$, then $\Pi^f(N, K, \delta) = \frac{(a-c)^2(-1+N(2+N)(-1+\delta)+\delta-4K^2\delta)^2}{((1+N)^3-(1+N)(1-2K+N)^2\delta)^2}$ is monotonic and continuous in δ. In this case, it is enough to prove that $\frac{\partial(\Pi^f(N-M,K,\delta)-(M+1)\Pi^f(N,K,\delta))}{\partial \delta}\Big|_{\delta=\overline{\delta}(N-M,K)} = \frac{(a-c)^2M(-1+2K+N)(-4K^2+4K(1+N)+(1+N)^2)^2}{16K(-1+K-N)^3(1+N)^3} < 0$. $\quad\square$

Proof of Proposition 6. If $N \geq 3 + 2\sqrt{2}$, collusion is partial and firms play the strategies described in (5), then the output produced by each firm is given by the following expression:

$$q = \frac{(a-c)(1+\delta+N(1-\delta)-2\sqrt{\delta})}{(1+N)^2-(1-N)^2\delta} \text{ if } \delta < \frac{(N-1)^2}{(N+1)^2} \text{ and } \frac{a-c}{2N} \text{ if } \delta \geq \frac{(N-1)^2}{(N+1)^2}.$$

If $\delta < \frac{(N-1)^2}{(N+1)^2}$, then price is given by: $p = \frac{a(1+N+(-1+N)\delta)+N(c(1+N)-c(N-1)\delta+2\sqrt{(a-c)^2\delta})}{(1+N)^2-(N-1)^2\delta}$ and $p = \frac{a+c}{2N}$ if $\delta \geq \frac{(N-1)^2}{(N+1)^2}$.

We note that $\frac{\partial p}{\partial N} = \frac{(a-c)N((a-c)((1+N)^2 - (N-1)^2\delta) + 2a((N-1)^2\delta) + 2(N^2-1)\sqrt{(a-c)^2\delta})}{\sqrt{(a-c)^2\delta((1+N)^2 - (N-1)^2\delta)^2}} > 0$. Thus, mergers increase price when full collusion is not sustained neither before nor after the merger. It is also easy to see that $\frac{(N-1)^2}{(N+1)^2}$ increases with N. Hence, the price increase is always strict except for high values of the discount factor such that full collusion is sustained before the merger. In this case, the price is equal to $\frac{a+c}{2}$ before and after the merger. $\quad\square$

Proof of Proposition 7. The minimum number of cartel firms that are involved in the minimum profitable merger is the (unique) root in M of the incentive for $M+1$ firms to merge. We denote it by M^*, and is given by the following expression: $M^* = N - \frac{\left(3\delta + \sqrt{(\sqrt{\delta}-1)^3(\sqrt{\delta}(4N-3)-4N-5)} - 1 - 2\sqrt{\delta}\right)}{2(\sqrt{\delta}-1)^2}$. Given that the incentive constraint is increasing in M around M^*, the result follows since $\frac{\partial M^*}{\partial \delta} = \frac{-1+\sqrt{(\sqrt{\delta}-1)^3(\sqrt{\delta}(4N-3)-4N-5)}+2\sqrt{\delta}-\delta}{(\sqrt{\delta}-1)^2\sqrt{(\sqrt{\delta}-1)^3(\sqrt{\delta}(4N-3)-4N-5)\sqrt{\delta}}} > 0$. $\quad\square$

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