

A nonlinear analysis of the Spanish economy

JOB MARKET PAPER

Federico Lampis*

Abstract

For many years the economic time series analysis has been dominated by the linear paradigm and the Box-Jenkins approach. Lately the influence of the nonlinear models have extended so far this kind of data, especially to study the business cycle in macroeconomics data. The aim of this paper is to analyze the nonlinearities of Spanish Economy, using a Self-Exciting Threshold Autoregressive model (SETAR) to estimate the Spanish Index of Industrial Production. The hypothesis of linearity in the model is refused, and the SETAR model improves the fitting of the series, moreover it allows to add information above the dynamic of the series. The relevance of the results lies in the analysis of the cyclical fluctuation of the Spanish economy. Finally it is performed an exercise of prediction where the results show that root-mean-square error of the SETAR is lower than the linear model.

Keywords: Spanish economy, nonlinear time series models, threshold autoregressive models.

*Department of Applied Economics III (Econometrics and Statistics) Faculty of Economics and Management Avda. Lehendakari Aguirre, 83 E-48015 BILBAO. E-mail: federico.lampis@ehu.es.

1 Introduction

For a long time the Box-Jenkins approach and the use of linear models dominated the times series univariate analysis. The long-lasting popularity of Box-Jenkins approach is justified by its usefulness in the analysis of time series data. The ARIMA models are very simple to specify, estimate and interpret. However, the empirical evidence of most of the existing literature outlines the limitations of the ARIMA models in case of time series generated by a nonlinear data generation process (DGP). This is logical, because the linear modelling is always the first representation of an unknown dynamic relationship, and when the dynamic is nonlinear the ARIMA model is unable to identify this nonlinear relationship, Tong (1990).

The nonlinear approach has a large tradition in the field of the finance and monetary markets, where the presence of nonlinearities and asymmetries are more evident. Meanwhile the use of nonlinear times series models has been neglected for a long time in the economic times series area, Franses and Van Dijk (2000). The most important feature of the nonlinear time series models is that their operation in the presence of different state of the world or regimes allows the possibility for the dynamic behavior to depend on the regimes occurring at any given point in the time (Tong, 1990).

The nonlinear models can be thought as a “piecewise” linear approximation of a nonlinear stochastic process via partitioning its state-space into several subspaces. The use of nonlinear model is highly related to the concept of business cycle, because they represent better than the linear models

the asymmetries present in the cycle. Although business cycle asymmetry is an old topic in economics (for example, Mitchell (1927), Keynes (1936)), until recently economists have generally neglected nonlinearities in empirical business cycle modeling preferring the use of linear time series specifications.

In spite of that lack of interest, the nonlinear analysis has extended its influence to macroeconomics data. The reason of this last fact is the capacity of the nonlinear analysis to recover the asymmetries in the business cycle to study the GDP or other macroeconomic data. The first study using nonlinear models for the analysis of the asymmetries in the US GDP was the work published by Hamilton (1989), that generated an important series studies in the same field. After this seminal paper more have been written in the same line research, as Tiao and Tsay (1991)¹, Teräsvirta and Anderson (1992), Potter (1995), Hansen (1997), Hansen (2000).

All these works study the nonlinearities in the USA economy and the asymmetry in its business cycle. Asymmetrical movements involve the possibility of differentiation between the the shape of expansive regime and the shape of recessive regime in amplitude and length. Moreover it involves that the function which represents the stochastic process may have a different dynamic around the turning points. The nonlinear models are the best models to represent this feature of the economic time series and this is the reason of their recent success. More recently a new research line using these results to

¹They estimate a TAR model for the rate of growing of American GDP with three regimes, in this way they can describe the existence of two regime that dominate the evolution of the GDP, one when the economy is in an expansive phase and another when the economy is in a phase of recession.

analyze the European economy has appeared².

In general these researchers use the same nonlinear models created ten years ago: Smooth Transition Models and Markov Switching Models. But they prefer to use monthly (or quarterly) coincident economic indicators of business cycles than the GDP or other real economic variables³. The introduction of nonlinear analysis suppose another important field of study: the forecasting accuracy. Clements and Smith (1997) declare that even if the nonlinear models can provide better estimations than the linear models, not ever can predict better. In this sense there exists a big debate between those who state the nonlinear models can predict better and those who state that they can not predict better.

The proposition of this paper is the analysis of the possible nonlinearity of the Spanish economy using a Threshold Autoregressive Model with two regimes following the model proposed by Hansen (1997), Hansen (2000). This nonlinear model is a strong instrument of analysis wich mixes both the simplicity and the capacity to describe very well a nonlinear series. How say Hansen (1997) “Threshold Autoregressive (TAR) models are relatively simple to specify, estimate, and interpret, at least in comparison with many other nonlinear time series models”.

²Among he recent articles published in this area we can cite Andreano and Savio (2002), Delli Gatti et al. (1998), Öcal (2000).

³There has been a large discussion on the usefulness of the GDP and the Industrial Index Production for the performance of the analysis of the business cycle or if it is better to construct an indicator ad hoc to better study the cycle in the economy.

The times series variable used to represent the Spanish economy is the Industrial Index Production, which is a good variable proxy to describe the whole economy. Our main goal is to establish if a simple nonlinear model can fit and forecast in a better way than a linear model. Our attention is focused on the univariate analysis of the IPI in order to observe if the times series dynamic can be approximated to a nonlinear stochastic process. All types of considerations related to business cycle which suppose the existence of the nonlinear dynamics are avoided in this paper. In the next the Threshold Autoregressive model is presented in brief. The third section deals with the preliminary analysis of the data to argue the opportunity to use a nonlinear model. In the fourth section are estimated a TAR model and a ARIMA model, comparing the two models. Finally a short forecasting exercise is performed in order to check the forecasting accuracy of the TAR model compared to the linear ARIMA.

2 Specification, estimation and diagnostic of the TAR model

In general the Threshold autoregressive model (TAR) can be considered as an AR model where the autoregressive parameters depend on the regime or state. A TAR model with k ($k \geq 2$) regimes is defined as:

$$y_t = \begin{cases} (\alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p}) + e_t, & \text{if } I(q_t \leq \gamma) \\ (\beta_0 + \beta_1 y_{t-1} + \dots + \beta_p y_{t-p}) + e_t, & \text{if } I(q_t > \gamma) \end{cases} \quad (1)$$

where q_t is the threshold variable that defines which regime is operating at the time t and the change from a regime to the other is determined by the

indicator function $I(\cdot)$,

$$I(\text{condition}) = \begin{cases} 1 & \text{if } \text{condition} \text{ is satisfied} \\ 0 & \text{otherwise} \end{cases}$$

Generally the threshold variable is a known function of the data $q_t = q(y_t, y_{t-1}, \dots, \dots y_{t-p})$, but a special interest arises when the threshold variable q_t is taken to be a lagged value of the time series itself, that is, $q_t = y_{t-d}$ for a certain integer $d \geq 0$. In this case the resulting model is called a Self-Exciting TAR (SETAR) model, (Franses and Van Dijk, 2000). The autoregressive order of the model is defined by $p \geq 1$ and γ is the threshold parameter. The parameters α_j are the autoregressive coefficients when ($q_t \leq \gamma$), and β_j are the coefficients when ($q_t > \gamma$). The error e_t is assumed iid $(0, \sigma^2)$ ⁴.

Equation 1 can be rewritten as follows:

$$y_t = x_t' \alpha I(q_t \leq \gamma) + x_t' \beta I(q_t > \gamma) + e_t \quad (2)$$

where: $x_t = (1 \ y_{t-1} \ \dots \ y_{t-p})'$ is the vector of the data, and the vectors of coefficients are $\alpha = (\alpha_0 \ \alpha_1 \ \dots \ \alpha_p)'$, $\beta = (\beta_0 \ \beta_1 \ \dots \ \beta_p)'$, or in a more compact form:

$$y_t = x_t(\gamma)' \theta + e_t \quad (3)$$

where the matrix of the data is

$$x_t(\gamma) = [x_t' I(q_t \leq \gamma) \quad x_t' I(q_t > \gamma)]$$

and $\theta = (\alpha', \beta')'$ is the matrix of the coefficients.

⁴This is a simplification, because it's really probably that the error could be conditionally heteroscedastic. In that case the theory follow being consistent assuming that the error e_t is a Martingala difference sequence with respect to the past history of y_t .

Once the TAR model is reduced to equation 3, the parameters to be estimated can be clearly seen: the vector of parameters θ , and the threshold parameter γ . However model 3 is nonlinear and discontinuous. It is possible to estimate this model via the ordinary least squared using sequential conditional least squared. For a given value of γ , the LS estimate of θ is:

$$\hat{\theta}(\gamma) = \left(\sum_{t=1}^n x_t(\gamma)x_t(\gamma)' \right)^{-1} \left(\sum_{t=1}^n x_t(\gamma)y_t \right) \quad (4)$$

the residuals of this model are $\hat{e}_t = y_t - x_t(\gamma)'\hat{\theta}(\gamma)$, and the residual variance:

$$\hat{\sigma}_n^2(\gamma) = \frac{1}{n} \sum_{t=1}^n \hat{e}_t(\gamma)^2 \quad (5)$$

Finally the resulting estimation of γ it will be that estimated $\hat{\gamma}$ minimizing the value of the equation 5. The model can be made more complex with the estimation of the autoregressive order p and the search of the threshold variable q_t . Simply it has to repeat the estimation process and changing the value of p and/or the variable q_t , used as threshold, up to minimizing the value of the equation 5.

It is important to remark that the modeling of the TAR by partitioning, allows to preserve the stationarity of the series. This contrasts the change-point models (Markov Switching models) where the regime switch is made on the basis of time, resulting in a non-stationary process. The problem of these models is that the traditional tools used to analyze the series are useless when the series is nonlinear: the autocorrelation function can not be used.

Therefore, as (Tong, 1990) suggests, other instruments based on data-

exploratory and data-analytic techniques such as various plots, background information, and nonparametric and semi-parametric techniques have to be employed. The very powerful feature of the TAR models lies in the easy and easy-to-use approximation of a more sophisticated nonlinear function of the piecewise function.

The p , which minimizes the AIC criteria, is chosen to establish the order of the autoregressive model. Obviously the AIC used in this paper is an AIC especially modified for the TAR models. Another technical problem of the TAR model is the asymptotic property of the estimators. But Hansen (1997, 2000) and others demonstrate how the distributions of these estimators can be known asymptotically via simulations. The test of nonlinearity is the last aspect to be considered in the estimation of a TAR model. It can be said that a univocal tool for measuring the nonlinear dependence does not exist which means that the nonlinear test is the starting instrument to check the nonlinearity in model estimation. There are many types of this test, and these can be divided into two categories: the ones which are based on the variance of a linear model (Portmanteau Test) without specifying alternative models and, the ones which are based on the specifications of the alternative models. In this paper the contrast procedure presented by Hansen (1997) is employed.

3 Preliminary Data Analysis

The establishment of the most suitable statistical indicator for the study of the business cycle of the economy has been a great controversy for many years. The first studies on the business cycle focused on the analysis of the

USA economy by using the GDP, as Hamilton (1989) and Potter (1995) did. More recently Öcal (2000), and Delli Gatti et al. (1998) have employed the GDP as a variable indicator to study the business cycle in some European countries. The use of the GDP has always presented several problems of practical nature: the delay in the data publication and the aggregation of its different components, presenting diversity in their cyclical behaviour.

The Industrial Production Index represents a good alternative to GDP: the data are quickly available, it is very sensitive to the cyclical fluctuation and it has a deep correlation to the GDP. At the same time the industrial sector plays an important role in the evolution of whole economy, especially the manufacturing. On the other side, rather than using a single variable, other authors such as Andreano and Savio (2002), prefer to employ a composite index to solve the problems of representativeness. Doing this, new problems like that of the solution of the aggregation method or the identification of the weights arise. The variable chosen for the study of the nonlinearities in the Spanish economy is the monthly Industrial Production Index (IPI).

Among the existing indexes, the selected IPI⁵ includes the whole industry and excludes construction so as not to mix the cyclical fluctuation of this sector with others. The sample starts on the first month of 1965 and finishes in the tenth month of 2007. This decision is based on the simplicity and practice on using the Industrial Production Index. Doing this the problem of calibrating a coincident indicator is then avoided. Following this solution,

⁵The data are drawn from the OECD data-bank, specifically from the Main Economic Indicator. The year base is 2000 and the original font is the Contabilidad Nacional Trimestral of Spain.

it is not necessary to know the different components of the GDP for the evaluation of its the several cyclical fluctuations. The decision related to the use of seasonal adjusted or unadjusted data is the second important decision in this preliminary analysis.

During many years in the literature has been discussed if, at the least approximately, the seasonal adjustment procedure is a linear data transformation. In the past, the answer to this important question was positive and researchers employed extensively seasonal adjusted data. They did not entertain the hypothesis that the seasonal adjustment could partly produce nonlinearities in the series. More recently, Ghysels et al. (1996) showed that this approach needs more caution. In their analysis they found that the standard seasonal adjustment procedure, X12-ARIMA, is far from being a linear data-filtering process. Therefore, if the use of linear filter to seasonalize the data can introduce involuntarily nonlinearities in the series, it may be more adequate to use unadjusted data, especially in the analysis of the business cycle. In this work, the use of non-seasonally adjusted data is proposed. This enables a more adequate evaluation of the superiority of a nonlinear model in comparison to a linear model.

The series has serious problems of autocorrelation and seasonality, thus a transformation is needed in order to be able to work with it. With the aim of avoiding these problems, the most frequently used transformation is the first difference of the variable. However this solution difficults the appreciation of the business cycle in the new resulting variable.

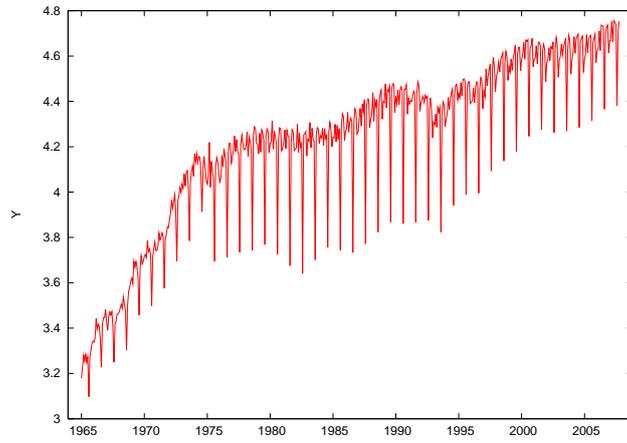


Figure 1: Spanish Industrial Production Index

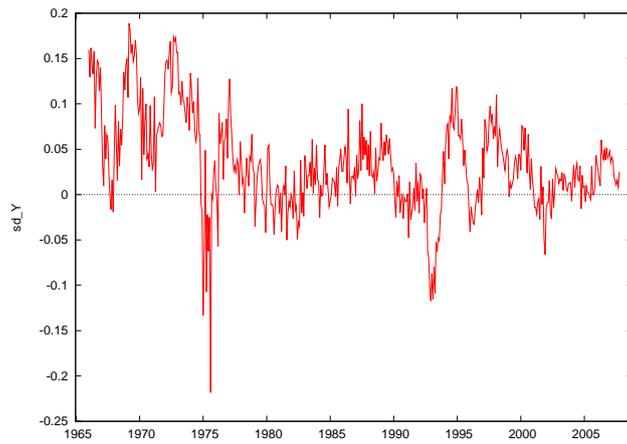


Figure 2: Seasonal difference of Spanish Industrial Production Index

In this paper, instead of the first difference, the seasonally difference is employed solving the problems of nonstationarity and seasonality. This facilitates the appreciation of the evolution of the cycle in the new resulting time series. Following the steps of other authors, (as Hansen (1997) and Potter (1995)), the series is multiplied by one hundred before transforming it. The new variable obtained is a new stationary and seasonally adjusted variable which represents an annual variation (annual rate of growth), see the Figures 1, 2.

The Spanish IPI does not present strong problems of asymmetry, the skewness and the kurtosis coefficients are not so far from the normal parameters of time series generated by a linear process (Table 1). However, the evolution presented by this index is considerably different during the first 10 years with regard to the rest of the sampling, as it can be observed in Figures 1 and 2. In the level and in the seasonal difference it is possible to appreciate the existence of two tendencies. During the first ten years the rate of growth is really high, whereas in the rest of the sample, the rate of growth is lower.

If the sample is divided into two subsamples, the asymmetries in the series appear more clearly. In Table 1 the statistics of the series in level and in the seasonal difference for two subsamples are reported, one for the period 1966-1975, the other for the period 1976-2007. The Spanish IPI means and

Table 1: Property of the Spanish IPI

	Min	Max	Mean	Med.	Ran.	Var	S.d	Kurt	Skew.
IPI	3.097	4.736	4.202	4.275	1.639	0.141	0.375	3.017	-0.897
$\Delta^{12}IPI$	-0.218	0.188	0.034	0.026	0.407	0.002	0.053	4.277	0.174

The range used is all the sample from 1966M1 to 2007M10 with the variable expressed in logarithms, Med. = Median, Ran. = Range, S.d. = Standard Deviation, Kurt.= Kurtosis, Skew.= Skewness.

variances are different in the two subsamples. In the case of the variable in difference, the second subsample presents a more regular evolution. The mean, the variance, the skewness and the kurtosis are smaller than in the first subsample. Consequently, it could be said that some nonlinearities in the series exist.

As Tong (1990) shows, these features can be well explained in terms of the so-called *regime effect*. Those nonstandard features cover: non normality, asymmetric cycles, bimodal distributions, nonlinear relationship between lagged variables. Another instrument to analyze a nonlinear series is the scatter plot of the series with all its lagged values. In Figure 3 all pairs of autocorrelation are represented. It can be observed that the correlation between the variable and its lags presents some nonlinear typical aspect. This is considered a method for checking the existence of autocorrelation, because in case of nonlinear dependence, the usual autocorrelation function is not useful. All these elements suggest the existence of nonlinear dynamic in the series. If this is true, the linear AR model is not able to detect it and to fit it.

Table 2: Property of the IPI in the two sub-samples

	Min	Max	Mean	Med.	Ran.	Var	S.d	Kurt	Skew.
IPI_1	3.097	4.219	3.719	3.720	1.122	0.087	0.295	1.778	-0.052
IPI_2	3.640	4.7559	4.382	4.391	1.116	0.0484	0.220	3.975	-0.751
$\Delta^{12}IPI_1$	-0.117	0.189	0.080	0.089	0.407	0.005	0.0682	5.546	-1.183
$\Delta^{12}IPI_2$	-0.218	0.128	0.019	0.0179	0.245	0.001	0.037	4.282	-0.264

The subsample used to compute the statistics of IPI_1 and ΔIPI_1 is (1966-1976), as the sub-sample used to compute the statistics of IPI_2 and $\Delta^{12}IPI_2$ is (1976-2007).

4 Threshold Autoregressive model estimated

4.1 Estimation and diagnostic

The first step in the process of building a TAR model is to establish the order of the autoregressive model p . Following the methodology proposed by Hansen, first a linear autoregressive model with 12 lags is estimated. Having monthly data, it is possible the existence of a dependence between the variable at $t = 0$ and its lags (from 1 to 12). The dependent variable to be used is the seasonal difference (of the logs) $\Delta_{12} \log(IPI_t)$ for the sample 1965-2006 (dropping down the last 10 observations that will be used for evaluating the prediction). From now on the dependent variable used is named $y_t = \Delta_{12} \log(IPI_t)$. After that, a new AR is estimated, dropping down from the initial AR(12) the lags whose t statistics are smaller than 1. The results of the estimation are reported in Table 4.

As threshold variable, the same dependent variable is employed. Following the procedure of Hansen (1997), two types of threshold variables are considered: the first $q_t = \Delta_{12} \log(IPI_{t-d})$ as a lagged value of the seasonal difference, the second $q_t = \log(IPI_t) - \log(IPI_{t-d})$ as a difference of order d . For $d = 1, \dots, 12$ a TAR model with the same autoregressive parameters specified for the AR model (1,2,3,6,7,12) is estimated. The sum squared error (SSE) for all the models estimated is reported in the Table 5. The value of d minimizing the SSE is $d = 0$ with $q_t = \Delta_{12} \log(IPI_t)$ as threshold variable. This means that the change between the two regimes is simply instantaneous and does not depend on a delay parameter. Now that the parameter of delay ($d = 0$) is known, a TAR model dropping down the lags 4, 5, 8, 9, 10, 11 is

estimated. The TAR model estimated is:

$$y_t = \begin{cases} (-0.0045 + 0.2063y_{t-1} + 0.3130y_{t-2} - 0.0137y_{t-3} \\ + 0.0398y_{t-6} + 0.1178y_{t-7} - 0.1922y_{t-12}) + \hat{\epsilon}_t, & \text{if } I(q_t \leq 0.028) \\ (0.0328 + 0.2343y_{t-1} + 0.1987y_{t-2} + 0.1439y_{t-3} \\ + 0.0741y_{t-6} + 0.1556y_{t-7} - 0.2047y_{t-12}) + \hat{\epsilon}_t, & \text{if } I(q_t > 0.028) \end{cases} \quad (6)$$

The estimations of the two regimes and the threshold variable are reported in Table 6. The threshold estimated is $\hat{\gamma} = 0.028167$. This splits the sample into two parts, with 253 observations for the lower regime, and 227 observations for the upper regime.

The test of linearity $LR_n^*(\gamma)$ proposed by Hansen (1997) refuses the hypothesis of linearity of the model at 1% of signification. This means that for the Spanish IPI, in the case of a univariate analysis, it is more appropriate to use a nonlinear model than a linear one. The other diagnostic tests reported in Table 6 confirm this sentence. In the first part of Table 6, the sum of squared errors, the residual variance and the joint R-squared are calculated for the complete TAR model and in the other two tables these parameters are calculated for each regime. It is possible to state some conclusions comparing the statistics of the whole TAR model reported in Table 6 with that of Table 4.

The use of a TAR model drops down the variance and the sum of squared errors, being the R-squared of the TAR model very high at the same time. So these tests confirm the good fitting of the Spanish IPI with the TAR model. The estimated coefficient, the variances of the parameters, the t-statistics

and the calculated 0.99 confidence intervals, as proposed by Hansen in 1997, are reported in the middle and in the bottom of the Table 6.

4.2 Some results

The first preliminary conclusion arisen from the observation of the estimated TAR model is the reproduction of the AR model in its upper regime. The structure of the coefficients is very similar, the sign of the parameters is exactly the same: only lag 12 is negative. This suggests that by this way the linear AR does not seem to have a mechanism strong enough to change the evolution of the series. The structure of the TAR model includes the existence of another regime that can explain more appropriately the evolution of the IPI. The lower regime presents three negative coefficients. Indeed this means that when $q_t < 0.028$ there is a force that pushes the series to stay in a low growing state. So the simple AR model does not pick up the dynamics produced by the lower regime. It seems that more importance is given to the observation where $q_t > 0.028$, which is more frequent in the first part of the sample.

Following Hansen (1997), Figure 5 represents the series in a scatterplot, where two regimes can be clearly distinguished. During the first ten years the upper regime is dominant, whereas the rest of the sample presents the dominance of the lower regime, only interrupted for short periods in 1985-1989, 1994-1995 and 1997-2000. In this way, it can be concluded that after the high rate of growth during 1965-1975, the Spanish economy had a large period of low growth between 1976-1985. This may be seen as a trivial statement, but the importance here is that the flexibility of the TAR model is

able to represent and fit this evolution of the time series much adequately than a linear AR model.

The last step in this analysis is the forecast of the estimated models. For this purpose the last ten observations of 2007 are employed and the prediction of the series in levels as well as in differences is done. From the results obtained, shown in Table 3, it is evident that the TAR model has a better capacity to predict the IPI. In this line, and in contract with other literature works ((Clements and Smith, 1997)), the nonlinear model predicts better than the linear AR. In levels and in differences the TAR model has a lower RMSE, MAE, MAPE⁶ than the AR model.

Table 3: Comparison of predictions

	TAR		AR	
	y	$\Delta_s y$	y	$\Delta_s y$
RMSE	0.0199	0.0183	0.0230	0.0207
MAE	0.0176	0.0162	0.0201	0.0183
MAPE	1,6296	68,218	1,8501	126,14

Δ_s : seasonal difference

$${}^6\text{RMSE} = \sqrt{\left(\frac{1}{h+1}\right) \sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2}, \text{ MAE} = \left(\frac{1}{h+1}\right) \sum_{t=T+1}^{T+h} |\hat{y}_t - y_t|, \text{ MAPE} = \left(\frac{1}{h+1}\right) \sum_{t=T+1}^{T+h} \left| \frac{\hat{y}_t - y_t}{y_t} \right|$$

5 Conclusions

The linear time series models have dominated the macroeconomic analysis for many years, being the only approach available. With time it has been observed that the empirical evidence has demonstrated the limitations of the linear analysis to explain the time series with nonlinear features. In this work, the nonlinear futures of the Spanish Industrial Production Index are analyzed in order to point out how the linear methods can hide an interesting economic structure. This paper uses the Threshold Autoregressive model proposed by Hansen (1997) in order to estimate and test the threshold parameters and to construct asymptotic confidence intervals for the parameters.

This model is employed to improve the confidence intervals for the parameters as well as solving the problem of nuisance parameters in the nonlinear test of the model. By doing this it is possible to make inference with the estimated model. The analysis is performed at a univariate level so as to compare the improvement that a nonlinear model produces with regard to a linear model, the latter being a simple ARIMA model. These results outline the presence of a complex dynamic structure that the linear model can not capture, and the nonlinearity test refuses the hypothesis of linearity. Overall the estimated Threshold Autoregressive model represents better the evolution of the Spanish IPI and its asymmetry, being capable of outlining two states of the Spanish economy.

The analysis on the Spanish IPI shows the existence of two different dynamics: one during a high growth phase and the other during a low growth

phase. Additionally, it is concluded that the TAR model gives better forecasting accuracy than that given by the ARIMA model. This result stresses the importance of using nonlinear models for the Spanish economy, moreover the need of adopting a new approach which is based on this kind of models to study its business cycle.

References

- M. Andreano and G. Savio. Further evidence on business cycle asymmetries in g7 countries. *Applied Economics*, 2002.
- M. P. Clements and J. Smith. The performance of alternative forecasting. *International Journal of Forecasting*, 1997.
- D. Delli Gatti, M. Gallegati, and D. Mignacca. Nonlinear dynamics and european gnp data. *Studies in Nonlinear Dynamics & Econometrics*, 1998.
- P. H. Franses and D. Van Dijk. *Nonlinear time series models in empirical finance*. Cambridge University Press, Cambridge, 2000.
- E. Ghysels, C.W.J. Granger, and P. Siklos. Is seasonal adjustment a linear or nonlinear data filtering process? *Journal of Business and Economics Statistics*, 1996.
- J. D. Hamilton. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 1989.
- B. Hansen. Sample splitting and threshold estimation. *Econometrica*, 2000.
- B. Hansen. Inference in tar models. *Studies in Nonlinear Dynamics and Econometrics*, 1997.
- J. M. Keynes. *The General Theory of Employment, Interest and Money*. Macmillan, London, 1936.
- W. C. Mitchell. *Business Cycles: The Problem and Its Setting*. National Bureau of Economic Research, New York, 1927.

- N. Öcal. Nonlinear models for u.k. macroeconomic time series. *Studies in Nonlinear Dynamics & Econometrics*, 2000.
- S. M. Potter. A nonlinear approach to u.s. gnp. *Journal of Applied Econometrics*, 1995.
- T. Teräsvirta and H. Anderson. Modelling nonlinearities in business cycle using smooth transition autoregressive models. *Journal of applied econometrics*, 1992.
- G. Tiao and R. Tsay. Some advances in nonlinear and adaptive modeling in time series analysis. *University of Chicago Graduate School of Business Statistics Research Center Report*, 1991.
- H. Tong. *Nonlinear time series, a dynamical system approach*. Oxford University Press, London, 1990.

6 Appendix

Table 4: Linear AR model

Variable	Estimate	St Error	t-statistic
Constant	0.006338	0.001822	3.478542
y_{t-1}	0.335545	0.069155	4.852093
y_{t-2}	0.323148	0.067345	4.798407
y_{t-3}	0.103607	0.064889	1.596687
y_{t-6}	0.121589	0.048885	2.487218
y_{t-7}	0.152578	0.063900	2.387684
y_{t-12}	-0.235781	0.055302	-4.263494

Obs. = 480, *DF.* = 473, *SSE* = 0.405500,
 $\sigma_\epsilon^2 = 0.000857$, $R^2 = 0.687907$

Table 5: Selection of the parameter of delay d

$q_t = \Delta_{12} \log(IPI_{t-d})$													
$d =$	0	1	2	3	4	5	6	7	8	9	10	11	12
SSE	0.27	0.38	0.36	0.37	0.37	0.36	0.37	0.36	0.37	0.38	0.37	0.39	0.38
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

$q_t = \log(IPI_t) - \log(IPI_{t-d})$												
$d =$		2	3	4	5	6	7	8	9	10	11	12
SSE		0.385	0.375	0.376	0.385	0.382	0.386	0.377	0.386	0.387	0.388	0.380
p-value		0.000	0.000	0.000	0.008	0.000	0.006	0.000	0.007	0.005	0.005	0.000

Table 6: TAR model estimated

$\gamma = y_t$	$\hat{\gamma} = 0.028167$.99C.I. = [0.027433, 0.040157]					
$SSE = 0.272670, \sigma_\epsilon^2 = 0.000585, R^2 = 0.790140, LR_n^*(\gamma)(p - value) = 0.000719$							
$q_t \leq 0.028167$							
Variable	intercept	y_{t-1}	y_{t-2}	y_{t-3}	y_{t-6}	y_{t-7}	y_{t-12}
α	-0.0045	0.2063	0.3130	-0.0137	0.0398	0.1178	-0.1922
s.e	0.0017	0.0893	0.0888	0.0850	0.0598	0.0867	0.0778
t.	-2.6465	2.3086	3.5242	-0.1610	0.6657	1.3585	-2.4699
low 95%	-0.0090	-0.0242	0.0838	-0.2331	-0.1144	-0.1058	-0.3929
up 95%	-0.0001	0.4368	0.5422	0.2057	0.1940	0.3414	0.0086
$Obs. = 253, DF = 246, SSE = 0.143660, \sigma^2 = 0.000584, R^2 = 0.67651$							
$q_t > 0.028167$							
Variable	intercept	y_{t-1}	y_{t-2}	y_{t-3}	y_{t-6}	y_{t-7}	y_{t-12}
α	0.0328	0.2343	0.1987	0.14399	0.0741	0.1556	-0.2047
s.e	0.0025	0.0611	0.0648	0.0549	0.0486	0.0613	0.0433
t.	13.3530	3.8360	3.0670	2.6254	1.5232	2.5394	-4.7305
low 95%	0.0265	0.0767	0.0316	0.0025	-0.0514	-0.0025	-0.31632
up 95%	0.0391	0.3919	0.3659	0.2855	0.1995	0.3137	-0.0931
$Obs. = 227, DF = 220, SSE = 0.129010, \sigma^2 = 0.000586, R^2 = 0.602057$							

Figure 3: Spanish IPI vs its lags

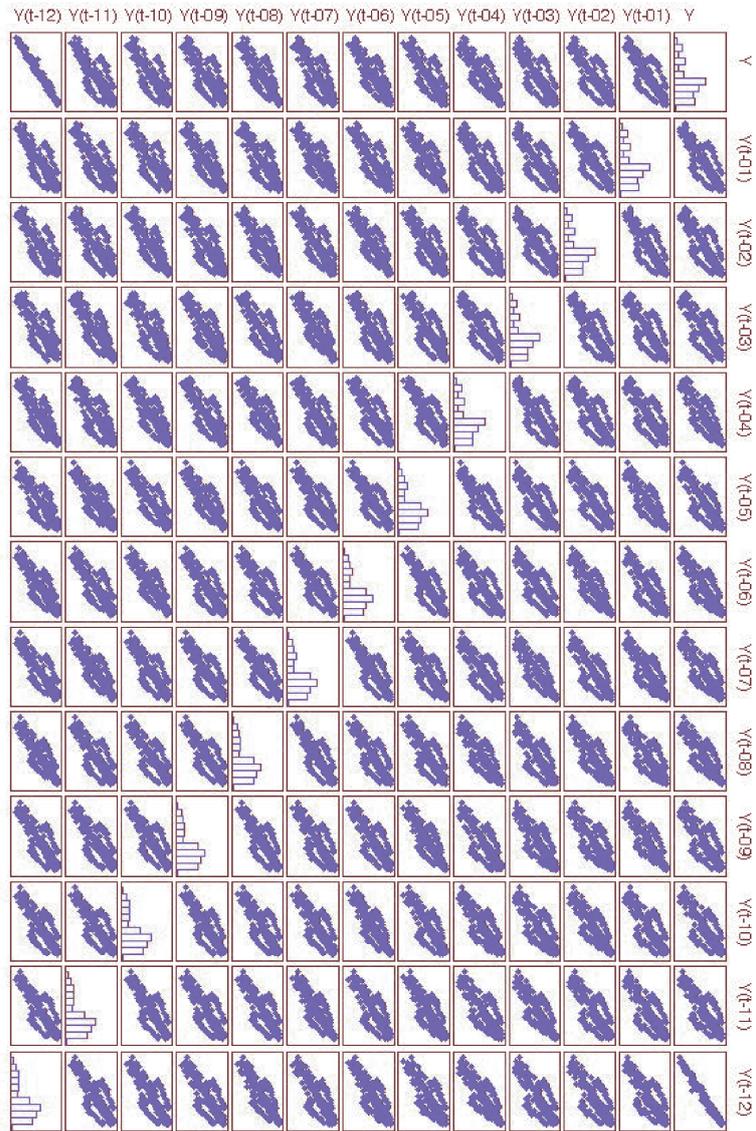


Figure 4: Confidence interval Construction for the Threshold

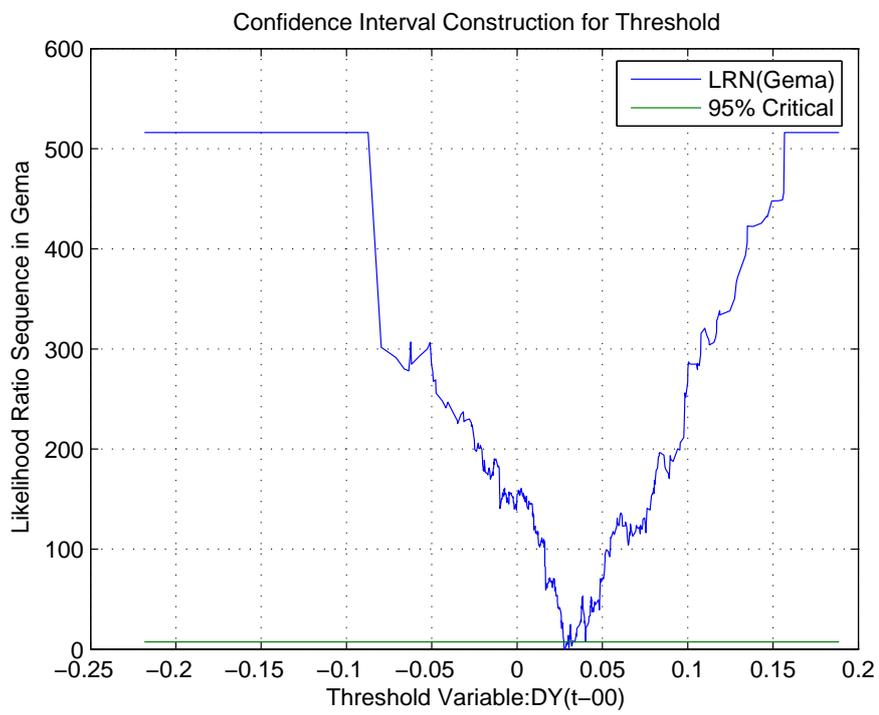


Figure 5: Classification by Regimes in the levels

